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UND IN DER GEGENWART

SYMPOSIUM DER LEIBNIZ-GESELLSCHAFT HANNOVER,
10. UND 11. NOVEMBER 1978

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ABSTRACTION, LOOKING-AROUND AND SEMANTICS

By
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This paper has two main parts. The first, sections (1) through (6), includes a general discussion of "definitions by abstraction". I claim (1) that under this label two quite different procedures are involved: the looking-around (2) and the abstraction method (3). The looking-around method is either unjustifiable or reduces to the abstraction method (4). The idea of the "definitions by abstraction" is correctly represented only by the abstraction method. The latter can be completed by some "looking-around" in a revised sense (5). The abstraction method is to be compared with abstraction theories in the history of philosophy and psychology at large (6).

In the second part of the paper I examine (7) Carnap's application of the looking-around method to semantics. Carnap's project is questionable not only qua instance of the looking-around method but it fails, more specifically, qua project for semantics as well. My claim is that Carnap's program can be rescued if reconstructed in terms of the abstraction method. (A parallel study, not attempted here, is conceivable in connection with the numerous applications of the looking-around method in set theory)*.

1. Definitions by abstraction

Let us consider the following text from Leibniz: «Au reste, j'ay fait icy à peu prés comme Euclide, qui ne pouvant pas bien faire entendre absolument ce que c'est raison prise dans le sens de géométries, définir bien ce que c'est mêmes raisons. Et c'est ainsi que, pour expliquer ce que c'est que la place, j'ay voulu définir ce que c'est que la même place» (GP VII, p. 401–2).

Similarly, Frege, after failing to grasp numbers ("Wie soll uns denn eine Zahl gegeben sein, wenn wir keine Vorstellung oder Anschauung von ihr haben können?"") decides that "um den Begriff der Anzahl zu gewinnen, muß man den Sinn einer Zahlenlelung feststellen" (Grundlagen, § 62).

This was generalized by modern logicians later on into the so-called "definitions by abstraction". What are these definitions?

* I am grateful to the Committee on Attendance at Meetings of Learned Societies, University of Texas, Austin, as well as to the Leibniz-Gesellschaft, Hannover, for helping me to attend the Symposium on "Die internationale Logik bei Leibniz und in der Gegenwart", Hannover, November 1978, where parts of this paper were read. Also, I am grateful to the several participants who contributed valuable comments on this paper, in particular M. Dascal, R. Kauppi, H. Scheipers, Ch. Thiel and F. von Kutschera. – GP = GERHARDT (ed.), LEIBNIZ, Philosophische Schriften, vol. I–VII, Berlin, 1875–1890.

There is only one thing that is perfectly clear and undisputed about the "definitions by abstraction". This is unfortunately only the fact that everybody considers an equivalence relation \( x \sim y \) over a certain domain of objects \( a, b, c \ldots \). For example, Frege considers lines with a relation of parallelism and concepts with a relation of one-one correspondence among the individuals falling under the concepts (Grundlagen, §§ 62–64), Peano considers fractions with a relation of having equal cross-products (cf. bibliography), and Carnap considers linguistic expressions with relations of "equivalence" or "L-equivalence" (Meaning and Necessity).

Apart from this, however, things are less unanimous. Logicians proceed according to two quite different methods, which I will call the looking-around and the abstraction method.

2. The looking around method

In our notation of an equivalence relation \( \sim \) defined on a domain of objects \( a, b, c \ldots \) the looking-around method may be described as the introduction of new singular terms \( \bar{a}, \bar{b}, \bar{c} \ldots \) in two stages: (i) we postulate \( \bar{a} = \bar{b} \iff a \sim b \), (ii) we "look around" for entities to be assigned to the symbols \( \bar{a}, \bar{b}, \bar{c} \ldots \) as their denotata. This second stage usually consists in looking for a function \( f(x) \) defined on the domain \( a, b, c \ldots \) in such a way that the symbol \( \bar{a} \) is assigned as denotatum the object \( f(a) \), the symbol \( \bar{b} \) the object \( f(b) \), etc. For example, \( f(x) \) may be the function assigning to each object \( x \) the corresponding "equivalence class", i.e. the set of objects that stand to \( x \) in the relation \( \sim \). But any other entity or function \( f_i \) will do as well, at least in principle, provided that the identity condition of stage (i) is respected, i.e. \( f_i(a) = f_i(b) \iff a \sim b \).

The qualification "at least in principle" is needed because of the possibility of additional restrictions that may occasionally rule out a certain \( f_i \), even though \( f_i \) satisfies the identity condition (i). For example, having stipulated that two sets have the same cardinal iff they are "similar" (\( x \sim y \) is here \( x \) is similar to \( y \) understood as one-one correlation of two sets \( x, y; \bar{a} \) is worded as "the cardinal of \( a \)"), one may consider the two following candidates: 1) the equivalence class of \( x \), 2) the first ordinal in the equivalence class of \( x \). Both perhaps satisfy the basic condition of stage (i), but in addition we may want to require that "the cardinal of \( a \)" be a set, a property which we may believe applies to the second candidate but not to the first one (Shoenfield p. 252).

We may visualize the "ontology" of the looking-around method by means of the following figure, which shows first the initial domain of objects and then an open list of functions (defined on the initial domain) whose values constitute the entities that may be assigned as denotata to the symbols \( \bar{a}, \bar{b}, \bar{c} \ldots \).
of sentences he proposed, but only as a conjecture, truth-values too ("Uber Sinn und Bedeutung". p. 34 and 35). The freedom of choice of denotata for ‘a’ has been particularly emphasized by Carnap (apart from Meaning and Necessity cf. his Introduction. 34 a) as well as by Quine (sections 12, 17, 22; "since we may construe the ordinals in any way that assures the required order, the best counsel is to construe them conveniently"), p. 152; p. 212–3).

3. The abstraction method

Given our equivalence relation \( x \sim y \) on the domain \( a, b, c \ldots \) the abstraction method consists in abstracting from any predicate \( A(x) \) that is not invariant with respect to the relation \( \sim \).

A predicate \( A(x) \) is invariant with respect to the relation \( x \sim y \) iff \( A(x) \sim A(y) \). This stipulation includes as invariant every \( A(x) \) that does not hold of any of the initial objects \( a, b, c \ldots \) For example, if the initial objects are fractions with the equivalence relation of equal cross-products, the predicate \( A(x) = \) ‘is a chair’ will be invariant. We may disregard these predicates as vacuously invariant. The plan of abstraction is to ignore non-invariant \( A(x) \) and to consider only the (non-vacuously) invariant \( A(x) \).

Having decided to do abstraction, we find it convenient to introduce some notation that signifies our intent of doing abstraction. For example, we may write \( \tilde{a}, \tilde{b}, \tilde{c} \ldots \)

Next we make the major discovery of the whole process: as a result of abstraction, any two objects \( a, b \) with \( a \neq b \) but \( a \sim b \) become indiscernible.

We take then the crucial step of reading our symbols ‘\( \tilde{a}, \tilde{b}, \tilde{c}, \ldots \)’ no longer as reminders (‘restrict yourself to invariant predicates!’) but now as singular terms standing for new entities, the abstract entities or abstracta: \( a, b, c \ldots \) In addition to \( a, b, c \ldots \) we have \( \tilde{a}, \tilde{b}, \tilde{c} \ldots \) with \( \tilde{a} = \tilde{b} \) iff \( a \sim b \), etc. We justify the identity statements \( \tilde{a} = \tilde{b} \), etc. on the basis of the indiscernibility of \( \tilde{a} \) and \( b \) (if \( a \sim b \)).

For example, the abstractum \( 2/3 \) generated from the fraction \( 2/3 \) by restriction to predicates that are invariant with respect to \( \sim \) construed as the relation of having equal cross-products is the new entity called the rational \( 2/3 \).

\footnote{Frege’s meticulous analysis of ‘a very odd kind of definition’ in Grundlagen § 63, published in 1884, is such that he may be regarded as a pioneer of the abstraction method. In this sense I agree with Tietel’s remarks in the discussion as well as with his paper (cf. bibliography). On the other hand, if one distinguishes abstraction method from looking-around method (as done, I believe for the first time, in the present paper), Frege in my view is to be classified in the latter rather than in the former. Surely Frege recognized that equivalence relations yield et was Gemeinsames (Grundgesetze II, § 146) but in the crucial applications: number, Werteverläufe, Bedeutung of sentences, he did not concentrate on this ‘something in common’ (practically the abstraction); he looked around for other things, such as equivalence classes and truth-values.

\footnote{But cf. for example Pasch’s reluctance to use equivalence classes, in FREGE, Briefwechsel, p. 173, as well as Russell’s cautious remark on ‘the contraction’, after choosing the equivalence class as ‘interpretation’ of the cardinal number, Principles § 284.}
What exactly is the new entity $\tilde{a}$, the abstractum? All we can say is that it is what is "left" of the object under the process of abstraction, but this, of course, does not add anything really new to what we already know. Perhaps more suggestively we may say that the abstractum $\tilde{a}$ is a qua restricted to invariant predicates. It seems hopeless to try to specify more precisely the "nature" of abstracta. On the other hand, we should not expect much intuitive clarity for objects that are per definitionem abstract, i.e. non-intuitive.

It may be useful to distinguish doing abstraction and reflecting upon abstraction. While doing abstraction our initial universe $a, b, c, \ldots$ is replaced by the new one: $\tilde{a}, \tilde{b}, \tilde{c}, \ldots$. When reflecting upon abstraction or when looking at abstraction "from the outside" what we see is that our initial universe $a, b, c, \ldots$ has been increased by the new entities $\tilde{a}, \tilde{b}, \tilde{c}, \ldots$.

In the reflective attitude we recognize the abstracta as new entities and we make statements about them ("$\tilde{a} = \tilde{b}^\sim$, $\tilde{a}$ is an abstractum") that in general do not hold of the initial objects (it may well be that $a$ is not an abstract entity).\(^4\)

Contrary to this realization of the novelty of abstracta, Lorenzen has emphasized the possibility of viewing abstracta as mere façon de parler, or of taking the corresponding singular terms as eliminable, in the sense that anything we say about $a$ may be reworded as being about $\tilde{a}$. From this point of view there is of course no new universe, no real change in our initial universe $a, b, c, \ldots$. The façon de parler approach requires of course disregarding the new statements of the type "$a$ is an abstractum" (in case $a$ is not an abstractum). This seems to involve, curiously, an additional abstraction; somehow we have to abstract from the fact that we have generated new, abstract entities.

Among the predicates that are invariant but not vacuously so (i.e. that hold of at least one of the initial objects too), some appear to be paradoxical. For example, the predicate "$x$ is a fraction" is non-vacuously invariant with respect to the relation of equal cross-products on the domain of fractions and consequently should be true of the abstracta (rational) $m/n$. However, as pointed out by Allan Back, if $m/n$ is a fraction, it must have a particular numerator (for example, $m$) and a particular denominator (for example $n$), which is incompa-

tible with the intended abstraction; in fact neither the predicate "$x$ has denominator $n$" nor the predicate "$x$ has numerator $m$" are invariant.

This "Böck's paradox" as well as some of the previous remarks may raise some doubts concerning the abstraction method. But it seems to me that the questions raised only call for caution and further analysis and do not justify a flat rejection of the method. (The looking-around method, on the contrary, appears to me untenable, cf. next section.)

One important reason for not surrendering too early in connection with the puzzles of the abstraction method is that the latter may be the only way of seriously preserving the notion of abstraction. How else, if not according to the analysis of the abstraction method, can we understand abstraction and abstract entities? (This question of course refers to genuine abstraction, not to pseudo-uses of the term, cf. below, 5).

It may be helpful to organize better our intuitions by means of the following simple theorems. T1. It is immediate that under the restriction to invariant predicates (at least the non-vacuously invariant): $A (\tilde{a}) \leftrightarrow A (a)$. T2. $\tilde{a} \sim \tilde{b} \sim a \sim b$. From left to right: the predicate $a \sim x$ is invariant with respect to $\sim$, because from the transitivity of $\sim$ we may assert $A x \forall y : a \sim y \sim x \sim y \leftrightarrow a \sim y :$, which is the invariance condition for the predicate $a \sim x$. By T1 and invariance of $a \sim x$ we obtain $a \sim \tilde{a} \sim a \sim a$, whose second member is true (reflexivity of $\sim$), hence $a \sim \tilde{a}$. In the same way we obtain $\tilde{b} \sim b$. Hence using the hypothesis $\tilde{a} \sim b$ and the transitivity of $\sim$ we obtain $a \sim b$. From right to left: we use again $a \sim a$ and $b \sim b$, which with the hypothesis $a \sim b$ and the transitivity of $\sim$ yield now $a \sim b$. If we are able to define $\tilde{a} = \tilde{b} = \tilde{a} \sim \tilde{b}$ then we have T3: $\tilde{a} = \tilde{b} \sim \sim \tilde{a} \sim \tilde{b}$ (but "$\tilde{a} = \tilde{b}$" is no longer eliminable into "$a = b$.")

The ontology of the abstraction method is much simpler than that of the looking around method; it contains just two rows: the initial objects and the abstracta:

<table>
<thead>
<tr>
<th>initial objects</th>
<th>a</th>
<th>b</th>
<th>c...</th>
</tr>
</thead>
<tbody>
<tr>
<td>(with relation $\sim$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>abstracta</td>
<td>$\tilde{a}$</td>
<td>$\tilde{b}$</td>
<td>$\tilde{c}$...</td>
</tr>
</tbody>
</table>

The crucial difference is that the symbols $\tilde{a}, \tilde{b}, \tilde{c}...$ here are not "unconstrained notation" but, from the start, stand for abstracta.

The basic insight into the abstraction method is found in Peano, who emphasizes that by considering in each object, $a, b, c...$ "toutes et seules les propriétés qu'il a communes avec les autres objets", we obtain a new object, "différent de tous ceux qu'on a jusqu'à présent considérés" (Notations, §§ 38,

---

\(^4\) The plausibility of relating abstraction to reduplication was rightly pointed out by M. Dascal in the discussion: I fully agree with him.

\(^5\) Expressions of the type "$x$ is an abstractum" have been classified by Lorenzen as Abstraktoren, not as Prädikatoren. In other words, they have not been recognized as new, genuine predicates. Cf. Logische Propédeutik, 2nd ed. § 7, Abstraktoren.
Abstraction, Looking-Around and Semantics

39). Peano regards this as an abstraction and this is why he introduces the phrase “definitions by abstraction”:

Le definizioni del tipo già considerato furono dette per astrazione nel mio opuscolo “Notations de logique mathématique”, a. 1894, pag. 45-48. La parola astrazione, di uso internazionale, fu introdotta in filosofia da Boezio (morto nel 524) per tradurre il greco ἀφαίρεσις. Applicando il linguaggio di Aristotele (Physica 1, 7) al nostro primo esempio (Peano refers to his favorite example of rational numbers as abstracta of fractions) dirremmo che come uno scultore, da un masso informe di pietra, fa l'astrazione (noi diremmo l'estrazione) del l'immagine o statua di Mercurio, così noi, ponendo in virtù d'une definizione $2/3 = 4/6$ (Peano, erroneously in my view, but agreeing here with Carnap, regards this identity, which corresponds to our $T_3$ above, as a definition), da una scrittura formata come tre segmenti $1/3$ otteniamo per astrazione l'idea del razionale $2/3$, conservando tutte e sole le proprietà, o note, che esso ha comuni coi suoi eguali, e sopprimendo quelle che lo distinguono. Sarà per essi, conservata la proprietà $2/3 < 1$; sono sopprimere le proprietà che il suo numeratore è 2, che è frazione irreduucibile, ecc.” (Le definizioni, appendice).

The same idea of abstraction appears later in Weyl (§2), who calls “invariante Eigenschaften” those properties to which we want to restrict ourselves. But the most systematic study of abstraction and the clearest recognition of its fundamental importance for theoretical thinking in general is found in Lorenzen’s works (cf. bibliography).

4. Criticism of the looking-around method

Two questions may be addressed to the looking-around method: 1) why should there be an unconstrained notation $\tilde{a}, \tilde{b}, \tilde{c}, \ldots$ in the first place? 2) why should the unconstrained symbol $\tilde{}$ be construed as a particular $f_1(a)$ rather than as any other $f_j(a)$?

The answer to the first question is that in practice there are no such totally unconstrained notations but phrases or expressions that are already at least vaguely meaningful and that one wants to explicate more rigorously. For example, Frege aims at clarifying the phrase ‘the number of the concept F’, Russell ‘the cardinal of the set S’, Carnap ‘the intension of the expression E’, etc.

To the second question it might be replied that it is not the purpose of the method as such to make any recommendation of any particular candidate $f_1(a)$ or $f_j(a)$ but only to set a wide framework within which any choice (or “explication” of the vague concept) can be equally accepted. Unfortunately, this only shifts the problem: someone still has to justify the choice of $f_1(a)$ rather than of $f_j(a)$.

How can this justification be provided? The only possibility seems to be that of producing conditions $C(x)$ such that candidates other than $f_1(a)$ are ruled out. For example, one might want “the cardinal of a” to be a set, which might rule out “the equivalence class of a” and lead us to prefer “the smallest ordinal” in that class as a candidate for $\tilde{a}$, i.e. for “the cardinal of a” (see Shoenfeld, p. 252). The trouble is, however, that one has to rule out all candidates except $f_1(a)$. The fact of having ruled out a particular $f_j(a)$ does not preclude the existence of a $f_k(a)$ that satisfies condition $C(x)$ and that is distinct from $f_1(a)$. To rule out all seems hopeless, unless the condition $C(x)$ that one chooses is such that $f_1(a) = 1 \times C_1$, in other words, unless one justifies the choice of $f_1(a)$ by means of the stipulation that the candidate for $\tilde{a}$ must be identical to $f_1(a)$. Clearly, however, such a stipulation amounts to an unjustified choice. This explains why we find either no justification at all or very unsatisfactory appeals to “convenience” or perhaps “beauty” (Quine, p. 152).

There is one and apparently only one way of overcoming the frivolity or lack of justification inherent in the looking-around method. This is to stress that the real spirit of the method does not call for an identification of $\tilde{a}$ with any of the particular candidates $f_j(a)$ “as such”, as an individual entity distinct from all the other legal candidates. Although a particular one is needed (if you want a pencil, you must pick a particular one) none of the candidates is needed in particular (there is nothing necessary about this pencil I have picked). All the legal candidates are good (legal means satisfying $\tilde{a} = \tilde{b}$ iff $a \sim b$).

This consideration meets the frivolity objection, but at the expense of reducing the looking-around method to the abstraction method. That $f_1(a)$ and $f_j(a)$, in spite of being announced as distinct, turn out to be regarded as “equal” can only mean that there is some abstraction somewhere. It is not difficult to figure out what is it.

Consider, for example, the relation $f_1(a) \bowtie f_j(b) \equiv a \sim b$, defined on the class of all candidates $f_1(a), f_j(b), \ldots f_j(a)$. The relation is reflexive: $f_1(a) \bowtie f_1(a)$ because of the reflexivity of the relation $\sim$. The relation $\bowtie$ is also symmetric and transitive.

If, trying to avoid frivolity, the supporter of the looking around method emphasizes that the method is not interested in this or that particular candidate $f_1(a)$ or $f_j(a)$ as such but only in so far as it satisfies the basic condition $\tilde{a} = \tilde{b}$ iff $a \sim b$, he means that he is viewing $f_i(a)$ and $f_j(a)$ in abstraction from predicates that are not invariant with respect to $\sim$. More concisely, he means that he is only interested in the abstracta $f_j^\prime(a), f_j^\prime(a)$ which are indistinguishable, hence identical, in case $f_1(a) \bowtie f_1(a)$.

As an effect of this “vertical” abstraction exerted on each of the columns of the diagram representing the “ontology” of the looking around method (cf. 2), the indefinite variety of candidates for the denotation of the “unconstrained notation” $\tilde{a}, \tilde{b}, \tilde{c}, \ldots$ drastically shrinks into a unique candidate in each column:
5. Abstraction plus some looking around

It must be emphasized that the frivolousness of the looking around method has to do only with the arbitrary, unjustified assignement of denotation to the symbols \( \tilde{a}, \tilde{b}, \tilde{c} \ldots \). The discovery and establishment of correlations between the initial objects \( a, b, c \ldots \) and values of functions \( f(x) \) (defined on the initial objects) in such a way that, for initial objects \( x, y, f(x) = f(y) \) iff \( x \sim y \), is in itself a challenging and interesting enterprise. For example, one once has defined for sentences \( s \) an abstractum \( \tilde{s} \) (let us call it "the Bedeutung of \( s \)" on the basis of an equivalence relation among sentences (for instance, interchangeability \( \text{salva veritate} \)). It is challenging to find out whether the following correlation can be established: "the truth-value of \( s \) = the truth-value of \( \tilde{s} \) iff the Bedeutung of \( s \) = the Bedeutung of \( \tilde{s} \)." Frege wrote a famous paper precisely in order to strengthen the plausibility of this conjecture of his (unfortunately he was on the looking-around side rather than on the abstraction side).

Thus the abstraction method may be completed by a challenging "looking around" for correlations of the described form. The "ontology" of the abstraction method may be extended then to include the correlated entities:

<table>
<thead>
<tr>
<th>initial objects (with ( \sim ))</th>
<th>( a )</th>
<th>( b )</th>
<th>( c \ldots )</th>
</tr>
</thead>
<tbody>
<tr>
<td>abstracta</td>
<td>( \tilde{a} )</td>
<td>( \tilde{b} )</td>
<td>( \tilde{c} \ldots )</td>
</tr>
<tr>
<td>correlations (1)</td>
<td>( f_1(a) )</td>
<td>( f_1(b) )</td>
<td>( f_1(c) \ldots )</td>
</tr>
<tr>
<td>correlations (2)</td>
<td>( f_2(a) )</td>
<td>( f_2(b) )</td>
<td>( f_2(c) \ldots )</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
</tbody>
</table>

The correlated entities do not have to be "identified" with the abstracta. For example, there is no reason compelling us to "identify" the truth-value of a sentence with the Bedeutung (in case the Fregean conjecture turns out to be true.) The interest of the correlations lies precisely in revealing that there are other, distinct entities which "go together" with the abstracta \( \tilde{a}, \tilde{b}, \tilde{c} \ldots \) or with the initial objects \( a, b, c \ldots \) in certain "isomorphic" ways. Nothing of course prevents doing a new abstraction leading to the "identification" of the abstractum \( \tilde{a} \) and the correlated entity \( f_1(a) \).
There is a large tradition, throughout the history of philosophy as well as in modern psychology, having to do with abstraction, or at least in which the word abstraction is mentioned. What is the relationship between these numerous references to abstraction made by so many authors and the method of abstraction invented by Peano, Weyl and Lorenzen? I will not attempt to answer this question here; I will restrict myself to a few remarks.

1) There are, especially in our contemporary philosophical literature, many pseudo-uses of 'abstraction'. For example, ‘abstract’ is frequently taken to mean the same as 'beyond space and time', 'intangible' (so Carnap in The Problem of abstract entities, Meaning and Necessity, Supp. A. 1) and 'abstraction' often stands for a mere manipulation of symbols, such as the closure or binding of free variables in an expression (Carnap, ibid. p. 3, Quine, p. 16). Contrary to this, it must be stressed that abstraction requires, in its genuine sense, some actual abstrac ting from something, a disregarding of some aspects of an object.

2) By contrast with the pseudo-uses mentioned under (1), the scholastic tradition (including, in this particular respect, among the scholastic such modern thinkers as Descartes, Locke, Leibniz etc.), phenomenology and more recent cognitive studies (Piaget, cf. for general references Battro) offer systematic theories of abstraction in senses that seem to be genuine. It is relative to these various traditions of schools that the comparison with the abstraction method should be fruitful.

3) Scholz in Scholz-Schweizer was the first to attempt to compare ‘classical’ abstraction with the so-called definitions by abstraction’ (as Scholz put it). It seems to me however that Scholz failed to appreciate the main point of the abstraction method, as one may gather from his remarks on Peano and Weyl (p. 33, 36). From the point of view of the abstraction method, the project has been undertaken for the first time by Schneider.

4) As an interesting example of the difficulties involved in the suggested historical study I refer to the passage from Leibniz quoted at the beginning of this paper. The passage has been mentioned by Weyl as an anticipation of abstraction, in the strict sense defended by Weyl. The problem is that in the quoted text Leibniz does not even talk of abstraction (there is one occurrence of the word 'abstrait' on the same page, but I fail to see the relevance) whereas in the frequent occasions in which Leibniz does refer to abstraction there seem to be no hints of Weyl's abstraction. Perhaps a text like the following may help to provide a link: 'Cum abstracta non sunt Entia, reducuntur ad veritates, e.g. Rationalitas hominis nihil aliud est quam veritas hujus Enuntiationis: homo est rationalis' (GP II, p. 472).

Carnap's method of extension and intension

Carnap's method of extension and intension is an instance, actually two instances, of the looking-around method. Carnap considers two equivalence relations: equivalence and L-equivalence, defined on designators (linguistic expressions). For example, if a and b are two predications (such as Human, Rational Animal) their equivalence consists in the truth of a x x is a + x is b, while their L-equivalence consists in the L-truth (logical or analytic truth) of that sentence. According to the looking-around method, as a prerequisite for the formulation of the biconditional (i), Carnap has to introduce pseudo-singular terms (pseudo in the sense that within the first stage of the looking-around method they do not stand for anything) corresponding to our symbols a, b, c, ... Carnap chooses, for each of his two parallel applications of the looking-around method, the phrases 'the extension of . . .', 'the intension of . . .'. Unfortunately, these two phrases already have meaning(s) in logic, independently of the definitions or stipulations made in stage (i) of the looking-around method. To avoid being misled by these other meanings, we must keep in mind that in stage (i) there is nothing that we might refer to as the extension or the intension of an expression. In fact, Carnap remarks: "note that the terms 'extension' and 'intension' have not been defined hereby but only the phrases 'have the same extension' and 'have the same intension'" (Meaning and Necessity, p. 23).

If we want to treat the expressions 'the intension of . . .' and 'the extension of . . .' as singular terms standing for a unique entity we have to move to stage (ii) of the method: "in order to speak about extensions and intensions themselves, we have to look for entities [...]. which can be assigned to designators in accordance with these definitions" (ibid., emphasis mine), "we have to look around for entities which might be taken as extensions or as intensions for the various kinds of designators" (Meaning and Necessity, p. 1, emphasis mine).

Carnap's procedure is of course subject to the general objections made against the looking-around method. This means that the "method of extension and intension" is either frivolous (unjustified) or reduces to abstraction. That is, the choice of designata for the expressions 'the extension of . . .', 'the intension of . . .' lacks justification or else amounts to choosing an abstractum (the f1(a), section 4).

Carnap's project fails also in a more specific sense, namely qua project for semantics. Carnap's method of extension-intension is presented as a new alternative to the old-fashioned "name-relation" method (Meaning and Necessity, preface). The failure of this proposal may be expressed as a dilemma: the extension-intension method either falls back into the name-relation method (which happens at the point in which particular entities are assigned to the new singular terms 'the extension of . . .', 'the intension of . . .') or remains, qua
program for semantics, totally void (which happens if one stays in stage (i) without moving to stage (ii): the singular terms 'the extension of...,' 'the intension of...,' remain denotationless, hence unusable.)

Carnap's procedure is rather ambiguous. On the one hand he affirms that all that matters is the equivalence relation and the corresponding stipulation of stage (i); he suggests that the entities introduced in stage (ii) are mere f a ç o n d e p a r l e r. On the other hand Carnap gives the impression of being genuinely interested in discovering the nature of the entities assigned as denotata of the phrases 'the extension of...,' 'the intension of...,' For example, he makes serious efforts towards understanding "what are" his "individual concepts". He suggests that they are functions from possible worlds to extensions (Meaning and Necessity p. 181), a view later on generalized into the conception of all intensions as functions from possible worlds to extensions (Kaplan 15.3, von Kutschera p. 24).

To the extent that Carnap seems to take seriously the chosen entities, his project seems to be really a project of semantics. But then it can be criticized for the above mentioned reasons: 1) the choice of entities is frivolous (not justified), 2) the assignment of particular entities to the phrases 'the extension of...,' 'the intension of...,' amounts to falling back into the name-relation method.

To the extent that Carnap emphasizes that all that really matters is the equivalence relation and the associated stipulation of stage (i), the two just mentioned objections do not apply (no choice being "really" made, the method is not frivolous: there is nothing to justify; no assignment being really made, there is no name-relation either). Other objections arise, however. If by saying that all that matters is the equivalence relation Carnap means that no entity is assigned to the phrases 'the extension of...,' 'the intension of...,' then 1) the project is no longer a project in s e m a n t i c s, 2) the looking-around method remains incomplete: one has looked around for n o t h i n g. If Carnap means that no particular entity, as such, really matters, and that all entities are equally welcome insofar as they fit with the stipulation of stage (i), then we have abstraction (unrecognized by Carnap though). The abstracts yielded by this abstraction correspond to the $f_1$ (a) described above, section 4.

What to do in view of this negative situation? The answer is that Carnap's project c a n be saved and preserved as a significant contribution to semantics if properly reconstructed in terms of abstraction. Under this reconstruction Carnap's method of extension-intension turns out to coincide with what has been done by Lorenzen since his Einführung 1955.

The rescuing proposal is to replace the looking-around procedure by the abstraction method. Accordingly the phrases 'the extension of...,' 'the intension of...,' are to be used from the start as singular terms denoting the abstracta generated by restriction to predicates that are invariant with respect to the equivalence relation under consideration (simple equivalence for extensions, L-equivalence for intensions.)

For example, the extension of a predicate $P$ becomes, in this approach, the abstract entity $\hat{P}$ that results from restricting our use of $P$ to statements in which $P$ is interchangeable salva veritate with any other predicate $Q$ such that $\Lambda x. P <--> Qx$. More accurately, we may designate the abstractum $\hat{P}$ by means of the notation $\hat{P}$. Then we may say $\hat{P} x = \in xQx --> \Lambda x. Px = Qx$, which of course strikes us as identical to Frege's famous axiom $\text{V}$ in his Grundgesetze (Frege does not seem to be aware of the conception of $\in xP$ as abstractum, cf. Thiel p. 263; he is trapped in the looking-around method.)

Those who claim to have straightforward insights into the meaning of predications or sentences will not be satisfied with the pale, abstract entities offered by Carnap's method reconstructed as abstraction. But then these logicians and philosophers do not need Carnap's method in any form, either reformed à la abstraction or in its original, looking-around presentation. They know how to go from predicates or sentences to their meanings without the aid of equivalence relations or of stipulations such as made in stage (i) of the looking-around method.

Obviously, Carnap's aim in designing his new method of extension-intension, as opposed to the old name-relation, was to provide a rigorous framework for those who feel insecure about the meaning of predicates and sentences. These people may end up with a defeatist equation of "meaning = use" ("I do not know what is "the" meaning but I know how to use the predicate or the sentence"). For such people, abstraction represents significant progress.

An interesting example of how abstraction helps is suggested by Prior's account of the development of the notion of proposition. After describing many unsuccessful attempts to "grasp" the "propositions in themselves" Prior hints at a new approach (p. 32), which consists in investigating the conditions under which two sentences represent the same proposition. Prior is unaware of abstraction, but his presentation confirms the conjecture that, for human beings lacking angelic insights, abstraction is the only way.

* Consider Carnap's statement: "In order to speak about extensions and intensions themselves, we have to look for entities, or at least for phrases apparently referring to entities..." (Meaning and Necessity, p. 23, emphasis mine).
Abstraction, Looking-Around and Semantics

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Weyl, H., Philosophie der Mathematik und Naturwissenschaft, 2nd ed., Leibniz Verlag, Munich, no date, first ed. 1928.