Abstract. In view of the confusion that exists in the 20th century with regard to the meaning of “abstraction” and “abstract” it is necessary to begin by recalling what is genuine abstraction and by stating what should be expected from a theory of abstraction today (section 1). Since Boethius and till the late 19th century abstraction enjoyed a peaceful life (except for some attacks from the British empiricists), within logic, metaphysics and psychology, under the reign of the great masters of abstraction (2). Aside from neo-scholasticism, only a few individual authors carried the torch of genuine abstraction in our century; for example: Husserl and Piaget; within modern logic: Weyl, and especially Lorenzen. Probably because of Frege and Russell, abstraction disappeared from the mainstream of modern logic and analytic philosophy (3). The void was filled by a proliferation of pseudo-uses of the terms “abstraction” and “abstract”: the usurpers (4). The survival of abstraction in modern logic (“modern abstraction”, Lorenzen) was unfortunately associated with nominalism (5). Nominalism shuns the challenge of having to say something about the nature of abstracta (6). But, thanks to nominalism, modern abstraction turns out to be immune to a recent criticism (7). Signs of a renewed interest in abstraction are mentioned (8). The final reflection is that philosophers have the right to reject abstraction, but then no pseudo-uses of the word should be introduced (9).

1. Making a few preliminary points about abstraction

The following conditions appear to be required in any discussion on abstraction.

1) The term “abstraction” should be used in the genuine sense. In the history of philosophy the word “abstraction” and cognate expressions (“abstract”, etc.) have had a genuine meaning, according to which abstraction involves an operation by which something is retained and something else is left out — to use Locke’s words (1959, III, 1, 9). In the special sense relevant for philosophy, the operation is intellectual, and the retaining and leaving out pertain to

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1 The present essay offers extensions and revisions of my previous writings on abstraction, starting with my (1979). The earlier papers, however, offer more details, and include references not cited here.
our mental consideration of things. This genuine meaning has been lost in the mainstream of the logical and analytic tradition of the 20th century and, what is worse, has been replaced by pseudo-uses.

2) The starting point of abstraction should be a set of (true) sentences. I will assume that the operation of abstraction can be represented as an operation on sentences — on sentences that one regards as true. Thus, in classical terminology, the terminus a quo of abstraction is a collection of (true) sentences: one decides to ignore, “hide”, abstract from, perhaps temporarily, some of them.

3) A technique of abstraction should be provided. Leaving aside the possibility of doing abstraction “at random”, any systematic abstraction should proceed according to some principle, reason or criterion. The latter, however, cannot remain as a generic guidance, but must be broken down into precise instructions as to exactly which sentences to retain and which to leave out.

4) The nature of the abstracta should be clarified. The obvious immediate output of abstraction is the set of sentences that are retained. However, sentences are about objects, and abstraction affects our view, or the way we view, those objects. The abstract view of an object resulting from the “hiding” of true sentences about it is the remote output of abstraction, or the abstractum — in the plural, the abstracta. Here the fundamental question arises: What is the nature of the abstracta? In linguistic terms: What statements are true or false about such abstracta?

These four points may serve as criteria to evaluate contributions to abstraction theory from different periods of the history of philosophy, past or present.

2. The great masters of abstraction

Berkeley writes:

It were an endless, as well as useless thing, to trace the Schoolmen, those great masters of abstraction, through all the manifold inextricable labyrinths of error and dispute, which their doctrine of abstract natures and notions seems to have led them into (1964, Introduction, p. 17, my emphasis).

In this issue, like in many others, the so-called “modern” philosophers — such as Berkeley — have had the regrettable effect of disconnecting philosophical thought from the state which it had reached about 1600.

Undisturbed by the disconnection, the artisans of abstraction continued to write, generally in Latin, well through the 20th century. One of their distinguished, recent representatives, Santiago Ramirez, devoted more than a hundred pages, in Latin, to (genuine!) abstraction in his four volume treatise De analogia (On analogy).

After a brief section on the notio abstractionis (the notion of abstraction),
which is presented as a motion from a *terminus a quo* (a starting point) towards a *terminus ad quem* (a terminal point), where the starting point is *aliquid compositum vel coniunctum* (something composite or conjoined) and the terminal point is the division or separation of what was united, Ramirez displays an enormous variety of *divisions* of abstraction. As a matter of fact, the author ominously announces that *abstractio dicitur multipliciter* (abstraction is said in many ways). The distinctions and subdistinctions of the meanings of the word *abstractio* are so exuberant that they require an extra folded sheet — a cartography of abstraction!

It is tiresome to reach, through the scholastic labyrinths and distinctions displayed by Ramirez, the kind of abstraction in which we are interested. However, one positive fact remains: even in the many meanings foreign to our concerns, the basic conception of abstraction as deletion or removal, today forgotten, is preserved.

Ramirez begins with abstraction *in ordine physico et corporali* (in the physical or corporeal realm). Here there is always a real separation *eius quod abstrahitur ab eo a quo abstrahitur: sicut navis abstrahitur a portu ad navigandum*... (of that which is abstracted and that from which it is abstracted, as the ship is abstracted from the harbour, in order to navigate). Among the scholastic authors, the examples are often amusing: *abstractio tunice ab habente* (abstraction of the tunic from him who has it, cf. Sebastianus de Aragonia 1992, p. 69), *cum ab ave pennae avelluntur* (when one plucks a bird, cf. Signoriello 1893).

Next, Ramirez moves to less physical abstractions, namely to abstraction *in ordine psychologico* (in the psychological realm). But again we stumble on a new distinction. Psychologically, abstraction is said in two ways (*dupliciter*): *primo in linea vitalitatis et per ordinem ad idem subiectum vivens; secundo in linea cognitionis et per ordinem ad idem obiectum* (firstly in the order of life and relative to the same living subject, secondly in the order of cognition and relative to the same object, cf. p. 757). While in the first of these two orders (in the order of life) we encounter strange things again (strange, at any rate, *vis à vis* our expectations), such as death, abduction (p. 758), etc., in the second (order of cognition) what we are looking for finally seems to appear.

However, a new subdivision is waiting for us. The abstraction in the order of cognition is twofold: *abstractio sensitiva* and *intellectiva*. The first has to do with the fact that one sense does not catch what other senses grasp in an object. This is close, but not yet what we want.

When we reach the *abstractio intellectiva* we are finally in business — although, alas, “business” means, in good scholasticism, still one more distinction, now the famous one between *formal* and *total* abstraction.

In total abstraction we recognize what has been usually called abstraction in logic and philosophy, namely the familiar move from individuals to univer-
sals, from Peter and John to man, and from man and horse to animal, etc. The word “total” refers to the universal regarded as a whole, of which the individuals, or inferior universals, are “parts”. One “abstracts from” the differences between Peter and John, or from the differences between man and horse — thus the universal concepts: man, animal, etc., emerge.

It is only with regard to formal abstraction that it becomes hard to understand what was meant by the Schoolmen. I will mention two difficulties I have encountered.

First, Thomas Aquinas, while generally indicating that in abstraction something is retained and something is left out (to use again Locke’s terms), says however that in formal abstraction both items remain, which seems to mean that nothing is left out:

Respondeo dicendo quod duplex fit abstractio per intellectum. Una quidem, secundum quod universale abstrahitur a particulari, ut animal ab homine; alia enim, secundum quod forma abstrahitur a materia; sicut forma circuli abstrahitur per intellectum ab omni materia sensibili. Inter has autem abstractiones haec est differentia, quod in abstractione, quae fit secundum universale et particulare, non remanet id, a quo fit abstractio; remota enim ab homine differentia rationali, non remanet in intellectu homo, sed solum animal. In abstractione vero, quae attenditur secundum formam et materiam, utrumque manet in intellectu; abstrahendo enim formam circuli ab aere remanet seorsum in intellectu nostro et intellectus circuli et intellectus aeris (1950, I 40, 3, respondeo, my emphasis).

An English translation:

I answer that, abstraction by the intellect is twofold — when the universal is abstracted from the particular, as animal abstracted from man; and when the form is abstracted from the matter, as the form of a circle is abstracted by the intellect from any sensible matter. The difference between these two abstractions consists in the fact that in the abstraction of the universal from the particular, that from which the abstraction is made does not remain; for when the difference of rationality is removed from man, the man no longer remains in the intellect, but animal alone remains. But in the abstraction of the form from the matter, both the form and the matter remain in the intellect; as, for instance, if we abstract the form of a circle from brass, there remains in our intellect separately the understanding both of a circle, and of brass (my emphasis).

A second problem I find with formal abstraction is that it is not clear whether the retained entity is an individual or a universal. If in considering a circle made of brass, we ignore the bronze and just retain the circle, as Aquinas wants, the question arises of whether this retained circle is an individual (accident) or a universal.

These two problems concerning abstractio formalis do not appear to be clarified in the very large literature, primary and secondary, devoted to abstraction, that I have consulted. A precise, critical analysis of formal abstraction remains, therefore, as a challenge for critically minded historians of scho-
Leaving aside this problem, let us evaluate the great masters’ performance relative to the four criteria defined in section 1.

With regard to the first point, we find, for example, that Aquinas emphasizes that abstraction applies to what is united in reality. “What is united in reality” can be reworded, linguistically, as “a true sentence”. Thus, the starting point of abstraction is, or may be construed as a collection of sentences (accepted as true).

With regard to the second point (abstractive techniques), Aquinas’ doctrine may be read as saying that to do abstraction is to “hide” some of the true sentences while keeping other true sentences visible. Cşmp tells us (1980, 4, p. 17) that Aquinas is quite emphatic about this: “le texte [Thomas Aquinas] ne cesse de rappeler que l’abstraction consiste dans une non-consideration” (the text does not stop reminding us that abstraction consists in a non-consideration). Crucial here is to keep in mind that to “hide” a sentence is not to deny it or to regard it as false: abstrahentium non est mendacium (those who do abstraction are not liars, cf. Wyser 1947, p. 475).

Aside from this, however, in general one should not expect techniques of abstraction prominently listed in the books of the great masters, since the abstractive activity, in the scholastic framework, appears to be conceived as a natural process, accomplished by the intellect (agens, possibile, etc.), rather than as an operation that we must perform according to given rules.

Perhaps the closest to an abstractive technique in the scholastic tradition is the neglected theory of reduplication, which mysteriously vanished from (non-scholastic) logic textbooks in the 20th century.

Reduplication is the linguistic trick by which we restrict our discourse, initially about \{a, b, c...\}, by attaching to the singular terms “a,” “b,” “c”... the particle “qua-F” (or equivalents). Now we talk not about a, but about a-qua F; for example not about Peter, but about Peter-qua-man. Of course the F is just one out of infinitely many choices. Once F is fixed, however, lots of true statements about a, b, c... are, in J. Lear’s felicitous phrase, “filtered out”, i.e. hidden, or abstracted from.

The masters of abstraction do not seem to tell us much either with regard to the nature of the abstracta in general, although they abundantly discuss particular cases of objects that they regard as products of abstraction, such as universals. Thus, through the particular case of universals we should expect to gain some insight into the Aristotelian-scholastic conception of the nature of abstracta in general. Consider, for example, the universal man (homo). Man

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2 Reduplication has been finally dusted off by A. Bick. Cf. his impressive volume, as well as some of my publications.
“considered in itself” is, for the Aristotelian-scholastic authors, a collection of other universals: \{animal, rational\}. From here we may perhaps generalize and say that abstracta are, for those authors, collections of properties.

One difficulty to be kept in mind in evaluating the great masters’ understanding of the nature of universals, or abstracta in general, is the following. Whatever intuitions the classical philosophers may have with respect to the nature of universals, or equivalently, with respect to determining the true statements that can be made about universals, the formulation of those intuitions will be hampered by the peculiarities of the Aristotelian (classical, pre-Fregean) predication theory. The Organon is not too interested in sentences such as “man is a universal” (or, to choose a good scholastic example, “man is an abstract entity in absolute consideration”), and does not give instructions about them.

3. Abstraction rejected from the logico-analytic mainstream

Abstraction is present, in different degrees, in most of the philosophers from the modern period. In fact, an interesting idea developed: one entity taken as representative of a class (Locke 1959, II, 11, 9). This is on the right track; for instance, in moving from fractions to rationals one may choose a particular fraction, say \(1/2\), as representative of the rational \(1/2\). Unfortunately, the consequences of this notion were not rigorously pursued and the project remained misleadingly half-way, unfinished, as if the consideration of the representative involved an exemption from looking beyond the initial, (relatively) concrete objects. To take the particular fraction \(1/2\) as representative (of the class of all fractions that are equivalent to \(1/2\), with equivalence defined as having equal cross-products) is tantamount to introducing a new entity in one’s domain of discourse, and correspondingly a new singular term (“\(1/2\) qua representative”). Here, requirements (3) and (4) listed in section 1 of this paper must be satisfied: precise rules for the selection of statements, and some explanation on the nature of the new entity must be offered. Aside from the self-defeating, muddled comments on “the general triangle”, neither issue was satisfactorily addressed by Locke or subsequent philosophers.

In the past hundred years, abstraction has been present in individual authors (Husserl, Piaget\(^3\), and within modern logic: Peano, Weyl, Lorenzen), but has been absent from the mainstream of mathematical logic and analytic philosophy, probably as a consequence of Frege’s and Russell’s negative view of it.

\(^3\) Cf. Battro (1966), articles on abstraction.
Frege speaks of abstraction in three senses (cf. my 1984): (i) the abstraction he calls “magical”, (ii) the common or ordinary abstraction, (iii) the definition by abstraction. The latter phrase occurs only once in Frege’s writings, in a letter to Russell, where Frege explains that the method of transforming a symmetric and transitive relation (similarity) into an identity (identity of the aspects in which the similar objects coincide) would perhaps be referred to by Russell as “définition par abstraction” (in French). This is suggestive, but deceitful: the phrase “definition by abstraction”, coined by Peano, corresponded, for the Italian mathematician, to a genuine interest in abstraction, but this was no longer the case in Russell, as we shall see. The notion of magical abstraction occurs within Frege’s vehement criticisms to the way in which the abstraction leading from a set to the number (cardinality) of the set had been analyzed by mathematicians like Cantor. According to these authors, that abstraction leading, for instance, from the set of the Apostles to 12 consists in ignoring the peculiarities of each individual member of the set. Frege rightly found this “impossible” – since the result would simply yield the concept Apostle. Only magic could do it.\footnote{Both Cantor and Frege were wrong however – cf. my (1984), §7, p. 468.} Under the heading of common or ordinary abstraction Frege deals with what can be regarded a simplistic version of the classical account of abstraction. Here Frege had no major objections, but no major enthusiasm either – it is as if he did not know what to do with that good, old abstraction. In sum, it is possible to affirm, on the basis of a careful study of the texts, that Frege’s final recommendation with regard to abstraction is not to waste one’s time with it, and pass it on to psychology. Given Frege’s anti-psychologist view, this amounts to a strong rejection.

In Russell, the abstraction story is different. As we see in The Principles, Russell considers a style of defining called by Peano “definizione per astrazione”, that Russell finds bad enough to declare that it is not valid (end of section 110) but at the same time good enough to try to improve it, and make it into something acceptable. Peano – according to Russell – assumes that any relation that is both symmetric and transitive (for instance, the similarity that obtains between Mary and Peter insofar as both are human beings) allows us, without further ado, to define, “by abstraction”, a new object that corresponds to what is usually called a “common property” of the given objects (in the given example, the property of being human). Russell first criticizes Peano for not securing the uniqueness of that something common to the initial objects (section 110); secondly, Russell objects to Peano’s lack of a proof of existence of that common entity (section 210). Russell says that he secures existence by a procedure called “principle of abstraction”, while uniqueness is trusted to a nominal definition (section 110). This principle of abstraction will become,
later on in *Principia Mathematica*, a beautiful theorem (72.66) – beautiful but no less deceiving for anyone interested in genuine abstraction. To be sure, theorem 72.66 does prove the existence of something common to Peter and Mary, given that they stand in the symmetric and transitive relation: “\(x\) is human \(\land y\) is human”. However, that common entity turns out to be, simply, the equivalence class relative to the chosen equivalence relation. Anyone expecting from theorem 72.66 an important and enlightening contribution to the theory of abstraction will be quite disappointed. No operation of abstraction is performed: all we do is move from human beings to a class containing them. To reply that that class is an abstract entity would require an explanation of the term “abstract”: if used in the pseudo-sense of “intangible”, it is not interesting for our purposes; if a genuine abstraction is involved, it should be properly explained and described. In sum, theorem 72.66 cannot be called “principle of abstraction”. In fact, Russell himself corrects his terminology later on, saying that rather than “principle of abstraction” one should speak of a principle that dispenses with abstraction – a principle that allows us to do well without worrying about abstraction (1956, p. 326).

### 4. The usurpers

Not only was abstraction exiled, but usurpers took its place in the mainstream of logic and philosophical analysis in the 20th-century. There have been, or rather there are, little usurpers and one big usurper.

#### 4.1 The little usurpers

The little usurpers are uses of the term “abstraction” (“abstract”, etc.) in which no reference is made to any properly abstractive process. There are two sorts of little usurpers: (1) uses of the adjective “abstract” that tend to mean some of the following: “non-accessible to the senses”, “intangible”, “neither spatial nor temporal”, or “not possibly being located in space-time”. (2) Use of the noun “abstraction” as tending to designate some notational operation which generates a term allegedly denoting an entity that is “abstract” in some of the just described, pseudo-senses.

A venerable, but no less wrong, example is provided by H. Scholz. If, à la Frege, the singular term 5 is deleted from the sentence “5 is prime”, we obtain a concept-word: “( ) is prime”, which denotes a concept. This “punching out” of singular terms has been called by H. Scholz die *logistische Abstraktionstechnik* (the logistic technique of abstraction, 1935, p. 15). I cannot imagine a worse violation of the first or third conditions listed in section 1 of this paper.
In Feys and Fitch’s *Dictionary* three senses of “abstraction” are distinguished: a syntactical, a semantical, and a real. Syntactical abstraction is the symbolic operation of “obtaining a name of a function by operating with the $\lambda$-operator on an associated form of the function” (40.3). The $\lambda$-operator is called an “abstractor”, and the result of applying such an abstractor to a wff $a$: $\lambda x a$, is an *abstract*. For semantical abstraction Feys and Fitch refer to Church’s *Introduction*, p. 22, where in fact we learn that “the passage from a form to an associated function (for which the $\lambda$-notation provides a symbolism)” is an abstraction. Needless to say, both the syntactical and the semantical senses are paradigms of pseudo-abstraction. In the final analysis, all they mean is that some linguistic expressions are intended to signify entities that are called “abstract” because they are intangible, not in space or time, etc.

The “real” abstraction mentioned by Feys and Fitch is described as “the process of finding the function itself, given suitable information about the values of the function for all its arguments” (40.2). Here the word “process” might hide some authentic, albeit undercover, abstractive activities, the consideration of which probably stems from Feys’ scholastic background. At the same time, however, I cannot fail to point out the slightly amusing nature of the terminology chosen by Feys: “real abstraction”, in the tradition of the great masters of abstraction, was rather applied to physical operations (cf. the example quoted in section 2: plucking the feathers of a bird).

In Quine, the usurpation becomes even more systematic and prominent. “Abstraction” is, for example, the title of an entire section (§24) of Quine’s *Mathematical Logic*, where we learn, again, that “the forming of class names by such prefixes (Quine refers to ordinary phrases as “the class of all elements $x$ such that” or a circumflex written on top of an $x$) will be called *abstraction*; and the result... will be called an *abstract*”.

Particularly interesting, and revealing, are the following passages from Quine 1961, pp. 117-119 (emphasis is mine, except for the word “inscription”)

> It may happen that a theory dealing with nothing but concrete individuals can conveniently be reconstrued as treating of universals, by the method of identifying indiscernibles. Thus, consider a theory of bodies compared in point of length. The values of the bound variables are physical objects, and the only predicate is “$L$”, where “$Lxy$” means “$x$ is longer than $y$”. Now where $\sim Lxy, \sim Lyx$, anything that can be truly said of $x$ within this theory holds equally for $y$ and vice versa. Hence it is convenient to treat “$\sim Lxy, \sim Lyx$” as “$x = y$”. Such identification amounts to reconstruing the values of our variables as universals, namely, lengths, instead of physical objects.

Another example of such identification of indiscernibles is obtainable in the theory

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5 S. Pollard drew my attention to these significant texts.
of inscriptions, a formal syntax in which the values of the bound variables are concrete inscriptions. The important predicate here is “C”, where “Cxyz” means that x consists of a part notationally like y followed by a part notationally like z. The condition of interchangeability or indiscernibility in this theory proves to be notational likeness, expressible thus:

\[(z)(w)(Cxzw = Cyzw = Czxw = Czyw).
\]

By treating this condition as “x=y”, we convert our theory of inscriptions into a theory of notational forms, where the values of the variables are no longer individual inscriptions, but the abstract notational shapes of inscriptions. This method of abstracting universals is quite reconcilable with nominalism, the philosophy according to which there are really no universals at all. For the universals may be regarded as entering here merely as a manner of speaking – through the metaphorical use of the identity sign for what is really not identity but sameness of length, in the one example, or notational likeness in the other example. In abstracting universals by identification of indiscernibles, we do no more than rephrase the same old system of particulars.

Unfortunately, though, this innocent kind of abstraction is inadequate to abstracting any but mutually exclusive classes. For when a class is abstracted by this method, what holds it together is the indistinguishability of its members by the terms of the theory in question; so any overlapping of two such classes would fuse them irretrievably into a single class.

Another bolder way of abstracting universals is by admitting into quantifiers, as bound variables, letters which had hitherto been merely schematic letters involving no ontological commitments. Thus if we extend truth-function theory by introducing quantifiers “(p)”, “(q)”, “(∃p)”, etc., we can then no longer dismiss the statement letters as schematic. Instead we must view them as variables taking appropriate entities as values, namely, propositions or, better, truth-values, as is evident from the early pages of this essay. We come out with a theory involving universals, or anyway abstract entities.

Here Quine observes that the quantifiers over sentential variables “happen to be reconcilable with nominalism if we are working in an extensional system”, so that “what seemed to be quantified discourse about propositions or truth-values is thereby legitimised, from a nominalist point of view, as a figure of speech”. However,

abstraction by binding schematic letters is not always thus easily reconcilable with nominalism. If we bind the schematic letters of quantification theory, we achieve a reification of universals which no device ... is adequate to explaining away. These universals are entities whereof predicates may be thenceforward be regarded as names.

Quine distinguishes an innocent abstraction and a bold abstraction. The former, also called by him “the method of identification of indiscernibles” – of which he gives two examples: lengths and notational forms – is practically the abstraction method (Peano, Weyl, Lorenzen) to which I refer below (section 5). Quine finds this method, in spite of its innocence, “inadequate”. I fail to see the force of this objection. One may simultaneously consider, although not
simultaneously perform, two different abstractions on the same domain of initial objects; for instance, given the domain of fractions, one may consider both an abstraction relative to the equivalence relation of having equal cross-products and an abstraction relative to the equivalence relation of having even numerators (“x’s numerator is even & y’s numerator is even”). Thus, 2/4 and 7/14 become indiscernible within the former, but not within the latter abstraction.

With regard to Quine’s “bolder method of abstraction” I should say that it may be bold — but it is not abstraction. No abstraction is involved in that method, except the fact of referring to, or naming entities that Quine calls “abstract” probably because they are intangible or inaccessible to the senses.

I do not know of any reason to explain the proliferation of all these pseudo-uses of “abstraction” and cognate words. They merely filled a void left by the removal of genuine abstraction.

4.2 The big usurper: looking-around or circumspection

I have coined the designation “looking-around method” for the simple reason that the phrase “looking around” is found in Carnap’s *Meaning and necessity*, in connection with a procedure to be examined in this section. If a more “serious” term is desired, the word “circumspection”, in its etymological sense, is a perfect equivalent, for my purposes.6

The motivation for those who, following Carnap, practise this method is to give a more exact sense to (“to explicate”) certain expressions from scientific or ordinary language, such as: “the number of a concept $F$”, “the extension of a concept $F$”, “the cardinality of a set $S$”, etc. Let us generally describe these expressions as being of the form “$f(a)$”, “$f(b)$”, etc. For the purpose of attaining a better definition, one pretends to forget the familiar meanings already associated with such singular terms, so that, officially, the latter become temporarily meaningless.

Next, instead of worrying immediately about the new meaning to be given to “$f(a)$”, “$f(b)$”, etc., one will become concerned about identity conditions for whatever entities are eventually assigned to these symbols. In fact, the formulation of identity conditions will be the first step of our method, previous to the assignment of any denotations to “$f(a)$”, “$f(b)$”, etc. While pretending to ignore, for example, the meaning of “the number of the concept $F$”, one stipulates that for any meaning, the number of the concept $F$ = the number of the concept $G$ iff $F$ and $G$ stand in the relation consisting in the one-one correspondence between the objects that are $F$ and those that are $G$. The chosen re-

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6 This was suggested to me by Jaime Nubiola.
lation must be, as in the given example, symmetric and transitive, i.e. an equivalence relation.

The second, and final, stage of the method consists, as Carnap tells us (1960, p. 1), in looking around for entities that are suitable candidates to play the role of denotata of our singular terms of the form “f(a)”, “f(b)”. By “suitable” is meant that the assignment must be compatible with the identity condition stated in the first part of the method. One popular choice of a suitable entity are the equivalence classes determined by the equivalence relation under consideration; in this case, for instance, the singular term “f(a)” receives, as denotation, the class of entities that stand to a in the given relation. But infinitely many other choices are equally suitable, and in principle, i.e. as far as the method is concerned, there is no reason to choose one rather than another. Equivalence classes somehow became the natural or first choice, but any other type of “suitable” entity is equally entitled to be selected.

Circumspection is the method that Frege uses in two crucial points of his theory: in the definition of number and in the definition of set (but, for a different opinion, cf. Thiel 1986, p. XLII). In *The Principles*, Russell transforms Peano’s definitions by abstraction (ignoring their authentic abstractive component) into the looking around method (continuing to call it, at least for some time, “the principle of abstraction”). The method is applied by Russell and subsequent authors, such as Carnap and Quine, in order to define – “explicate” – several fundamental concepts: cardinal number, extension, intension, etc.

It is hard to understand, as already Husserl pointed out in his *Philosophie der Arithmetik* in connection with the philosophical première of the method offered by Frege, how such a procedure may have been regarded as progress in logical theory. It is a frivolous, irrational method – one more hara-kiri of reason (Weyl 1928, p. 41), since no reason or justification is provided for the choice of entities (among the infinitely many that are “suitable”).

How can we regard circumspection as an usurper of abstraction’s place, given that, according to the above description, the circumspection method has nothing to do with abstraction?

The answer is quite simple: circumspection, at a certain point, either in Peano himself or among his associates, began to be called “definition by abstraction”, a phrase originally coined by Peano with genuine abstraction in mind. The semantical mutation (or usurpation) was facilitated by the fact that Peano’s abstraction, as made fully clear, later on, by Lorenzen, needs as infrastructure a previously defined equivalence relation (for instance: the equality of cross-products of fractions, leading, by abstraction, to the rationals). Instead of using the equivalence relation as an abstraction ladder, logicians employed it as a platform for the circumspection method. A full disentanglement of the complex circumstances and confusions that surrounded this tale of two cities (abstraction and circumspection) is still an open task in the historiography of
modern logic.

While at no point do the rules of the circumspection method prescribe the performance of any genuine abstraction, it must be observed that the fact that all suitable candidates are “equal” can be construed as an abstraction. This is true – but it only shows that the circumspection method badly needs a serious overhaul. The abstraction involved in the fact that the differences among the suitable candidates can be ignored (abstracted from), and that it does not really matter which of the suitable candidates one finally chooses, should be explicitly highlighted and duly analyzed (just as Locke should have done with his “representative” view of abstract objects). Without such a radical transformation of the looking-around method into an abstraction method, the former cannot be called “abstraction” – neither “definition by abstraction” (as often done in the 20th-century) nor “logical abstraction” (as Dummett proposes, cf. next section).

Some authors, like Russell, intelligently realized that it was preposterous to talk of abstraction where nobody plans to do abstraction, and corrected the terminology (cf. section 3, above). Others, like Beth, unwisely kept the terminology while inconsistently admitting that no abstraction is intended (1959, p. 94). Still others, the worst (in this respect) from a philosophical standpoint, believed (in view of the fact that the favourite entities for the looking around are the equivalence classes) that the method was really an analysis of the old notion of abstraction in terms of the new set or class theory; so Reichenbach: “the notion of class finds an important application in the interpretation of a logical operation which traditionally has been called abstraction” (1956, section 37).

4.3 The final blow: total semantic depletion

In Dummett’s (1991), the main theme in the discussion of Frege’s answer to the question “What is number?” is abstraction. Philosophers of arithmetic are classified by Dummett into “bad” and “good” depending on their using or not using, respectively, abstraction in their definitions of number; this evaluation is presented together with a strong attack on abstraction. Dummett thinks that Frege is a champion of anti-abstractionism – I disagree: properly, Frege just criticizes the misuse of abstraction by Cantor and others in their attempts to define number. Finally, Dummett thinks that Frege does, after all, practise abstraction – but in a new, good sense of the term, which Dummett calls “logical” abstraction, in contrast with the bad, “psychological” abstraction. Dummett characterises these two abstractions in the following manner:

Both types of abstraction aim at isolating what is in common between the members of any set of objects each of which stands to each of the others in the relevant equivalence relation: Frege’s logical method by identifying the common feature
with the maximal set of objects so related to one another and containing the given objects; the spurious psychological operation by deleting in thought everything except that common feature (pp. 167-8).

Frege’s “logical method” – so much praised by Dummett – is what I have called the looking-around or circumspection method, a frivolous procedure that cannot be referred to as “logical abstraction” or as abstraction of any sort simply because there is no abstraction in it, except in the remote sense mentioned in the preceding section. The method is not really interested in any of the infinitely many, different particular types of objects that are “suitable”; all that matters is their common suitability, i.e. the compatibility of the chosen objects with the previously established condition (naturally, each author may add extra requisites). But then of course this obscure desire for doing abstraction should be properly expressed, and the equivalence class could no longer be put forward by the “circumspective” authors as the denotatum; rather the equivalence class-qua-compatible with the previously established condition should be the denotatum. Here, however, the denotatum of the singular term “the equivalence class-qua-compatible with the previously established condition” is, and should be recognised, as quite a new entity, an abstractum in a genuine sense. A rigorous handling of this abstractum and of the corresponding abstraction would require, first, disassembling the looking-around technique, and secondly, reassembling it according to some abstraction method (probably à la Lorenzen, cf. below, section 5).

Dummett’s phrase “logical abstraction” is a striking example of how the word “abstraction” has lost, in contemporary philosophy, and especially in the logico-analytic tradition, any connection with what is essential to abstraction: “leaving out” something and “retaining” something. Dummett’s text quoted above is perhaps a unique item in the collection of pseudo-uses of “abstraction” of the past century: no other text is known to me where the operation of “deletion” (the “leaving out”) is explicitly excluded from the notion of abstraction.

Dummett’s study of Frege’s philosophy of mathematics represents a final blow to the concept of abstraction, as well as the total semantical depletion of the term.

4.4 Victims of the usurpers

Authors using the term abstraction (or related expressions) in a non-genuine sense may be false prophets of abstraction (actively contributing to the usur-

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7 Curiously, “logische Abstraktion” occurs in Kant-Jäsche (1800), Allgemeine Elementarlehre, § 15. But there abstraction is genuine: it means Absonderung (deleting, leaving out), cf. ibid. § 6.
pation) or mere victims of the false prophets (just misled by the false prophets). Here are some instances of the latter.

The main victim has been H. Scholz, in his very important and equally neglected Definitionen durch Abstraktion. The first part is a survey of what Scholz calls “classical” abstraction theory (which is in fact medieval, since Aristotle has little to do with it). Scholz believes that classical abstraction should be replaced by “something better” (1935, § 1,1). Unfortunately, this something better turns out to be the circumspection method.

Other examples of distinguished philosophers misled by the usurpers are the following. Körner talks of “Frege’s principle of abstraction” and explicitly commends the application of this terminology to the presentation of Frege’s views: “the name given to the principle which is used to justify the definitions, the principle of abstraction, has been well chosen”.8 G. Küng, in his study of universals in recent philosophy writes: “The operation of constructing (or discovering) the equivalence classes of things with equal concrete properties corresponds to what is traditionally called abstraction” (1967, p. 173). More recently, Patzig (1983) describes Frege’s procedure in defining number as follows: “Frege schlägt hier einen Umweg ein, der als “Definition durch Abstraktion” folgenreich und berühmt geworden ist” (Frege recommends here an approach that has become famous and fruitful as “definition by abstraction”).

5. Quiet survival (modern abstraction)

The negative view with regard to abstraction in the logico-analytic tradition has one exception in the work of Paul Lorenzen, who has continued, in my view, hints found in Peano and in H. Weyl. To be sure, the interest of these scholars in genuine abstraction has been largely unnoticed.

Lorenzen’s main merit, with regard to abstraction, is that he was the first to dare to overhaul the circumspection method in the direction indicated above (section 4.2), turning it upside down, bringing out, and subjecting to a rigorous analysis, the abstraction that obscurely underlies the “equivalence” of all “suitable” candidates.

The following is the key text for anyone interested in understanding the confused interaction – started in Peano’s writings – between (genuine) abstraction and circumspection in the history of modern logic:

Dadurch [that is according to what is customary since Frege and Russell] soll die Abstraktion auf die Einführung von “Klassen” zurückgeführt werden. Wir werden

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jedoch weiter unten sehen, dass die Klassen nichts anderes als ein spezieller Fall von abstrakten Objekten sind (Lorenzen 1955, p. 101).

An English translation:

Thereby [that is according to what is customary since Frege and Russell] abstraction is to be reduced to the introduction of “classes”. However, we will see below that classes are nothing else but a special kind of abstract objects.

First of all, it is remarkable that in this 1955 text both the noun “abstraction” and the adjective “abstract” are used in the genuine sense. This should be a major surprise for readers familiar with modern logic. Moreover, the passage is not an isolated, casual remark but belongs in a section titled: Abstraktion, Relationen und Funktionen.

The great significance of the brief text may not be obvious for a reader unaware of the tensions between abstraction and circumspection since Peano’s time.

In the first sentence Lorenzen points to the fact that in “Frege and Russell” (I would more generally say: in the circumspection method at large) abstraction was reduced to classes (I would say more generally: abstraction was replaced by the looking-around method, where classes, equivalence classes that is, happen to be the most popular among the “suitable” entities).

Thus, the first sentence means much more than what it says.

On the other hand, the second sentence says more than what I think is right. That classes can be understood as abstract objects (again, genuine abstraction!) is obvious to anyone who seriously reads, for instance, Frege’s definition of Wertverlauf (Axiom V) in the light of Frege’s own comments in 1962, II, § 146. But I would not say that classes are “nothing else but a special kind of abstract object”, since other – let us say “intuitive” – conceptions of class are possible.

Having chased away the main usurper, and having restored genuine abstraction, Lorenzen moves to the description of a technique for abstraction, conceived as an operation on sentences. This is as follows.

We start with a universe of discourse: \(a, b, c,\ldots\) on which an equivalence relation, i.e. a relation (transitive and symmetric) is defined. At a certain point we decide to do abstraction. This means that we decide to restrict our lan-

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9 To be sure, Frege’s comments are not a full statement to the effect that his classes are abstracta (in the genuine sense). All one can say is that Frege somehow realizes that one begins with an equivalence relation defined for predicates (equiextensionality in his case), and that any two predicates standing in that relation have etwas Gemeinsames (something in common), which is the class. Unfortunately, this remarkable observation was never developed by Frege into a theory of classes as abstracta, and Frege contented himself with the “circumspective” reading of the “something in common” as some “suitable” entity (truth-values in this case).
guage, our discourse about the objects $a, b, c,...$ in such a way that we will use only predicates that are non-vacuously invariant\(^\text{10}\) with respect to the chosen equivalence relation. The obvious effect of this linguistic decision is to remove from circulation a number of (true) sentences and to obtain that two initial objects $a, b$, different before doing abstraction, become now indiscernible while doing abstraction.

In a large historical perspective, the novelty of all this is that while the great masters of traditional abstraction trusted the entire operation to the natural mechanisms of the intellectus agens or possibilis, in the just described modern abstraction it is our responsibility to perform the selection of sentences to be hidden and of sentences to be retained, and we have precise instructions on how to go about it. Thus, in the modern version, abstraction has been transferred from nature to art (technique).

Once genuine abstraction has been restored, and a technique for abstraction as an operation on sentences has been explained (points (1), (2) and (3) from the list given in section 1), it remains to discuss the most difficult issue, namely what is the output of abstraction.

Indeed, we should begin with an even more fundamental question, namely whether there is any output at all to the endeavours of modern abstraction. To use Peano’s and Lorenzen’s favourite example, when we restrict our discourse about fractions to statements that are invariant with respect to the equality of cross-products, obviously the old domain of fractions (where for example 1/2 and 36/72 are so different) evaporates. We are not talking anymore about fractions, since we do not want to say that, for instance, 1/2 is not further simplifiable. This is easy to accept, but the question comes next: Does a really new domain emerge? H. Weyl encourages us to lean towards an affirmative answer, when he writes that abstraction creates einen neuen Objektbereich (a new domain of objects, 1928, p. 9).

If Weyl’s insight is accepted and followed up, there will be a justification for introducing new singular terms – for the members of the alleged new domain. To continue the example of abstraction on fractions, we may write “[1/2]”, or, with ordinary words: “the rational 1/2” to denote what is left (retained, not abstracted from) the fraction 1/2. The rational 1/2 is quite different from the fraction 1/2: for instance, the sentence “the rational 1/2 is not further

\(^{10}\) Cf. my (1979), § 3. My “non-vacuously” clause was intended to correspond to Lorenzen’s condition (4.5) in (1962): “Wird $A(a)$ behauptet, so wird damit auch $A(a)$ behauptet” (If $A(a)$ is affirmed, by that very fact is therefore $A(a)$ affirmed). The significance of that clause will appear below, in the examination of Siegwart’s critique (section 7). The usual formulation of invariance: $\forall x \forall y (A(x) \land x \neq y) \rightarrow A(y)$), as an instruction on how to perform abstraction, has a loophole: it lets in vacuously invariant predicates. This is of course against the intentions of anyone who wants to do abstraction: the idea is to drop predicates, not to add predicates.
simplifiable” is not affirmed – it has been “hidden”.

Once we have introduced singular terms as designations for the abstracta, the ontological question of what the entities they stand for are, i.e. what are the abstracta, or its linguistic version of how the new singular terms should be used, cannot be further postponed. This question has been clearly stated by Lorenzen (1978, p. 41):

Man kann ein Wort an die Tafel schreiben, aber man kann keinen Begriff an die Tafel schreiben. Ein Wort, so lautet die übliche Erklärung hierzu, ist etwas Konkretes, ein Begriff aber etwas Abstraktes. Wer kritisch ist, wird sich mit dieser Erklärung nicht zufriedengeben. Er wird fragen, was abstrakte Gegenstände (kürzer: Abstrakta) seien. Oder schon etwas gewitzter: Wie wird das Wort “Abstraktum” verwendet? (emphasis mine).

An English translation:

It is possible to write a word on the blackboard, but a concept cannot be written on the blackboard. A word, as the usual explanation goes, is concrete, while a concept is abstract. Nobody who is critical can be satisfied with such an explanation, and will ask: What are the abstract objects (briefly: the abstracta)? Or in a subtler way: How is the term “abstract” used? (emphasis mine).

Lorenzen’s answer is, unfortunately, not as good as his question. In fact, it would be more accurate to say that his answer ruins his question. Lorenzen takes the output of abstraction to be a mere façon de parler (manner of speaking). This is a consequence of a strong nominalism (in his lectures, Lorenzen would write a class-expression on the blackboard and then would warn the audience not to look for anything “behind” it). In the quoted passage two questions are asked: one is: What are the abstract objects?, the other is: How is the term “abstract” used? The nominalistic answer for the first question is: “nothing”, and for the second question: “as a manner of speaking”.

Either because of the paralyzing effects of nominalism or for other reasons (philosophers at a certain point in their careers may want to concentrate their efforts on new topics), the fact is that a “correct description of the phenomena” pertaining to the nature of abstracta never became one of Lorenzen’s leading concerns. The new singular terms for abstracta that Lorenzen introduces are only apparent, they are just “abstractors”, i.e. reminders that we are committed to abstraction in our discourse about the initial concreta. Thus,

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11 Remembering Husserl’s impressive statement: “die Schwierigkeit liegt in den Phanomenen, ihrer richtigen Beschreibung, Analyse und Deutung” (the difficulty lies in the phenomena, their correct description, analysis and interpretation, Husserl 1891, p.142).

12 (1973), III, §7: Abstraktoren: “Abstraktoren sind nicht Prädikatoren, die wir Gegenständen zusprechen können, sondern lediglich Zeichen, die anzeigen, dass Aussagen in einer bestimmten Weise verstanden werden sollen” (Abstractors are not predicates that we can ascribe to objects, but only signs that indicate that sentences must be understood in a certain way).
the singular terms “[1/2]” or “the rational 1/2” do not denote any new entity but only remind us that they (they? or just the old “1/2”?) can fill the blanks only of predicates that are non-vacuously invariant. Alternatively, we may say that the singular terms “[1/2]” or “the rational 1/2” are totally eliminable in favour of just “1/2” or of any other “m/n” such that l.n = 2.m. Lorenzen’s favourite abstractor was a tilde “~”; thus, to indicate that a singular term “u” is subject to abstraction, he would write “ã”.

This nominalistic approach is wrong. The very statement, in the preceding paragraph, that the new singular terms are “just reminders” or “abstractors” indicating that we are committed to abstraction “in our discourse about the initial concreta”, reveals the incoherence. The truth is that “our discourse about the initial concreta” is a matter of the past – now we are talking (or at least trying to talk) about something else.

In spite of such nominalistic shortcomings, which hinder the proper study of point (4) (in the abstraction program sketched in section (1) of this paper), it must be said, in fairness, that Lorenzen’s contribution with regard to points (1), (2), and (3) (restoration of genuine abstraction against circumspection, and Entpsychologisierung of abstraction), remains outstanding.

6. On the difficulties of not being nominalists (with regard to abstracta)

There is also the other side of the coin: it is easy to complain about nominalism but difficult to build something better, i.e. to produce a good answer to the ultimate question: What is the output of abstraction? Here the phenomena – the abstracta – are hard to describe.

It should be helpful to distinguish a merely practical and a theoretical knowledge with regard to the abstracta. There are prestigious examples of the existence of the former without the latter.

Consider the abstraction that takes place by restricting one’s language on sets to properties that are non-vacuously invariant relative to bijection. One may be quite confident that something definite is attained by that abstraction, and one may give a name to it: “cardinal number”. Nevertheless, one may content oneself and (worse) one’s readers, with a half-way notion and a half-way account of the nature of the output of abstraction, describing the abstractum as “an arbitrary representative” (ein beliebiger Repräsentant) (Kamke 1962, § 8) of sets equivalent with respect to bijection. The author of the classical set-theory textbook I am referring to, Kamke, does not explain what is this notion of an “arbitrary representative”. The notion is, as Locke’s “representative”, on the right track – but needs further analysis, without which the reader wrongly believes that the universe of discourse continues to be the same: sets, rather than quite new entities: abstracta. Kamke, the practising mathematician,
is not interested in illuminating the nature of such *abstracta*, or does not know
how to go about it, or is unaware of the issue. Still, on behalf of Kamke, it
must be said that he does better than his illustrious predecessor, the founder of
set-theory. In fact, Cantor’s definition of cardinal number (quoted in a footnote
to the last section of this paper) cannot be offered as a good example of even
merely practical knowledge of *abstracta*, since the technique of Cantor’s ab-
straction is on the wrong track from the very outset: he suggests that the ab-
straction leading from a set to its cardinality consists in ignoring the differ-
ences among the elements of the set (cf. my 1984).

By “theoretical” knowledge of the abstracta I mean that at least some hints
are offered with regard to the internal nature of abstracta. Peano (1958, append-
dice) appears to say a bit about the interior of abstracta when he writes that
from the fraction 2/3 “otteniamo per astrazione l’idea del razionale 2/3, con-
servando tutte e sole le proprietà, o note, che esso ha comuni coi suoi eguali, e
sopprimendo quelle che lo distinguono” (we obtain by abstraction the idea of
the rational 2/3, keeping all and only the properties, or marks, that it has in
common with its equals, and suppressing those that differentiate it), emphasis
mine.

Is then the *abstractum* (in Peano’s example: the rational 2/3) a bundle of
properties – all those of the (relatively) *concretum* (the fraction) minus the
non-invariant ones? Or, by elaborating on Kamke’s “representative”, and con-
sidering that to take a *concretum* as representative of its equivalents is really to
take a slice of it, is the *abstractum* rather a mutilated *concretum* (amusingly, in
Peano’s example: “a fraction” of a fraction)?

7. A recent criticism of modern abstraction

Siegwart’s recent criticism of “constructive” abstraction – the derivation of a
contradiction – does not apply to modern abstraction (as I prefer to say instead
of “constructive abstraction”) as far as (my version of) Lorenzen is concerned.
Siegwart uses the predicate “*x* is identical to an abstractum” or “*x* is an abstract
entity”, which is only vacuously invariant in the initial domain of concreta –
hence beyond consideration.

Here one may appreciate both the convenience and the narrowness of
nominalism. While the non-consideration of predicates as “*x* is an abstract en-
tity” protects Lorenzen’s modern abstraction from Siegwart, it also reveals
Lorenzen’s self-inflicted, nominalistic unwillingness to recognize, in the ab-
stracta, anything beyond the mere façon de parler (manner of speaking). Not
having crossed the Rubicon – not having faced the problem of the ontology of
the abstracta – the nominalist has played safe, but has lost the opportunity of
conquering the abstract world.
Unfortunately, Lorenzen insisted on “reconstructing” (his favourite term) the predicate used by Siegwart as an Anweisung (indication, reminder), “deconstructing” its predicative content: “Freilich, Sätze wie ‘der Begriff ‘Revolution’ ist ein abstractum’ (ein abstrakter Gegenstand) sind keine solche Prädikationen, sonder wieder nur Anweisungen für den Gebrauch des Wortes ‘Begriff’” (“Surely, sentences as ‘the concept ‘revolution’ is an abstractum (an abstract object) are not such predications, but again only instructions for the use of the word ‘concept’”) (1973, §7, p. 102). In 1977 I had the opportunity of asking Lorenzen what he thought about saying of an abstractum that it is abstract — the question was rather dismissed, and my impression is that it was regarded as a Sophisterei.

Certainly, the predicate “x is an abstract entity” is only vacuously invariant and while doing abstraction should not be kept; but there is something else aside from doing abstraction: to reflect on the abstraction that has been done. Within this reflective context, the non-nominalist ontologist will try to reconstruct predicates similar to “x is an abstract entity” as genuine Prädikationen, not just as wieder nur Anweisungen (again only instructions). Of course, those predicates should not be “eliminable”: being abstract should be true of the abstract entity, not of the concretum from which the abstract entity was extracted (thus, there should not be a reason for the derivation of Siegwart’s contradiction).

Curiously, the suggested reconstruction points in the direction of something reminiscent of medieval semantics, where some statements about an essence (nature, etc.) descend to the individuals (homo est animal), whereas others do not (homo est species, which amounts to saying homo est abstractum). Analogously, in modern abstraction we would have statements $F(\tilde{a})$ of two sorts: some eliminable (that is, $F(a)$ can be asserted), others not (when for instance $F(x)$ = “x is abstract”).

Incidentally, Siegwart appears to believe that there is, in addition to “constructive” (modern) abstraction, a classical theory$^{13}$ (Lehre) of abstraction, which he calls set-theoretical (1973, p. 251). He does not explain what he means by “classical theory of abstraction” but his brief comment suggests that he has in mind the usual move from members of equivalence classes to the equivalence classes. If this is really so, Siegwart must be counted as one more logician among the many who believe that circumspection deserves being called “abstraction”. Once again, it must be emphasized that looking-around or circumspection is not abstraction. As said above, circumspection does, obscurely and unofficially, involve abstraction to the extent that none of the

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$^{13}$ Methode was Lorenzen’s preferred term with regard to abstraction, rather than Lehre or Theorie.
particular chosen entities matter, as long as they are “suitable”, but this only shows that circumspection must be overhauled in terms of modern abstraction, as is done, for example, in Lorenzen (1955) (cf. the preceding section).

8. Current (real and only apparent) attempts to return to the foreground

Outside philosophy, genuine abstraction seems to be a topic increasingly attractive for computer scientists. Even within the logico-analytic tradition there are recent, albeit individual, examples: J. Lear (who practically reaches modern abstraction through the analysis of Aristotle’s reduplicative “qua” statements) and N. Cocchiarella (who discusses Piaget’s “reflective abstraction”\(^\text{14}\)).

However, not all expressions of interest in abstraction correspond to an interest in genuine abstraction. One example is W. Künne, who complains (1982) that in analytic philosophy, while there is much discussion on “abstract objects”, the question: “What is an abstract object?” is almost completely ignored. Such an excellent and promising remark, unfortunately, does not appear to be followed up in the rest of Künne’s work (for instance 1983, 1988), where one misses the consideration of the essential feature of genuine abstraction (“leaving out”, “retaining”).

9. Concluding reflection

From the great masters of scholasticism to the founder of set-theory abstraction has been regarded as a method, a way to reach such entities as universals\(^\text{15}\) or cardinals.\(^\text{16}\) In spite of these awesome historical credentials, and the fact that to do abstraction seems to be such a simple matter, it may turn out that the ghostly and elusive nature of the abstracta finally discourages us from trying to

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\(^{15}\) Eustachius (1616): Ut universalia cognosci possint, necesse est ut per operationem intellectus abstrahantur a suis inferioribus (in order that the universals be knowable, it is necessary that they be abstracted, by the operation of the intellect, from its inferiors), p. 30, quarta conclusio.

\(^{16}\) Cantor (1962): “Mächtigkeit oder Kardinalzahl von M nennen wir den Allgemeinbegriff, welcher mit Hilfe unseres aktiven Denkvermögens dadurch aus der Menge M hervorgeht, dass von der Beschaffenheit ihrer verschiedenen Elemente m und von der Ordnung ihres Gegebenseins abstrahiert wird”. A slightly modified version of Russell’s translation (in 1956) is this: “We call the power or cardinal number of M that general idea which, by means of our active faculty of thought, emerges from the set M, by abstracting from the nature of its diverse elements m as well as from the order in which they are given”. Interestingly, Cantor expresses this double abstraction by writing a double bar on top of the name “M” of the set in question.
fulfil, properly, the fourth requirement mentioned at the beginning of this paper, so that, short of joining the nominalists, no alternative is left but to forget completely about abstraction in philosophy (meeting the fourth requirement seems necessary for a philosophy of abstraction). However, if this happens, one should not continue to use the terminology of “abstraction” in any of the pseudo-senses (basically two: intangibility, looking-around), creating the false impression (as in Dummett or Siegwart) that, after all, abstraction ("good" or "logical" abstraction in Dummett, “classical set-theoretical abstraction” in Siegwart) has a place in one’s theory.

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