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Foreknowledge, Bivalence, Freedom

Freedom, or liberty, or free will, has been threatened, since Antiquity, by the principle of bivalence (PB) as well as by the notion of (divine) foreknowledge. On this logico-theological topic there is a large list of primary sources, from Aristotle’s famous chapter in De Interpretatione through much of the early modern period, and there is also a new, increasingly complex secondary literature. My purpose in this essay is limited to some tentative reflections on a few authors from the primary literature. In (1) the threat to freedom is presented as a twofold dilemma: logical and theological. The main reaction to the logical dilemma has consisted in some restriction on PB; this is examined in (2), where a constructive interpretation of that restriction is outlined. The main reaction to the theological dilemma has been to reaffirm the compatibility of freedom and foreknowledge – not done, however, in a rationally satisfactory way (3). My opinion (of a Cartesian type) is expressed in the concluding remarks (4).

1 The logical and the theological freedom dilemmas

Let us recall that dilemmas are arguments with a basic, «disjunctive» premiss that lists an allegedly exhaustive number of cases (usually two), followed by a number (equal to the number of disjuncts of the basic premiss) of other premises which are conditionals whose antecedents are each of the disjuncts of the basic premiss. The conclusion is the disjunction of all the consequents. Such dilemmas are of course valid arguments, so that given a dilemma it only remains to accept its disjunctive conclusion or to attack its soundness, i.e. to show that at least one premiss is false. Successfully attacking the disjunctive premiss is described as escaping between the horns, and showing that a conditional premiss is false is described as grasping the dilemma by a horn («Cette façon d’argumenter est si dangereuse, que les Anciens l’ont appelé du nom de Belier, pour que par les deux membres d’une proposition disjonctive, comme avec deux cornes, il hante celui qu’il veut perdre», Abillon, p. 166).

1 This style of reasoning is so dangerous that the ancients have called it the ram, because by the two members of a disjunctive proposition, as with two horns, it damages the opponent.
The following diagrams represent the two dilemmas that have emerged in the history of philosophy with regard to freedom and the principle of bivalence (PB), and freedom and divine foreknowledge (F). Each dilemma has three premises: \(\{1, 2a, 2b\}\), followed by the conclusion (3). The conclusions are equivalent to, respectively, «we cannot have both freedom and logic», «we cannot have both freedom and God».

\[
\begin{align*}
\text{PB} & \quad \text{not-PB} \\
2a & \quad 2b \\
\text{no freedom} & \quad \text{no logic} \\
\text{freedom is lost or logic is lost} & \\
3 \\
\text{F} & \quad \text{not-F} \\
2a & \quad 2b \\
\text{no freedom} & \quad \text{no omniscient God} \\
\text{freedom is lost or God is lost} & \\
3
\end{align*}
\]

The history of the impact of the logical version has been far less dramatic than that of the theological version - if one restricts, for the sake of freedom, the use of PB one becomes, at worst, a «deviant» logician, but tampering with divine omniscience is another matter. However, the two dilemmas go hand in hand, and it is not surprising that the traditional strategy of weakening PB in terms of «indeterminate» truth-values (cf. § 2) has been regarded by some as if it was, at the same time, a weakening of divine foreknowledge, adversus fidem nostrum (Rubio, p. 405).

2 Reactions to the logical version of the dilemma

Aristotle and his commentators reject the truth of the right horn (2e), suggesting that a restricted PB does not necessarily ruin logic. The Aristotelian and prevailing scholastic view is, however, far from clear and requires some interpreting.

Aristotle on the one hand asks us to accept \(p \vee \neg p\) as logically true. On the other hand, for \(p = \) future contingent, he says that the disjunction is not to be «divided» (οὐ μένοι διελόντα, De Interpretatione, 19a 29).

For centuries, this twofold Aristotelian gesture has been developed into a theory according to which, for sentences \(p\) about future human events, while the disjunction \(p \vee \neg p\) is true, \(p\) as well as \(\neg p\) are «indeterminately» true (false). Consider, for example, Petrus peccabit (Peter will sin): the totum disjunctum (disjunctive whole) Petrus peccabit aut non peccabit is determinate verum (determinately true). Nevertheless, authors claim that stramnabit contradictoriam per se sumpsit [«Petrus peccabit», «Petrus non peccabit»] non esse determinate verum, aut falsam (either contradictory [Peter will sin, Peter will not sin] taken by itself is not determinately true or false, Rubio, p. 400).

What is an indeterminant truth (falsehood)? It is frustrating to look for a satisfactory answer, even when the terminology is in the hands of a prominent character in the saga of the theological version of the dilemma: Molina. As paraphrased by our contemporary Molinist scholar Freddoso, to be indeterminately true amounts to not being «unpreventably» true (in Molina: On divine foreknowledge, p. 166, n. 4). I fail to understand this mixture of truth and preventability. What does it mean to say that «Peter will sin» is true but not unpreventably so?

One attempt to clarify the muddled notion of indeterminate truth is to read the latter as a third truth-value (Łukasiewicz, p. 126). I will sketch another, in my view better interpretation, which I call constructive.

The central issue is to make sense of the Aristotelian twofold remark: i) \(p \vee \neg p\) is not or cannot be divided, nevertheless ii) \(p \vee \neg p\) is true. Accordingly, one first question concerns the nature of the «dividing»; another, second question has to do with how to understand, and preserve, the truth of the disjunction \(p \vee \neg p\) once the impact of the «non-dividing» on it has been interpreted in one way or other.

i) What does the non-dividing mean? Two senses of the «non-dividing» have to be excluded. A) An epistemic-bivalent sense: I do not divide or cannot divide the disjunction because I do not know which disjunct is true – but I assume that one of them must be true; B) a modal sense: the disjunction \(p \vee \neg p\) is necessarily true but neither \(p\) nor \(\neg p\) are necessarily true. Neither sense, A or B help. Assume sense A. The sentence Petrus peccabit (Peter will sin), regardless of my knowledge, has a truth-value now, which means that the allegedly contingent future event, is already (now) «locked», i.e. it is nonsense to refer to it as contingent. Sense B is of course what one reads in Aristotle's text, which is, however, quite deceiving. What matters, and all that matters is whether now, at this point of time before Peter's sinning, the sentence Petrus peccabit has a truth-value or not. If it has a truth-value, the future event is already (now) locked, and adding the clause «not necessarily» is totally irrelevant, indeed deceiving. Once (A) and (B) are excluded, short of adopting the Łukasiewicz third-value approach, only the following interpretation seems available: C) for sentences about future human events I am not allowed to use the principle of bivalence: I put this law of thought, PB, on hold.

ii) Without PB available, how can one still defend the truth of the disjunction \(p \vee \neg p\)? For one thing, it is forbidden to speculate à la truth-table (there are two possibilities: \(p = T\), \(p = F\), in either case the disjunction comes out T). What does
the disjunction $p \vee \neg p$ still say, once PB has been put on hold? In the following Lorenzen's question we see the answer:\footnote{3}

Soll die Annahme, jede Aussage sei entweder wahr oder falsch – da sie nicht bedeutet, dass diese Disjunction effektiv entscheidbar ist – etwa nur bedeuten, dass keine Aussage zugleich wahr und falsch ist? (§ 2).

The answer to our question is that without PB the traditional insistence on the truth of $p \vee \neg p$ can only mean that this disjunction is not false. For $p \vee \neg p$ to be false, both $p$ and $\neg p$ ought to be false, which violates the principle of non-contradiction, which has not been put on hold. It turns out that $\neg(p \vee \neg p)$ is logically false, and indeed $\neg(p \vee \neg p)$ is logically true. Here, the amazing phenomenon, discovered by Brouwer (§ 1) but not by the previous logical tradition, is that we have good reason to assert $\neg(p \vee \neg p)$ but no reason at all for the assertion of $p \vee \neg p$.

Traditional authors continue to say that _navale bellum erit vel non erit est necessaria simpliciter_ (that a sea-battle will occur or will not occur is necessary simpliciter), i.e., they persist in regarding $p \vee \neg p$ not only as true but even as necessarily true – but why? Because its contradictory _non (navale bellum erit vel non erit), esset simpliciter impossibilis, ut patet per aequivalentem, quae est copulatio facta ex partibus contradictiorum: ut navale bellum erit et navale bellum non erit_ (Niphus, p. 33). Excellent acquaintance with the De Morgan equivalences, but no trace of the Brouwerian insight.

The just proposed, constructive interpretation, meets the challenge of making sense of the twofold Aristotelian claim ($p \vee \neg p$ is not or cannot be divided, yet $p \vee \neg p$ is true) at the expense, obviously, of a departure from the letter of that claim (the above mentioned modal sense B). The disjunction _Petrus peccabit aut non peccabit_, alas, can no longer be asserted; it could be defended only if one knew that Peter will sin or if one knew that Peter will not sin. (The conceivable objection, that in constructive logic the conditional $\neg p \rightarrow p$ is asserted although it cannot be «divided» either, and that this is exactly what Aristotle does with $p \vee \neg p$, is not good: the conditional $p \rightarrow p$ can be defended, namely by producing a proof of $p$ if an opponent has produced a proof of $p$, but nothing analogous exists for $p \vee \neg p$). Within the proposed constructive interpretation only $\neg \neg(p \vee \neg p)$ is available for assertion, on purely formal or logical grounds. To anyone who objects that the constructive interpretation is a departure from the Aristotelian or scholastic texts, the counter-objection is simple enough: some departure is needed, since it is nonsensical to claim a compound sentence, in our case a _totum disjunctum_, to be true without any reason at all.

The proposed constructive interpretation successfully attacks horn (2b) of the logical dilemma. Logical theory is not obliterated by the restriction on PB: the entire Aristotelian categorical syllogistic remains untouched and, moreover, nothing prevents reinstating PB whenever future contingents are not under consideration. Besides, putting PB on hold is a more «cautious» move than creating a third truth-value; it is an «abstention», better reflecting the reality of the limitation of human knowledge (by the same token, the constructive interpretation may be less subject to being seen as _adversus fidem nostrum_, cf. end of § 1). I modestly «abstain», I do not affirm the indetermination of «Peter will sin».

3 REACTIONS TO THE THEOLOGICAL VERSION OF THE DILEMMA

One may anticipate that the acceptance of the truth of all the premises of the theological dilemma should not be popular, since it leads to either reject God (by denying foreknowledge) or freedom: one possible or partial example is Hobbes (3a). Far easier to find are authors who take the theological dilemma to be unsound (3a), and either attack the disjunctive premiss (Aquinas) or intend to refute the left horn (Molina).

3a The theological version regarded as sound (Hobbes?)

As reported by Augustine, Cicero accepted the soundness of the dilemma and was consequently forced to choose between God and freedom: in order to preserve the latter he ended up rejecting the former.

Hobbes is an interesting, difficult case. Some texts can be read as clear statements in favor of the soundness of the dilemma. In fact, the critical left horn (2a) of the theological version is fully endorsed by the author of _Leviathan_ in the following passages: «whatsoever God foreknoweth shall come to pass, cannot but come to pass, that is, it is impossible it should not come to pass ... But whatsoever was impossible should be otherwise, was necessary: for the definition of necessary is, that which cannot possibly be otherwise» (Questions p. 19). Equivalently: if «a man is free to will, it followedeth that the prescience of God is quite taken away» (Questions, p. 17-18). To this extent one may guess that Hobbes, having to accept the conclusion, will end up choosing God and rejecting freedom. There is however a complication, generated by the (at least) two ways of understanding «freedom». As explained by Molina (Concordia, I Disp. 2: _Quid nomine liberi arbitrii intelligendum sit_): what should be understood by the phrase
free will» libertas » means (1) mere lack of coactio (coercion), or (2) that, assuming that all conditions for the action in question are given, one can act or fail to act («do or forbear»). When it is time to define freedom, Hobbes prefers the first sense (not being hindered to do what one wills to do, Leviathan I ch. 21), and even thinks that the second sense is inconsistent (My opinion, thesis 8). To the extent that, as it seems reasonable to assume, the first sense of «liberty» does not destroy (for Hobbes) divine prescience, Hobbes has to be classified together with those who, like Augustine, Aquinas, Molina, reject the soundness of the theological dilemma. Of course, Hobbes being together with, say, Molina, means very little in this connection. Molina would object that Hobbes manages to refute the left horn only because of having adopted an insufficient definition of freedom as «lack of impediments».

3b THE THEOLOGICAL VERSION REGARDED AS NOT SOUND (AQUINAS, MOLINA)

Aquinas confidently grants the truth of the left horn: contingens, ut futurus est, per nullam cognitionem cognosci potest, cui falsitas subesse non possit; unde cum divinae scientiae non subsit falsitas nec subesse possit, impossibile est quod de contingentibus futuris scientiam habereet Deus, si cognosceret ea ut futura sunt5 (De veritatis, q. 2 a. 12). Aquinas rightly believes that foreknowledge locates the future – he is not questioning the left horn. He attacks the disjunctive premise. The last sentence of the quoted passage suggests that God can know future events as future or as something else. The immediately following text explains (à la Boethius) that this something else is to know future events as present. Thus, the list of possible cases to be considered includes a third case: knowledge of what we call future that is not fore-knowledge. There cannot be any temporal sequence of the type «past-future» (ordo praeterit ad futurum: order from the past to the future) in the divine cognition. For the divine vision, which is outside time, there is no future (divinae visionis, quae est extra tempus, futurum non est: for the divine vision, which is outside time, there is no future, ibid.). This is a successful escaping the theological dilemma between the horns. However, an unexpected fact makes the Thomist victory over the dilemma useless. To appreciate this, one should consider the possibility that, at a time t earlier than the time t* at which the allegedly contingent event E occurs, somehow the divine knowledge can be «downloaded» into a human mind, e.g. a prophet. Then, the future event E would become known as future (which is the way humans know future events).

By Aquinas’ own principles, E would no longer be contingent but would become necessary, and freedom would be destroyed. It is correct to reject the disjunctive premise of the theological dilemma: God has no »fore-knowledge. The list of possible cases is not sufficiently described by »there is foreknowledge or there is not foreknowledge«. A third case is to be added: »or there is knowledge of future events that is not fore-knowledge«. In the end, however, prophets make this further distinction useless.

Molina, in the Pars Quarta: De praescientia Dei of his awesome Concordia promises, many times indeed, that he will reconcile divinam praescientiam cum nostris arbitriis libertate remanente contingenti (the divine foreknowledge with our free will and the contingency of things, Disp. 47, 1). Does Molina deliver what he repeatedly promises, namely some acceptable argument against the left horn (2a)? The answer is negative. This is not surprising, given Molina’s declaration that Augustine ex eodem capite quo nos conclutur arbitrii libertatem cum praescientia (reconciles freedom of choice with foreknowledge on the same foundation as we do, Disp. 53, II, 23), and the fact that Augustine fails to offer any reason (aside from faith) to support his emphatic claims (De civitate, V, 9–11, De libero arbitrio) that the left horn is false.

In order to show that the promised concordia remains undelivered, three items that seem to be (rational, not religious) arguments, justifications, or proofs of the concordia have to be cleared away. These items have generated the false appearance that Molina’s rejection of the left horn is backed up by some reasoning independent of faith and revelation. The three items are: the notion of scientia media, the »double-because« argument, and an argument involving modalities. Reviewing these three items shows that they are irrelevant for the desired rational justification of the concordia.

For the appreciation of the scientia media is it necessary to go back to Aquinas, and to introduce a subtle distinction in his attack on the disjunctive premiss. One thing is 1) the sheer statement that »fore-knowledge« is nonsense when applied to God, and another thing is 2) to move one step forward and say that the future is present to God. The latter may be called »presentialism« and is not strictly needed for the purpose of escaping between the horns.

Molina’s war is against Aquinas’ overgrown presentialism, and this war he fights well, and successfully, by bringing in three biblical examples indicating that God knows future contingent events that neither have occurred nor will occur, so that it is nonsense to speak of their being «present» to God rather than future. One of the examples (Disp. 49, 9) is the following: »Woe to you, Chorazin! Woe to you, Bethsaida! For if the mighty deeds done in your midst had been done in Tyre and Sidon, they would long ago have repented« (Matthew 11, 20–24). These events are called futurabilia in the jargon found in scholastic manuals, or perhaps more commonly »conditional future contingents« (futurabilia seu futura libera conditionata, i.e. ea, quae re vera non sunt futura, sed essent, si aliqua conditio,
the present, theological case, what matters, and all that matters is that because of being foreknown by God, the sentence Petrus peccavit is true at a time t earlier than the time in which the event of Peter’s sinning happens. The foreknowledge of Peter’s sinning, quite regardless of Molina’s modal experiments, locks, freezes Peter’s course of action. To use Sartre’s brilliant language, to look for freedom once foreknowledge has been assumed, amounts to chercher du vide dans un récipient qu’on a préalablement rempli jusqu’aux bords (to look for emptiness in a container that one has totally filled up in advance, L’être et le néant, IV, 1, 1, p. 516).

4 Concluding remarks

For the non-believer only the logical version is a concern, and with the constructive interpretation (§.3) there is no further problem. For religious persons, of course, the theological version matters as well.

An intelligent man by the name of Descartes briefly suggests that the issue is to be transferred from natural theology to sacred theology. The title of Principia I, 41: Quamodo arbitrari nostri libertas et Dei praecedentario simul conciliatur (How our free will and God’s foreknowledge can be reconciled), may lead the reader to expect a Cartesian rational argument in favor of the concordia, but this is not the case. Descartes rejects the truth of the left horn of the theological dilemma only on religious grounds: the concordia is just not accessible to our finite minds, all a religious person can do is accept it as one more “mystery of the faith” (e.g. Incarnationis et Trinitatis, Principia I, 25).

Was the (apparent) champion of the rational justification of the concordia, Molina, really and ultimately convinced that he had rationally refuted the left horn of the theological dilemma? At the end of his Disputation 52, right after triumphantly claiming that he has solved the concordia problem, Molina suddenly wonders if his proclaimed solution will be understood by the masses; in fact, he acknowledges that the matters of foreknowledge and freedom are “beyond the understanding of many”: soldiers, farmers, sick people. It is important to notice here that the difference with Descartes is only that the latter says: “beyond everyone’s understanding”. Molina fears that these common humans, not having grasped the complex concordia theory, and falling into the mistake of assuming that all future events are already pre-established, will stop doing their best in their lives. For such masses Molina writes in a different mode: Quare nihil omnino de divina praescientia solliciti iuxta consilium Divi Petri [2 Peter I:10] satagamus ut bonis operibus certam nostram vocationem factamus6 (Disp. 52, 39). He gives as

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6 The *suumuilia* or conditioned free future things, that is those things which actually are not going to happen but would happen if a certain condition, actually never posited, was posited.

7 There is some textual uncertainty about this Augustine passage. The Loeb edition adds a *non*: *neque enim idio non peccat homo…*. The Augustine scholars Patrick Gerard (*Peter*) Walsh and Gerard O’Daly have kindly informed me (Spring 2006) that they prefer to omit the *non*.

8 Therefore, with not a worry at all about the divine foreknowledge, let us, in accord with the advice of St. Peter’s, busy ourselves so that by good works we might do what we are called to do (Freddoso’s translation).
a good example to follow the devil (diabulus), *qui longe melius quam nos novit
devam omnia praecipere, nihil de divina praecognitione sollicitus nullum non movert
lapsium, diligentissime obviamulant et currit terram quarum quem devoret*9. Can
one perhaps suspect that what led Molina to writing such a touching finale for his
Disputation 52 was an occasional doubt, on his part, about the rational provability of
the concordia?

Glances at the most recent literature, not examined in this paper, show authors
who still want to argue – rationally, not à la Descartes – that divine foreknowledge
is compatible with human freedom. I would rather leave, sceptically, the discussion
of such attempts for the Festschrift in honor of Christian Thiel’s 80th birthday.

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Part IV titled *De praecognitione Dei*, pp. 293–405 in Rabeneck, has been translated, with
an introduction and notes, by Alfred J. Freddoso, with the title: *On divine foreknowledge: part
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9 The devil, who has understood far better than we have that God foreknows all things, caring not a
bit about the divine foreknowledge, leaves no stone unturned and carefully roams about and circles
the earth, seeking whom he might devour (Freddoso’s translation).