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FRÉGE'S NOTION OF "Bedeutung"

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The aim of this paper * is to contribute to a better understanding of Frege's notion of Bedeutung. The paper is divided into four parts. In 1 I quote the crucial texts, in 2, I present an analysis of them. In the analysis several problems arise, listed at the end of Section 2. In 3 I propose an interpretation which I apply to solve those problems. Some critical remarks are added in 4 by way of conclusion.

Abbreviations: BP = the first principle of Bedeutung, BP₂ = the second principle of Bedeutung, V = the conjecture (Vermutung) V, RS = the rule of substitutivity of identicals. The letters ρ, q, r are used to abbreviate three statements concerning the relationship among Bedeutung of sentences, truth-value of sentences, and singular terms occurring in sentences. For all these see Section 2. Principle B or the principle of Bedeutung is explained in Section 3.

1. The texts


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1 All the quoted texts in this section are from FREGESUB, pp. 32–36.
A. Ein solcher Satz enthält einen Gedanken (Ich verstehe unter Gedanken [...] dessen objektiven Inhalt [...]). Ist dieser Gedanke nun als dessen Sinn oder als dessen Bedeutung anzusehen?

A3. Nehmen wir einmal an, der Satz habe eine Bedeutung!

A4. Ersetzen wir nun in ihm ein Wort durch ein anderes von derselben Bedeutung, aber anderem Sinne, so kann dies auf die Bedeutung des Satzes keinen Einfluss haben.


A6. Der Gedanke kann also nicht die Bedeutung des Satzes sein.

A7. Vielmehr werden wir ihn als den Sinn aufzufassen haben.


B3. Aber sicher ist doch, dass jemand, der im Ernst den Satz für wahr oder für falsch hält, auch dem Namen “Odysseus” eine Bedeutung zuerkennt, nicht nur einen Sinn;

B4. denn der Bedeutung dieses Namens wird ja das Prädikat zu- oder abgesprochen. Wer eine Bedeutung nicht anerkennt, der kann ihr ein Prädikat weder zu- noch absprechen.

B5. Nun wäre aber das Vordringen bis zur Bedeutung des Namens überflüssig; man könnte sich mit dem Sinne begnügen, wenn man beim Gedanke stehenbleiben wollte. Kämme es nur auf den Sinn des Satzes, den Gedanken, an, so wäre es unnötig, sich um die Bedeutung eines Satzteils zu kümmern;

B6. für den Sinn des Satzes kann ja nur der Sinn, nicht die Bedeutung dieses Teils in Betracht kommen. Der Gedanke bleibt derselbe, ob der Name “Odysseus” eine Bedeutung hat oder nicht.

B7. Dass wir uns überhaupt um die Bedeutung eines Satzteils bemühen, ist ein Zeichen dafür, dass wir auch für den Satz selbst eine Bedeutung im allgemeinen anerkennen und for dern.

B8. Der Gedanke verliert für uns an Wert, sobald wir erkennen, dass zu einem seiner Teile die Bedeutung fehlt.

B9. Wir sind also wohl berechtigt, uns nicht mit dem Sinne eines Satzes zu begnügen, sondern auch nach seiner Bedeutung zu fragen.


B13. Das Streben nach Wahrheit also ist es, was uns überall vom Sinne zur Bedeutung vorzudringen treibt.

B14. Wir haben gesehen, dass zu einem Satze immer dann eine Bedeutung zu suchen ist, wenn es auf die Bedeutung der Bestandteile ankommt; und das ist immer dann und nur dann der Fall, wenn wir nach dem Wahrheitswerte fragen.

B15. So werden wir dahin gedrängt, den Wahrheitswert eines Satzes als seine Bedeutung anzuerkennen.

C1. Wenn unsere Vermutung richtig ist, dass die Bedeutung eines Satzes sein Wahrheitswert ist, so muss dieser unverändert bleiben, wenn ein Satzteil durch einen Ausdruck von derselben Bedeutung, aber andern Sinne ersetzt wird.
C2. Und das ist in der Tat der Fall. Leibniz erklärt gradezu: "Eadem sunt, quae sibi mutuo substitui possunt, salva veritate".

C3. Was sonst als der Wahrheitswert könnte auch gefunden werden, das ganz allgemein zu jedem Satze gehört, bei dem überhaupt die Bedeutung der Bestandteile in Betracht kommt, was bei einer Ersetzung der angegebener Art unverändert bliebe?

D1. Es soll nun die Vermutung, dass der Wahrheitswert eines Satzes dessen Bedeutung ist, weiter geprüft werden.

D2. Wir haben gefunden, dass der Wahrheitswert eines Satzes unberührt bleibt, wenn wir darin einen Ausdruck durch einen gleichbedeutenden ersetzen; wir haben aber dabei den Fall noch betrachtet, dass der zu ersetzende Ausdruck selber ein Satz ist.

D3. Wenn nun unsere Ansicht richtig ist, so muss der Wahrheitswert eines Satzes, der einen andern als Teil enthält, unverändert bleiben, wenn wir für den Teilsatz einen andern einsetzen, dessen Wahrheitswert derselbe ist.

2. Analysis of the texts

In the initial pages of SUB, that is, up to text A1, Frege explains the notion of Bedeutung only for the special case of singular terms: Die Bedeutung eines Eigennamens ist der Gegenstand selbst, den wir damit bezeichnen (p. 30).  

As soon as we reach text A1 we observe that Frege has more ambitious plans: he wants to extend the notion of Bedeutung from singular terms to sentences. In A2 Frege refers to the Gedanke (thought) as the Inhalt (content) of the sentence. The question arises whether the Gedanke is the Bedeutung of the sentence. Any unprejudiced reader, who imagines that Frege is looking for an entity that stands to the sentence in the same relation in which objects stand to their singular terms, will probably guess that the Gedanke is the Bedeutung of the sentence. In A6, however, Frege affirms that this is not possible. He presents this assertion as a conclusion from three premises. The first (A3) is the assumption that the sentence has a Bedeutung. The second premise (A4) says that substitution of words of equal Bedeutung inside a sentence cannot affect the Bedeutung of the sentence. The third premise (A5) claims that the substitution of terms of equal Bedeutung in a sentence does not preserve the identity of the Gedanke expressed by the sentence.

We may ignore the third premise (A5) as well as A7, which have to do with Sinn. The second premise (A4) is the one that is very important for us. Just as it stands, it is a statement about the Bedeutung of sentences. Frege, however, is not supposed to know anything at this point about the Bedeutung of sentences, except what follows from the notion of Bedeutung in general. Thus we have to construe the second premise as an application to the particular case of sentences of some general principle for Bedeutung that Frege has in mind but that he has not written down. Let us call this the first principle of Bedeutung (BP):

> if two expressions E and E' have the same Bedeutung, and E occurs in an expression A(E) which has a Bedeutung, then the result of substituting E' for E, A(E'), has the same Bedeutung as A(E).

Why should we accept BP1, what is its justification or indeed its meaning? Clearly, for a reader who only knows about Bedeutung Frege’s explanations of the Bedeutung of singular terms in the initial pages of SUB, BP1 makes hardly any sense at all. This is our problem 1.

Next we have a series of texts B, whose main objective is to establish the conjecture (Vermutung, cf. texts C and D) that the Bedeutung of a sentence is identical to its truth-value. Frege’s presentation of his reasoning is not up to his reputation; some benevolent reconstruction

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8 BP, reappears in other writings: Die Bedeutung eines Satzes muss etwas sein, was besteht bleibt, wenn einer seiner Teile durch etwas Gleichbedeutendes ersetzt wird (Nachlass I, p. 251); wenn man in einem Satze oder Satzteile einen Bestandteil durch einen gleichbedeutenden [. . .] hat der abgeänderte Satz oder Satzteil dieselbe Bedeutung wie der ursprüngliche (Nachlass I, p. 276); wenn der Gedanke die Bedeutung des Satzes wäre, so änderte er sich nicht, wenn einer seiner Teile ersetzt würde durch einen anderen Ausdruck von derselben Bedeutung (Nachlass II, p. 235); die Bedeutung des Satzes muss etwas sein, was sich nicht ändert, wenn wir ein Zeichen durch ein anderes ersetzen, das dieselbe Bedeutung [. . .] hat (ibid.); da man in den Bedeutungen ihrer Bestandteile vollkommen übereinstimmen, müssen auch sie dieselbe Bedeutung haben (Nachlass II, p. 245).
is needed. I propose the following. Let us consider these three statements about an arbitrary sentence $s$:

$p$) every singular term in $s$ has a *Bedeutung*,

$q$) the sentence $s$ has a truth-value,

$r$) the sentence $s$ has a *Bedeutung*.

In B1 Frege says that $r$ does not necessarily hold of all sentences: there may be sentences without *Bedeutung*—just as there are singular terms without it. In B2 we find (1) $\neg p \rightarrow \neg r$. In B3 we are told that (2) $q \rightarrow p$, which is supported by the remark in B4 that there can be no prediction if no object is there. B5 may be analyzed as (3) $\neg r \rightarrow \neg p$. Leaving aside B6, which has to do with *Sinn*, we find in B7: (4) $p \rightarrow r$. B8 is vague—what does *Wert* mean? *Wahrheitswert* or *Erkenntniswert*? ⁴ In B9 Frege feels that he can draw the conclusion that it is justified to look for the *Bedeutung* of sentences. In B10 we have two questions: (i) why do we want *Bedeutung* of singular terms? (ii) why do we want *Bedeutung* of sentences? Both questions are answered at once: we want *Bedeutung* exactly to the extent that we are concerned with the truth-value of the *Gedanke*. This might be expanded into the following: (5) $p \leftrightarrow q$, (6) $r \leftrightarrow q$.

In B11 it is pointed out that we are not always interested in the truth-value of sentences. When we are not, B12 tells us that we are not interested in the *Bedeutung* of the singular terms either: (7) $\neg q \rightarrow \neg p$. All this indicates that there is a close relationship between truth and *Bedeutung* (B13). B14 summarizes in two parts the preceding considerations: (8) $p \rightarrow r$, (9) $p \leftrightarrow q$. Apparently, the conditional $r \rightarrow p$ is left out.⁵ Finally, B15 formulates the thesis that the truth-value of a sentence is the same as its *Bedeutung*. We are forced (gedrangt) to accept this thesis, if not as a demonstrated truth, in any event as a conjecture, *Vermutung*, cf. texts C and D. The only reason why Frege continues his paper beyond

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⁴ The two words occur, for example, in the last two paragraphs of SUB.

⁵ Curiously, in Nachlass I, pp. 250–251 Frege repeats the same reasoning of our texts B1–B15 and again seems to overlook the $r \rightarrow p$ half of $p \leftrightarrow r$, although it is obviously needed: Wenn es uns also darauf ankommt, dass der *Eigenname* *Ae* etwas bezeichne, wird es uns auch auf die *Bedeutung* des ganzen Satzes ankommen. Dass der Name *Ae* etwas bezeichnet, ist uns aber immer dann und nun dann von Wert, wenn es uns auf die *Wahrheit* im *wissenschaftlichen Sinne* ankommt. Eine *Bedeutung* wird also unser *Satz* dann und nur dann haben, wenn der in ihm ausgedrückte *Gedanke* wahr oder falsch ist. These three sentences are $p \rightarrow r$, $p \leftrightarrow q$ and $r \leftrightarrow q$, respectively. The last is presented as a conclusion from the first two, which obviously requires the additional $r \rightarrow p$.

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⁶ B15 is to increase the plausibility of this conjecture, to which I will refer as conjecture V.

In analyzing the sequence B1–B15 we cannot fail to recognize in the biconditional $p \leftrightarrow r$ an application, to the particular case of sentences, of another general principle of *Bedeutung* which Frege, again, takes for granted rather than presenting properly. This second principle of *Bedeutung* may be called the principle of *existence* of *Bedeutung*: all the expressions occurring as parts of a complex expression have a *Bedeutung* if the complex expression has a *Bedeutung*.⁶ No reader of SUB can be expected to make any sense of this principle on the basis of Frege’s explanations concerning the *Bedeutung* of singular terms at the beginning of the paper (problem 2).

There are further difficulties in the sequence B1–B15. First, the conjecture V itself is hard to understand. For readers who, again, about *Bedeutung* only know that the *Bedeutung* of a singular term is the object denoted by it, it is surely very awkward to learn that the *Bedeutung* of a sentence is its truth-value (problem 3). To the reader’s distress, Frege insists elsewhere that the truth-value is the *Bedeutung* of the sentence just like (ebenso wie) for example number 4 is the *Bedeutung* of ‘2+2’ or London is the *Bedeutung* of ‘the capital of England’ (FUB, pp. 13, 16 ⁷).

Additional difficulties in the sequence B1–B15 are also related to the conjecture V. How is V suggested by texts B1–B14? The closest to V in these sentences is the biconditional $r \leftrightarrow q$. Thus it seems appropriate to view the entire sequence B1–B14 as oriented towards the establishment of $r \leftrightarrow q$ as a conclusion from the two premises $p \rightarrow r$ and $p \rightarrow q$.⁸ But why should $r \leftrightarrow q$ suggest, let alone imply V? (problem 4). Moreover, why a mere conjecture and why not a full assertion or stipulation? If V has to be proved, how can it be proved? (problem 5).

Let us now approach texts C and D. Each of them represents an attempt to confirm the conjecture V by showing that a sentence implied by V is true.

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⁸ This trend of reasoning may be observed in other passages, such as, for example, Nachlass I, pp. 250–251 (see footnote 5). Related texts are in Nachlass II, pp. 235, 240, 247, Nachlass I, pp. 210–211.
In C1 Frege asserts a conditional whose antecedent is the conjecture V and whose consequent is the thesis that singular terms of equal _Bedeutung_ are interchangeable _salva veritate_. In C2 Frege claims that this thesis is true. The thesis is often formulated as a rule: the rule of 'substitutivity of identicals' (cf. _Angelelli_, 1976). Let me use the abbreviation RS to refer to the thesis or to the rule, as the context may require. Thus the conditional asserted by Frege in C1 is: V → RS. Because of believing that RS is true (C2), for Frege the conditional V → RS is true, but this makes it only 'materially' true. By adding the principle BP1, we obtain the logically true conditional: BP1 ∧ V → RS. BP1 says that the substitution of singular terms of equal _Bedeutung_ does not touch the _Bedeutung_ of the sentence in which they occur; RS says the same, except that it has 'truth-value of the sentence' instead of 'Bedeutung of the sentence'; V secure the identity of truth-value and _Bedeutung_ of sentences.

In C3 Frege states his belief that there is no other entity available to perform the duties of _Bedeutung_ of a sentence apart from the truth-value of the sentence.

In D1 Frege announces his plan of testing V a second way. D2 restates RS while pointing out that we have not yet considered the case where sentences are substituted for one another. In D3 we have a conditional again: if the _Bedeutung_ of a sentence is identical to its truth-value, then substitution of a sentence by another of same truth-value inside a compound sentence should not change the truth-value of the compound.

As it stands, this conditional is not logically true. Addition of BP1 to the antecedent gives a logically true conditional: the consequent becomes an instance of BP1 if 'truth-value' is replaced by 'Bedeutung', i.e. if the conjecture V is used. The remaining pages of Frege's paper aim at proving if not the truth at least the plausibility of the consequent (mit hinreichender _Wahrscheinlichkeit_, p. 49).

To sum up, let me list the difficulties encountered in the analysis of the texts: (1) How can we make sense of BP1? (2) How can we make sense of BP2? (3) How can we understand the Fregean doctrine that truth-values stand to sentences like objects to singular terms? (4) How does r → q help to conjecture the identity of truth-value and _Bedeutung_? (5) Why does Frege restrict himself to a mere conjecture and how is the conjecture to be proved?

### 3. Interpretation

As a first step in the interpretation I will distinguish two meanings of the term 'Bedeutung': (i) _Bedeutung_ in the semantical sense, semantic _Bedeutung_ (for example, the _Bedeutung_ of the singular term 'Caesar' is the object Caesar), (ii) _Bedeutung_ in the sense of importance, _Bedeutung_-importance (for example, the _Bedeutung_ of Caesar is enormous). Of course, in the case of _Bedeutung_-importance one has to explain relative to what is the importance understood. 9

The distinction is not made by Frege when he uses the word technically (from the time of SUB on) but the two meanings are recognizable in his pre-technical use of the term. 10 To show this, as well as to suggest how the Fregean notion of _Bedeutung_-importance is to be approached, I will quote a few texts from the _Begriffschrift_:

(1) _Deshalb ist auf den Ausdruck alles dessen verzichtet worden, was für die Schlussfolge ohne Bedeutung ist. Ich habe das, worauf allein es mir ankam, in § 3 als begrifflichen Inhalt bezeichnet (EG, Vorwort). 11_

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10 Frege refers to SUB as the first paper where the term 'Bedeutung' occurs in a technical sense (Nachlass II, p. 41, p. 96).
11 For a related text cf. Nachlass I, p. 37, footnote: _eine Verschiedenheit nur dann einen logischen Wert hat, wenn sie die möglichen Schlussfolgerungen berührt._
FREGE’S NOTION OF "BEDEUTUNG"

he used the word as a technical term (from SUB on) it seems convenient to make the abstraction that leads to Bedeutung-importance relative to the relation of interchangeability salva veritate rather than to ‘behaving equally in inferences’. In other words, it seems convenient to conceive the Bedeutung-importance specifically as importance for truth rather than as importance for inferences. This choice not only seems to work but it is justified by text (4), at least in the sense that in text (4) Frege takes equal importance for logical inferences to entail equal importance for truth (substitutivity salva veritate).  

Let me then propose, as starting point of my interpretation, the following statement: The Bedeutung-importance of an expression $E = \text{the Bedeutung-importance of an expression } E' \text{ if } E \text{ and } E' \text{ are interchangeable salva veritate. I will call this the principle of Bedeutung, briefly principle B.}$  

Although principle B is suggested by the Begriffsschrift, it does not seem to occur in Frege’s writings. Still, it is a very Fregean principle. To show this, let us represent its general form by $\bar{a} = B \text{ iff } a \sim b$, where $\sim$ is a relation defined on the domain of objects $a, b, \ldots$. If the objects are predicates and $\sim$ is the relation of being true of the same individuals, then $a$ is the Wertverlauf of $a$ and the principle becomes Axiom V of Grundgesetze. If the objects $a, b, \ldots$, are again predicates but now $\sim$ is the relation of being gleichzahlig (equinumerous, GRL, § 68), then $\bar{a}$ is the number of $a$ and the principle becomes another fundamental thesis of Frege’s philosophy (GRL, § 62 ff).  

In the given formulation of principle B I have not made the interchangeability salva veritate relative to any specific set of sentences;  

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13 Frege describes what is relevant to logic not only as ‘what matters for inference’ but also as ‘what has to do with truth’: Die Logik betrachtet ihre Gegenstände sofern sie wahr sind, Nachlass I, pp. 2, 3. In GED: so weist ‘wahr’ […] der Logik die Richtung (p. 58). One might speculate that the two characterizations of logic in terms of inference and in terms of truth amount to the same, given that good inferences are defined as (formally) truth-preserving. Frege himself, for example, views the active-passive transformations both as not affecting inferences (BG, text (2)) and as not touching ‘what is true or false’ (GED p. 64).  

14 Principle B is certainly akin to Frege’s principle that words have Bedeutung only in context (GRL). In ANGELELLI, (1967), 2.7, last paragraph I have referred to the relationship between this principle and the so-called “definitions by abstraction”.  

15 This relativity is exemplified by Leibniz, when he observes, in connection with the terms ‘triangulum’ and ‘trilaterum’ that in omnibus propositionibus ab Euclide demonstratis de Triangulo substitui posset Trilaterum et contra salva veritate (p. 236).
Frege, in fact, envisages the totality of language; his notion of Bedeutung is, so to speak, 'absolute'.

Principle BP₁, read with 'Bedeutung-importance' becomes trivially true under the assumption of principle B. Referring to the formulation of principle BP₁ in Section 2, suppose that the expressions E and E' have the same Bedeutung and assume principle B. Consider any statement C(A(E)) in which A(E) occurs; it will have the same truth-value as C(A(E')) by principle B. But this amounts to saying that A(E) and A(E') have the same Bedeutung as well, because of C being any (solution of problem 1).

In principle BP₂, the phrase 'having a Bedeutung' means the same as 'standing in the relation of interchangeability salva veritate with some expression'. In other words, an expression E has a Bedeutung iff there is an expression X such that X and E are interchangeable salva veritate. Accordingly, principle BP₂ becomes the following: all the expressions that are parts of a compound expression belong to the field of the relation of interchangeability salva veritate iff the compound expression belongs to that field.

Atomic expressions do not belong to the field of the interchangeability salva veritate relation just by virtue of their 'shape'. To be admitted, they must fulfill certain conditions. Atomic singular terms must have a semantic Bedeutung, otherwise any sentence in which they occur lacks a truth-value (cf. text B4) and no substitution salva veritate is possible. 'Odysses', for example, is not admissible, it is not interchangeable salva veritate with any expression (of course, not even with itself), it lacks Bedeutung (importance), because it lacks semantic Bedeutung. Atomic predicates must have the property of being 'sharply defined' (scharf begrenzt, for every object x, either x falls under the predicate or not), otherwise any sentence in which they occur lacks 'Sinn' (GRG II, pp. 69-70), hence evidently cannot have a truth-value and again there is no possibility of substitution salva veritate. The predicates 'Haufe' ('heap', BG, p. 64) and

16 Cf. his remarks towards the end of SUB, where he says in connection with the difficulties involved in the second test of the conjecture V: es ist schwer, alle in der Sprache gegebenen Möglichkeiten zu erschöpfen (p. 49). An exceptional reference to interchangeability relative to specific sets of sentences ('mathematical compounds') in GEFFGE, last page.

17 A possible hint at a relative Bedeutung in HUSS, pp. 319-320: Für den Mathematiker, who is interested in die Sache selbst in die Bedeutung der Worte, equiestensional concepts are 'equal'.

'Christ' ('christian', GRG II, p. 69) seem to be for Frege examples of non-admissible predicates. To be admissible (zulässig), i.e. to be bedeutsamvoll, a predicate must be sharply defined (GRG II, p. 77). I would add (without textual proof) that these necessary conditions on atomic expressions (singular terms and predicates) are also sufficient in order to have a Bedeutung-importance.

Frege, in connection with his system of Grundgesetze, assumes that the atomic expressions are bedeutsamvoll, have a Bedeutung-importance or, equivalently, in our interpretation, that they are interchangeable salva veritate with some expression. Having assumed this, he concentrates on the proof of the left-to-right half of BP₂, namely that the rules of formation of his system preserve the property of having a Bedeutung-importance (GRG I, § 28-32). The other half of BP₂, i.e. if a component lacks Bedeutung, then the compound lacks Bedeutung as well, follows from the existence of Bedeutung for atomic expressions in conjunction with the demonstrated left-to-right half of BP₂: atomic components have Bedeutung; compound components, if well-formed, have Bedeutung too.

While these considerations represent a clarification of the meaning of BP₂ (solution of problem 2), the truth of BP₂ depends of course on the particular system relative to which it is claimed; for example, Frege's attempt to prove it for Grundgesetze has been regarded as unsuccessful (cf. THEIL, 1965).

Our clarification of the meaning of BP₂ requires taking 'Bedeutung' in the importance sense. This seems to create a new problem concerning the biconditionals p ↔ r, q ↔ p from texts B1–B15, used by Frege to infer r ↔ q. The biconditional p ↔ q, to be intelligible, needs 'Bedeutung' in the semantic sense (it is because of 'Odysses' standing for nothing that no sentence with 'Odysses' is true or false) but p ↔ r, insofar as it is an instance of BP₂ must have 'Bedeutung' in the importance sense, both in r and in p. Thus the argument p ↔ r, q ↔ p therefore r ↔ q seems to lose the uniqueness of its 'middle term' p. This difficulty is overcome by considering that Frege 'identifies' (in a sense to be explained below) the Bedeutung-importance and the semantic Bedeutung of singular terms.

Before moving to the solution of the remaining problems I would like to mention another feature of the notion of Bedeutung-importance as explicated by principle B. Two expressions cannot have the same Bedeutung-importance if their mutual substitution does not preserve wellformedness (this is an obvious prerequisite for the preservation of truth). It is not clear that anything analogous should necessarily apply to semantic Be-
Deutung. Thus, the following text, obscure and unconvincing with semantic Bedeutung, becomes perfectly intelligible with Bedeutung-importance:

Die Worte 'der Begriff Quadraturwurzel aus Vier' verhalten sich [...] in Hinsicht auf ihre Ersetzbarkeit wesentlich anders als die Worte 'eine Quadraturwurzel aus Vier' [...] d.h. die Bedeutungen dieser beiden Wortverbindungen sind wesentlich verschieden (BGGE, p. 201).

The solution of the problems (3), (4) and (5) requires some further speculation on the nature of the principle B. We have reached the latter starting from the hints given in the Begriffschrift. According to these, the Bedeutung-importance of an expression is an abstractum and the principle B is a theorem about abstract entities. Unfortunately, this is not the way in which Frege conceives and makes use of statements of the form of principle B, after the Begriffschrift and within the program of what he calls eine sehr ungewöhnliche Art der Definition (GRL, § 63). 18

Let me again represent statements of the form of principle B by the schema $a = b$ iff $a \sim b$. Frege proceeds in two stages; I have referred to this procedure (sometimes wrongly called 'definition by abstraction') as the 'looking around method' (ANGELELLI, 1979). First, Frege stipulates (GRL, § 62-67) $a = b$ iff $a \sim b$ without assigning any denotation to the singular terms 'a', 'b', ... Secondly, he looks around (CARNAP's phrase, 1956, p. 1) for entities that are suitable to become such a denotation — suitable in the sense of being compatible with the stipulation made in stage one. All suitable entities are equally eligible to become denotata of the symbols 'a', 'b'.

Frege does not emphasize, as Carnap does, the freedom of choice of entities, limited only by the requirement of being 'suitable'. He chooses the so-called 'equivalence classes' in the case of number (GRL, § 68) and perhaps in general (Nachlass II, p. 195-196). It is clear, however, from his considerations in GRL, § 69 that there is for him nothing necessary about the equivalence classes. As a matter of fact, all he wants from the equivalence classes is that they satisfy the general condition $a = b$ iff $a \sim b$; other properties of the equivalence classes are rather a hindrance (GRL, § 69, especially last paragraph). Moreover, he remarks, in con-

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18 Surely GRG II, § 146 may be read again in the sense of abstraction. But I do not think Frege has been sensitive to this. He heard of 'abstraction' from PEANO (1896, Nachlass II, p. 192) but in his reply he insists on choosing the equivalence class, i.e. he continues to proceed according to the looking-around method (pp. 195-196).
**Bedeutung** of singular terms) are suitable to perform the duties of **Bedeutung**-importance of singular terms (solution of problem 3).

The significance of the biconditional $r \iff q$ relative to the conjecture $V$ is obvious; it assures Frege that the truth-values are available exactly when they are needed, if Frege wants them to play the role of **Bedeutung** of sentences (solution of problem 4). Naturally, $r \iff q$ does not yet secure the suitability of the truth-values; it remains to be established that the truth-value of a sentence $s$ = the truth-value of $s'$ iff $s$ and $s'$ are interchangeable salva veritate. While from right to left there is no problem, the left-to-right part is far from evident, especially if claimed with respect to the indefinite totality of ordinary language. This is what keeps Frege from asserting (at least at the beginning) the identity of truth-value and **Bedeutung** of sentences. Frege has to prove the left-to-right part of the suitability condition. The attempt to do this makes up the largest portion of SUB (solution of problem 5).

The just mentioned left-to-right conditional is the consequence of the conditional used by Frege in texts D for a second test of his conjecture; let us abbreviate it by "$T$". A proof of $T$, or an argument proving that $T$ is highly plausible (the latter is what Frege thinks of his own achievement towards the end of SUB) has of course a much weaker effect within the second test of the conjecture planned by Frege in texts D than within the looking-around method. Within the second test, the truth of $T$ does not establish the truth of the conjecture, but only confirms the conjecture to the extent that something logically implied by the conjecture is shown to be true. Within the looking-around program, the truth of $T$ is sufficient to establish the suitability of the truth-values (suitability relative to principle B) for being **Bedeutung** of sentences. It might be counted as an objection against our interpretation of Frege in the sense of the looking-around method the fact that he ‘wastes’ the power of $T$ (at least in the paper SUB) by using $T$ within the second test rather than for a proof of the suitability of the truth-values. It must be admitted that there is an oddity here. I do not think, however, that this is sufficient to disprove that Frege proceeded according to the looking-around method. By the time of writing SUB, it was only a few years that he had first referred to the method in GRL. He was still breaking new ground; the looking-around method could not be as clear to him as it was to CARNAP in **Meaning and necessity** (1956).

The distinction of two **Bedeutungen** in conjunction with the derivation of BP, from B leads us to question the previously asserted logical truth of the conditional BP, $\land V \dashv RS$ (from text C1). BP, says that expressions of equal **Bedeutung** may be interchanged without touching the **Bedeutung** of the whole. Now the phrase ‘equal **Bedeutung**’ has to be read here in the sense of **Bedeutung**-importance, because of our interpretation of BP, as derived from B. But the principle RS seems to refer to equal **Bedeutung** in the sense of singular terms standing for, denoting the same object, which is semantic **Bedeutung**. If BP, is read with **Bedeutung**-importance and RS with semantic **Bedeutung**, the conditional BP, $\land V \dashv RS$ seems to be no longer logically true. The difficulty, however, is removed by taking advantage of the ‘equivalence’ of semantic **Bedeutung** and **Bedeutung**-importance in the case of singular terms, which constitute the category of expressions intended by Frege in text C1 (as opposed to D1–D3, where the substituted expressions are sentences).

After basing the interpretation of Frege’s notion of **Bedeutung** upon a distinction of two meanings of the word, the issue must be faced of why Frege never mentions the existence of these two meanings. I would like to make the following comments on this.

(a) It is hardly believable that Frege has not been aware of the ambiguity of the German word as used by him for example in the Begriffsschrift. Consider the following remarks, in a letter to Russell (who had previously complained about the awkwardness of sentences denoting truth-values 'just like' names denote objects, Nachlass II, p. 233): *jeder wahre Satz durch jeden wahren Satz unbeschadet der Wahrheit ersetzt werden kann, und ebenso jeder falsche durch jeden falschen. Und damit ist gesagt, dass alle wahren Sätze dasselbe bedeuten oder bezeichnen und ebenso alle falschen Sätze* (Nachlass II, p. 247). Undoubtedly ‘bedeuten’ is here of the importance-for-truth type: Frege cannot have failed to realize that such a ‘bedeuten’ is not the same as ‘bezeichnen’.

(b) Frege may have aimed at fusing the two meanings into one, as revealed by the just quoted text: *bedeuten oder bezeichnen*.

(c) Frege’s silence with regard to the two meanings is not really surprising. Frege’s theory of **Bedeutung** does not contain, in fact, any actual discrepancy between the two meanings. In the case of singular terms there is coincidence: the semantic **Bedeutung** may even play the role of **Bedeutung**-importance. In the case of sentences Frege really considers only one of the two meanings, the **Bedeutung**-importance, so that no discrepancy
can arise. In the case of predicates the situation is more involved. Predicates *bedeuten* concepts,\(^1\) predicates also *bedeuten* extensions, as we saw, but concepts and extensions are not the same thing for Frege.\(^2\) The second ‘*bedeuten*’ is of the importance type. If the first ‘*bedeuten*’ is of the importance type too, we have two kinds of *Bedeutung*-importance and a second, alternative version of principle B is needed. If the first ‘*bedeuten*’ is semantical, then we have a conflict between the two meanings of *Bedeutung*. Neither situation, however, ultimately becomes fully real for Frege, because of his peculiar doctrine ruling out the possibility of talking of ‘the’ *Bedeutung* of predicates.

4. Conclusion

While solving the problems raised by the analysis of the texts, the notion of *Bedeutung* in the proposed interpretation turns out to be disappointing. What is the *Bedeutung* of an expression? The answer is: *anything, any* entity that is ‘suitable’ relative to principle B.\(^3\) Indefinitely many entities may be suitable to play the role of *Bedeutung* of an expression. Frege is aware of the possibility of indefinitely many choices in connection with *Wertverlauf* (GRG I, §10). Curiously, with regard to *Bedeutung* of sentences he seems to expect that the truth-value of the sentence is the unique suitable candidate (text C3 as well as later writings\(^4\)). But just the opposite of the truth-value of the sentence would be equally suitable. Now, given that there are many candidates, *why should* one in particular be chosen rather than any other? No sufficient reasons *can* be offered here by the looking-around method. At best, reasons of ‘convenience’ or ‘beauty’ may be mentioned (Quine, 1963, p. 152). This is the essential defect of the looking-around method, hence of Frege’s notion of *Bedeutung* in the proposed interpretation. It might be replied that from the point of view of the method no particular entity matters *per se* but only *to the extent* that it complies with principle B. This, however, only shows that the method badly needs being reconstructed in terms of abstraction, as originally suggested in the Begriffsschrift.

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\(^1\) *Ein Begriffswort bedeutet einen Begriff* (Nachlass I, p. 128); *der Begriff ist Bedeutung eines grammatischen Prädikats* (BGGE, p. 193, footnote); *diese Worte bedeuten einen Begriff* (BGGE, p. 194); *das Wort ‘Planet’... bezeichnet einen Begriff* (UGG, p. 308).

\(^2\) Ich meine hiermit nicht, dass *Begriff* und *Begriffsumfang* dasselbe sind (HUSS, p. 320).

\(^3\) Cf. Quine (1963), p. 209: "A cardinal number simpliciter is anything that is $x$ for some $x$". See also p. 152.

\(^4\) Nachlass I, p. 211: *Das einzige aber...*; Nachlass II, p. 240: *was kann das Anderes sein, als der Wahrheitswert?*