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Studien zu Frege II
Logik und
Sprachphilosophie

Studies on Frege II
Logic and
Philosophy of Language

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21. Friends and Opponents of the Substitutivity of Identicals in the History of Logic

1. The different approaches to substitutivity

In order to fix our terminology, let us call the following the "rule of substitutivity of identicals" (RS): if the sentences "a is the same as b" and "A (a)" are true, then "A (b)" is true as well. The letters a and b represent proper names or descriptions, A (a) is a context (sentence) in which a occurs, and A (b) results from A (a) by substitution of b for a at one or more places. This brief reference should be sufficient for the aims of the present paper, except that I would like to emphasize (1) that RS is not an ontological thesis about things and their so-called properties but a rule for substitution of singular terms in linguistic expressions, (2) that there are no restrictions at all on RS, that is, the capital letter A represents any context from the given language (Greek, Latin, German...).

There is a deceptive similarity between our RS and some formulas or rules of inference from the predicate calculus with equality ( Pence). This requires some special comments. Consider the rule of inference in PCE: a = b ⇒ A (a) → A (b), where now a and b are object variables and A (a) represents any formula in which a occurs, while A (b) results from A (a) by replacing b for a at one or more places. This rule seems to coincide exactly with our RS, except for the greater abstraction of having formulas instead of "real" sentences. Insofar as the PCE rule is sound or truth-preserving, one might

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"The essentials of this paper were presented in a meeting of the Sociedad Argentina de Analisis Filosofico (Sadaf) in Buenos Aires, August 1972 as well as in a seminar at The University of Texas at Austin in Fall 1974.

1 In recent discussions (cf. Cartwright [1], Linsky [2]) a so-called principle of identity has been brought forward in contrast with the rule of substitutivity. The principle of identity talks about things and their properties if two things are [not two but] the same, then they have exactly the same properties. This is appealed to as a position safer than the untenable RS and as somehow revealing the source of the fact that people have taken RS to be plausible.
imagine that RS is equally correct. However, the soundness of the PCE rule is relative to the semantics usually chosen for the PCE, according to which one replaces the two object variables a and b by one single symbol, say \( \sigma \), representing the object \( \alpha \) from the universe of discourse under consideration. At this point, the conclusion of the PCE rule really becomes a tautology: \( A(\sigma) \rightarrow A(\alpha) \). No wonder that the PCE rule is truth-preserving. But the fine point about RS is totally missed, namely that pace the asserted identity \( a = b \), the move from \( A(\sigma) \) to \( A(\alpha) \) does involve some change — innocuous and purely notational as it may appear to be.

For this reason, it is convenient to abstain from referring to RS when one is talking about PCE formulas as well as from referring to PCE formulas when one is talking about RS. Examples of the contrary are frequent in the literature. Thus, we find in Kalish-Montague [1] the following comment: „The pattern of inference corresponding to T308 \( [x = y \rightarrow (Fx \leftrightarrow Fy)] \) is essentially Leibniz’ principle of the indiscernibility of identicals […] which asserts that if two things are identical, then anything true of one is also true of the other“ (p. 223). The authors themselves had already taken care to imply that T308 is not what the text just quoted claims, that is, not the grand principle allowing substitution in any context („anything true of one is also true of the other“). In fact, a few pages earlier, the class of sentences that we can replace for the „P“ in T308 had been drastically reduced. The reduction was such that it is not surprising that T308 „works“: „At this point we must relinquish consideration of a large class of English formulas — those we shall call nonextensional. An English sentence \( \varphi \) is said to be extensional if, whenever a name occurring in it is replaced in one or more occurrences by another name designating the same object, the resulting sentence has the same truth-value as \( \varphi \)“ (p. 215). An example of the converse kind is in Rescher [2], where the statement of the Leibnizian principle \( \text{Eadem sumt quorum unum in alterius locum substituti potest salvus veritate} \) is accompanied by the following footnote: „The principle so formulated might be construed either as the thesis \( x = y \equiv (\varphi) \ [\varphi x \equiv \varphi y] \) or as the rule if \( x = y \) then from \( \varphi x \) we may infer \( \varphi y \), for any \( \varphi \), and conversely“ (p. 8).

In earlier publications I have shown that Aristotle regarded RS as a fallacy, that Leibniz restricted RS (so that it is not correct to keep referring to RS as „Leibniz’s law“) and that Frege’s initial insights into identity were such that his acceptance of RS is rather unexpected. In this essay I intend to look more generally into the history of RS.

2 To my knowledge, Aristotle’s rejection of RS as a fallacy and Leibniz’s restriction of the so-called „Leibniz’s law“ have been first pointed out in Angelelli [1] (date of acceptance by the Journal: March 1965). Mates [1] adds a fourth text to the three passages showing Leibniz’s restriction in my [1]. On „Leibniz’s law“ cf. also Curley [1], Kappi [1] and [2], Feldman [1]. I have also referred to this topic in my [2], ch. 2.

The various subgroups of (1) represent the „friends“ of RS. This „friendship“ obviously has to do with how one feels about the philosophical or intuitive
significance of RS and it does not imply that one accepts RS in practice — members of (1a) are "friends" of RS even though they do not accept RS in practice. Those in (1b) are "critical friends" while those in (1a) are "naive friends". Regarding (2), its members are "non-friends" of RS, but to members of (2a) we may apply the stronger label of "opponents" of RS. The lack of intuitive appreciation of RS, characteristic of group (2), does not exclude that in practice some restricted version of RS is accepted, and it may happen that this restricted version coincides with what members of (1a) have or want to settle for. But even in this case I want to maintain the distinction between the two groups. Also, I want to distinguish between an analysis of language that reveals that failures of RS are only apparent made from the standpoint of (2) and the same analysis made from the standpoint of (1b). Let us now try to find examples of these groups in the history of logic.

2. Non-friends of RS

In Quine's *Word and Object* there is a suggestion that both Aristotle and Aquinas were friends of RS, in the sense that both have stated and apparently accepted our rule:

On the other hand Aristotle had the matter straight: things are identical (ταὐτά) when "whatever is predicated of the one should be predicated of the other" (*Topics*, Bk. 7, Ch. 1, [...]). Aquinas says the same, *Summa Theologica*, Part I, question 40, art. 1. 3. Cf. Peano, *Opere Scelti*, vol. 2, pp. 258, 417, whence these references, (Quine [2], p. 116.)

Let us begin with Aristotle. There is no doubt that the passage from *Topica* represents a clear, apparently unrestricted statement of our RS. In the plan of discussing how sentences of the form a = b can be disproved, Aristotle tells us that "one ought to be on the look-out for any discrepancy anywhere in any sort of predicate of each term [and in the things of which they are predicated]" and the reason he gives is precisely the validity of our RS: δι' αὐτὸν ἕκαστον καταγεγραμμένην καὶ ἔκαστον καταγεγραμμένην δι', καὶ δὲν ἕκαστον καταγεγραμμένην καὶ ἕκαστον καταγεγραμμένην δι'. 

(Aristotle [1], 7,1, 152 b, 25—30). A modern reader may disregard the bracketed portions of the text; the a' and b' of our RS, even if singular terms, could be regarded as predicates of some other subject in the pre-Freggian, "traditional" predication theory.

Notwithstanding this endorsement of RS, the *Organon* elsewhere rejects our rule. The rejection of RS can be recognized in Aristotle's denial of the validity of the transitivity of predication (*Sophisti Elekndi*, 179 a 25 — b 7). The transitivity of predication is one of the forms of the fallacy of accident. Aristotle's examples are (with unessential changes): (1) the man who is approaching is Coriscos, you know Coriscos, hence you know the man who is approaching, (2) the man with his face covered is Coriscos, you know Coriscos, hence you know the man with his face covered, (3) I am about to ask you what the good is, you know what the good is, hence you know what I am about to ask you. The form common to these examples is: M is Q, Q is P hence M is P, that is, transitivity of predication. But the "conclusions" of the "arguments" may be false while the premises are true, which leads Aristotle to assert: φανερὸν γὰρ ἐν ἡμῖν τοῦτοι δὴ οὐκ ἀνάγηκ τὸ κατὰ τοῦ συμβαθμοῦ καὶ κατὰ τοῦ πράγματος ἐλπισθεναι (179 a, 35). That is, there is no necessity that a predicate of a predicate of x be a predicate of x. I write 'predicate' for 'accident', following Alexander [1] p. 37 and commentators in general. All this becomes a restriction on RS as soon as we see that the copula 'is', in the a is b' premise of the three examples, represents for us, Freggian readers, an identity rather than a predication. Thus we are led to reread Aristotle's view as follows: there is no necessity that whatever is true of a thing given under a certain description, be true of the same thing given under another description. The reason that there is 'no necessity' lies in the word 'another' and ultimately depends on the diversity of predicates used in the construction of the two descriptions of the same object: τὸ δ' ἄνδρον οὐ καὶ λέγεται ἄνδρον νὰ τὸν ἔχων νὰ τὸν ἔχων ἐξοφλεῖαι, ἀλλὰ τὸ περιτέλλει τὴν ἀντικειμενικὴν προσῳχῆν τε νὰ τὸν καὶ Ἀδριανοῦ (179 a, 39). Thus, to be approaching is not the same as to be Coriscos; the description 'the man who is approaching' and 'the man whose name is Coriscos' or briefly 'Coriscos' may well designate the same object, but still the diversity of the predicates is such that οὐκ ἀνέγινη, it is not necessary.

5 "It is clear that in all these instances it does not necessarily follow that the predicate which is true of the accident is also true of the subject."

6 "In the case of the good, it is not the same thing to be good and to be about to be the subject of a question, nor in the case of the man who is approaching (or whose face is covered) is to be approaching the same as to be Coriscos."
that everything true of the man who is approaching be true of Coriscus or vice versa.

This is in my view the subtle point that justifies the inclusion of Aristotle (as far as the Sophistici Elenchi are concerned) in the group (2) of opponents of RS. In contrast with all those who insist on the plausibility of RS (Fregna, Carnap, Quine, etc.) the Aristotle of the Sophistici Elenchi may be interpreted as a logician who does not find RS to be plausible at all. For Aristotle RS is (or would be, had he explicitly written about the substitutivity of identities) a fallacy of accident, and as such a mere consequence of our "inability to perceive the distinction between the identical and the different" (169 b, 2). The only plausibility that may be assigned to RS from the standpoint of Aristotle qua author of the Sophistici Elenchi is the plausibility of fallacies; indeed, the fallacy of accident, and hence RS, seems to be for Aristotle the most "plausible" of all, such that even the experts are deceived (168 b, 1).

Aristotle seems to accept a restricted substitution principle: μόνος γὰρ τούτης ἰδιότητας καὶ ὑπὸν ἰδιότητος καὶ ἐν ὑπὸν ἑπάνω δοκεὶ ταῦτα ἐπάγγεια (179 a, 38—40). Hence RS seems to be restricted to the case of a "b" expressing an "essential" identity. Obviously, the identity expressed by "the man who is approaching is Coriscus" is not of the "essential" type. With reference to the various sorts of identity distinguished in Topica 1, 7, this means that the third type of numerical identity, exemplified by "Socrates is the man who is talking" or "Socrates is the man who is sitting" (103 a, 38), does not qualify as a premise of RS. This may be disturbing only to readers of Aristotle who think that without RS there is no identity (such is the case of White [1], who is led to affirm that the "sameness in number" of Topica 1, 7 cannot be identity because for one of its kinds, namely for the accidental sameness, the restricted substitutivity principle of Sophistici Elenchi is not valid, cf. p. 183).

We are justified in reading RS in the passages from the Sophistici Elenchi, but we should not lose sight of the fact that Aristotle's plan is not formaliter the examination of our RS but the discussion and qualification of the transitivity of predication as one of the forms of the fallacy of accident. Thus, while we view the text as an attack on RS, Alexander and the anonymous Greek commentator view it as a striking restriction on the transitivity of predication suggested and even endorsed by other parts of the Organon: οὐ γὰρ ὑπὸ κατὰ τὸ κατηγορομένου λέγεται, τάδει πάντα καὶ κατὰ τὸ ὑποκειμένου ἔμβαςται ὡς ἐν ταῖς Κατηγορίαις ἐμάθομεν (Alexander [1], p. 38, 25. Alexander obviously refers to Categories 3 b, 4, to be carefully distinguished from 1 b, 10, where a crucial, restrictive clause is attached to the statement of the transitivity of predication).

If we now turn to Aquinas, we find the reference reproduced in Quine's footnote to be wrong simply because it is not Aquinas who asserts the validity of RS; RS is mentioned by him as a possible objection against a theological thesis. As a matter of fact, Aquinas denotes the validity of RS in his "replies" and joins group (2 a). RS is formulated by the author of the Summa Theologicae as follows: Quaecumque sunt idem, ita se habent, quod quidquid prae dicatur de uno, prae dicatur de aliqua (Aquinas [1], 1, 40, 1, 3). In the replies Aquinas says: Dicendum quod quaedam et quaedam sunt idem re, differentiam secundum rationem. Unde non oportet quod multiplicantu uno, multiplicantur reliquia (11). Leaving aside what has to do with the particular theological issue examined by Aquinas, we may easily recognize here a rejection of RS and, more specifically, the view that "a b" does not qualify as a premise of our rule if it merely expresses an identity re, not also an identity secundum rationem. Some of the commentators on this passage of the Summa confirm this interpretation and add that with identity both re et ratione, RS is valid. The most impressive commentary is the following: Iliud axioma [= RS] non est verum nisi in quae sunt eadem et et rationem (Aquinas [2]). Also: Quandoque aliquid sunt idem secundum rem et secundum modum significandi, vel secundum rationem, tum verum est quod omnia quae praedicatur de uno ipsorum, praedicantur etiam de altero. Si vero sint idem tantum secundum rem, non est necessarium (Hieronymus de Medicis [1]).

The crucial feature of these Thomist rejections and restrictions of RS is that their authors do not show any sorrow over the lack of success of the rule. For them referential opacity is not an infirmity worth worrying

7 "For it is only to things which are indistinguishable and one in essence that all the same predicates are held to belong."

8 "For it is not true that whatever is said of the predicate is also said of the subject, as we learned in the Categories."

9 "Identicals are such that whatever is predicated of one is also predicated of the other."

10 "It should be said that person and property are the same in reality but are different according to the reason. Hence it is not necessary that if one is multiplied, the other is multiplied too."

11 "That axion is not true except in things that are identical both in reality and according to the reason."

12 "Whenever things are identical both according to reality and according to the mode of signification or according to the reason, then it is true that all the
about" (for the opposite view cf. Quine [1] p. 145). On the contrary, referential opacity is what one has to expect in view of the different \textit{rationes} expressed by the singular terms \textit{a} and \textit{b}, even when \( a = b \). The infirmity lies precisely in the acceptance of or in being deceived by RS, which is one of the fallacies threatening users of ordinary Latin. The view of RS as a fallacy ("of accident") appears in the treatise \textit{De Fallaciis} (attributed to Aquinas by some scholars) as well as in general in the scholastic literature. An impressive example of how this view has survived the scholastic tradition is the following: "All the fallacies which attempt the substitution of a thing in one form for the same thing (as it is called) in another, belong to this head [= fallacy of accident];", in De Morgan [1], p. 252.

Apart from the fallacy of accident, there are other \textit{laci} in traditional logic literature where we may expect to recognize our RS. One of them includes the discussions on the principle "all things identical with one and the same thing are identical with one another". On the face of it, this axiom expresses the transitivity of identity and is a particular case of RS. One should be very cautious, however, in interpreting the controversies about this principle as necessarily having to do with the transitivity of identity or even with RS. The reason is that in pre-Fregelian logic predication and identity are not well distinguished. Already Aristotle in \textit{Sophistici Elenchi} 168 b, 32—35 regards the principle as a source of fallacies, but we discover that what he means is quite different from our expectations as soon as we see his example: snow is white, swan is white, hence snow is swan, which has nothing to do with the transitivity of identity or RS. Thus, for example, it is hard to ascertain what is the real target of Caramuel's forceful attack on the traditional "transitivity of identity" (it is tempting, nevertheless, to literally apply to our RS now his diagnosis of the axiom in the 17th century: "Morbus, de quo multum Medici disputant, nec in remedio adhibendo convenient, mortalis est: et hoc Axioma incurable laborat morbo, siquidem tot emplastris appositis nondum valet. An non utilius permitteretur emori, quam ingenia vexare? Sic sentio", Caramuel [1] p. 411).

The work of some medieval logicians is relevant to RS in an interesting and intriguing way. Buridan, for example, anticipates Frege in observing that certain contexts (\textit{cognosco}, etc.) have the power of "bringing out" the \textit{rationes} (Frege's \textit{Sinn}) of words: "faciant terminos sequentes appellare suas rationes" (Buridan [1] p. 50). Hence in such contexts not all equi-referential terms can be replaced but only \textit{synonyma} (presumably: words denoting

predicates of one of them are also predicating of the other. But if things are identical only according to reality, this is not necessary". 

the same \textit{res} and connoting the same \textit{ratio}: same \textit{Bedeutung} and \textit{Sinn}) or terms whose \textit{ratio} is included in the \textit{ratio} of the term being replaced (Buridan [1] p. 61). Should we include Buridan in our group (1b) together with Frege?

We saw that both Aristotle and Aquinas seem to accept some restricted rule of substitutivity, expressed by additional conditions on the premise \( a = b \), such as the condition of being an "essential" identity or of being an identity \textit{re} and \textit{ratione}. A more radical view is conceivable. In fact, even if the singular terms \textit{a} and \textit{b} do not express any different \textit{rationes}, that is, even if the sentence \( a = b \) is true \textit{re et ratione}, and between \textit{a} and \textit{b} there is only a notational difference, one may hesitate to subscribe to the wild claim that \textit{a} and \textit{b} are interchangeable \textit{salva veritate} in all contexts of some ordinary language. This extreme approach could have been implicit in Husserl's remarks on "Leibniz's law" as found by him in H. Grassmann's \textit{Lehrbuch der Arithmetik} ("gleich heißen zwei Dinge, wenn man in jeder Aussage statt des einen das andere setzen kann"): "Von dem Werte dieser Definition können wir uns nicht überzeugen [..] Solange noch ein Rest von Verschiedenheit vorhanden ist, wird es Urteile geben, in welchen man die betreffen- den Dinge nicht vertauschen darf salva veritate (Husserl [1] p. 97). In a more recent author, Linsky, the extreme view is clearly stated:

Not only is it possible to produce counterexamples to Leibniz's law; it can be shown that no two terms obey it. Let \( t \) and \( t' \) be different terms and consider any true statement of the form:

(4) Jones explicitly denied that \( t = t' \).

Surely one cannot substitute \( t \) for \( t' \) in (4) in order validly to obtain

(5) Jones explicitly denied that \( t = t \).

No statement of the form (5) follows from the corresponding statement of the form (4), even though \( t = t' \) be true. If one insists that the principle of substitutivity is analytic and explicative of the concept of identity, one is faced with the peculiar consequence that only trivial statements of the form

\[ t = t \]

are true statements of identity. A true statement of identity can never be informative (Linsky [1] p. 139).

Incidentally, Linsky describes his own approach to RS in a way that perfectly fulfills the definition of group (2) and specifically (2a):

What do all these cases in which substitutivity fails have in common? As far as I can see there is nothing they have in common that can be appealed to in explaining the failure of Leibniz's law. I do not find this distressing, for I cannot see why anybody ever thought the "law" was true (Linsky [1] p. 145).
Also, Rescher, while admirably describing the approach of members of (1b), for whom failures of RS are “wicked”, places himself, at least as far as I can tell from the following passage, in group (2a):

Once we move from pure mathematics to its applications to contingent matters of fact, or from pure logic to the realm of epistemic concepts, there seems to be no real alternative but to accept modifications of Leibniz’s Rule. In view of this, it seems to me pointless to insist that the unrestricted inter-
substitutivity of identicals is somehow “good” and that restrictions upon substitution are somehow “wicked”. Nor can I take satisfaction in attacks upon modal logic on the sole ground that it involves a conflict with Leibniz’s Rule. Only if there were some impeccable fundamental reason why Leibniz’s Rule must be accepted in rigidly unqualified form — and such a reason has yet to be produced — could a conflict between this rule and the logic of modalities appropriately be used as justification for rejecting modal logic. Until such a reason is provided, any polemic along these lines involves a sacrifice of resources. There is surely more than enough work to be done in developing various alternative systems of logic attuned to differing needs and requirements (philosophical as well as mathematical) (Rescher [1] p. 167).

Apart from these exceptions, however, examples of group (2) are very scarce in the recent literature, and indeed in the tradition dominated by Leibniz, Frege, and mathematics. It is in fact surprising to find opposition to RS at all within this tradition. The newest example I know is in Belnap [1], where Leibniz’s Law is referred to as “Leibniz’s Lie”. Perhaps RS’s prestige is beginning to decline.

3. Naive friends

Although it may be conjectured that all in all the history of logic until Leibniz does not favour RS, it is likely that the *Topica* texts where RS is accepted have exerted an influence of their own and have generated “friends” of substitutivity, naive or otherwise. Besides, it is conceivable that some logicians became friends of RS not merely *ex auctoritate Aristotelis* (*Topica*) but by their own insights.

One example of pre-Leibniz RS is found in the seventeenth century philosopher Derodon, in whose treatise we have an impressive set of *axiomata* for identity, one of which reads as follows: „Sextum axioma sic formatur: quaecunque sunt eadem, habent eadem accidents, ideo habent omnia eadem praecipitata“ 13 (Derodon [1] p. 67). To my knowledge, Derodon was a naive friend of RS.

13 „The sixth axiom is as follows: identicals have the same accidents and indeed absolutely the same predicates.“

Acceptance of RS, however, does not seem to become a systematic business for logicians until the eighteenth century, among authors mainly dependent on Wolff and Leibniz. About Leibniz we know at least that he was not a naive friend of RS throughout his life, but for the other authors, whose logic treatises exhibit substitution of equals as the basic operation of reasoning, it is not clear whether they were aware of the apparent failures of RS or not. A more detailed research may confirm the conjecture that Reusch, Baumeister, Wallerius, etc. were all naive friends of RS (or that they did not exactly mean RS, that is, in these authors we may have pseudo-
ocurrences of RS). It should be emphasized that these are not at all negligible authors.

For example, Reusch’s *Systema Logicum* is an impressive treatise of one thousand pages on the logic of ordinary Latin. From the outset, the idea of *substitution* plays a fundamental role: „Eadem sunt, quae sibi possunt substitiri [...] Entia, quae respectu omnium suorum praedicatorum possunt substitui, dicuntur perfecte eadem“ 14 (Reusch [1] § 7). The syllogism is viewed as a substitution process: „Totum scilicet negatium ratione ordinariae absolvitur substitutione idearum loco subjecti vel praedicati propositionis fundamentalis, quam aliqui vacant seuationem cogitationum“ 15 (§ 507). In this connection, substitution is regarded as a generalization of the old *dictum de omni* (§ 506), anticipating Jevons. The more emphasis is laid by Reusch on the idea of substitution, the more we wonder about his unawareness of the failures of RS, and the puzzle is even greater when we consider his careful discussion of reduplication (contexts governed by *qua*, etc. § 506, where Leibniz had detected exceptions to RS) as well as his section on the fallacy of accident (§ 722). I have not been able to recognize any hint at the possibility of fallacies related to substitution. In § 725 (p. 820) I found the word „substitutur“ in the context of the discussion of the fallacy *dictum simpliciter- dictum secundum quid*, but this does not lead anywhere either. Unless I am missing some basic point in the work of men such as Reusch and his contemporaries, they seem to belong to (1a) unless, as said, their writings contain only pseudo-occurrences of RS, a possibility not to be discarded (cf. Jevons below). Still, there are cases where RS is really

14 „Identicals are such that they can be substituted one for the other [...] Entities that are interchangeable relative to all their predicates, are called perfectly identical“.

15 „The whole process of ordinary reasoning is carried out by substitution of ideas in the place of the subject or predicate of the fundamental proposition . . . “
meant. Let us consider one such example, occurring in a mid-eighteenth century attempt to prove that there cannot be two objects perfectly identical:

Principium enim indiscernibilium est propositio enuntians omnia possibilia esse a se invicem diversa, sicut ne duo quidem dari possibilitas perfectae eadem [...] Pone nimirum duo haec perfecte eadem A et B, tunc necesse est, ut ea omni respectu pro se invicem possint substitui, per consequens etiam quoad omnia praedicata. Sed absurdum inde oritur. 1) A est A, 2) A non est B, utraque propositio est vera. Si igitur A et B sunt perfecte eadem, etiam hoc respectu pro se invicem substitui queueris; unde haec orientur propositiones: 3) B est A, 4) B non est B, utraque propositio est absurda. Nihil hoc loco feci, quam quod B vel A tanquam perfecte idem substituerum, ex quo autem aperta fluit contradictione" (Wedel [1] § 118, p. 169—171).

The assumption a ≠ b and the assumption that a and b have all their predicates in common are incompatible. Take the predicate of a; to be same as a' or the predicate of a; to be distinct from b'. The object b should have these predicates, i.e. b = a, b ≠ b, but the first sentence is incompatible with our other assumption that b ≠ a and the latter is clearly false. Skipping Boole and other authors, let us turn to the logician who seems to be the most enthusiastic supporter of RS in the history of logic: W. S. Jevons. For Jevons the "substitution of similars", that is the principle that "whatever is true of a thing is true of its like" (Jevons [1] p. 15) is an admissible rule for ordinary English and in fact the most important law of logic:

The fundamental principle of reasoning authorizes us to substitute the term on one side of an identity for the other term, wherever this may be encountered, so that in whatever relation B stands to a third thing C, in the same relation A must stand to C. Or, using the sign $\equiv$ to denote any possible or conceivable kind of relation, the formula

\[ A = B \equiv A \;
\]

$\equiv$ hence $\equiv$

C C


One can hardly imagine a stronger formulation of RS. On the other hand, Jevons does not seem to be aware of any failures of his principle. Thus, on the face of it, Jevons appears to be the best example of our group (la).

Some doubts arise, however, when we examine more closely Jevons' intentions. His plan is to improve the traditional "dictum de omni". The various forms of the latter enable us to pass from the predicate to the subject, and to affirm of the subject whatever we know or can affirm of the predicate" (Jevons [1] § 10). In view of the conception of the proposition as an identity that "reformers of logic" before Jevons have introduced, the old dictum de omni should now be completed into: "whatever is known of either term of the proposition is known and may be asserted of the other." (§ 11). Thus Jevons' main point is that the old dictum de omni should be applied "in both directions, now that the two terms are indifferently subject and predicate" (ibid.) and now that propositions are equations.

To the extent that Jevons' "substitution of similars" is meant by him to be an improvement of the dictum de omni, all Jevons achieves by introducing his new principle is to replace the old Barbara

\[ \forall x. \, Sx \to Px. \]

\[ \forall x. \, Px \to Rx. \]

\[ \forall x. \, Sx \leftrightarrow Rx. \]

by a "two ways" one:

\[ \forall x. \, Sx \leftrightarrow Px. \]

\[ \forall x. \, Sx \to Px. \quad \text{or} \quad \forall x. \, Px \to Rx. \]

\[ \forall x. \, Px \leftrightarrow Rx. \]

and to this extent we have in Jevons a pseudo-occurrence of RS, while Jevons' principle seems to be RS only because of the traditional rather than Fregean predication theory according to which it is formulated. The whole point of emphasizing that Jevons' principle, as meant by him, becomes to a significant extent a "mere" two ways Barbara, is that the latter is a harmless, undisputed law of first order logic, free of any of the questions raised in connection with RS. Here, a difficulty has been pointed out by John D. Stone. Apparently the ordinary language counterexamples to RS can be schematized in the form of the two ways Barbara or dictum de omni. Take, for example, Sx = x is Coriscos, Px = x is approaching, Rx = you know x. More generally, RS itself can be schematized as a harmless two-ways Barbara:

not B: these two propositions are true. Hence if A and B are perfectly identical, they can be mutually substituted also in this respect, and we obtain thus the following propositions: (3) B is A, (4) B is not B, and the two propositions are absurd. Here all I did was to replace B for A, insofar as they are assumed to be perfectly identical, wherefrom an open contradiction flows."
Thus, there seems to be no point in saying that Jevons' principle reduces to a two ways Barbara and is no longer the real RS. As a reply, let us first see that the validity of the two ways *dictum de omni* or Barbara, their harmless, trivial nature (in the above first order logics formulation) amounts to the fact that for any object *o* we may wish to choose, the set (So ↔ Po, So → Ro) even truth-functionally implies the conditional Po → Ro, and analogously for the one-way dictum. At the beginning of this paper I emphasized that RS should not be identified with any externally similar rule from the predicate calculus with equality. For the same reason, RS cannot be represented by a two ways Barbara (in the first order logic formulation given above), because in so doing the *a* and *b* of RS are replaced by one singular term *o*, which amounts to putting aside all the significant differences in the meaning of *a* and *b*, whereas these differences are precisely what makes RS questionable. In the example of Coriscos, \( \text{\wedge}_x, x \text{ is Coriscos} \leftrightarrow x \text{ is the one who is approaching} \), *does not exactly reproduce the sentence Coriscos = the one who is approaching*, inasmuch as through the quantification we are taken from *a* and *b* to a single, *x*.

A deeper study of Jevons may lead to a more precise evaluation of how much is and how much is not really RS in his writings. For the time being we know that not all is real RS. But even knowing Jevons' intentions of launching the *substitution of similars* as an improvement of the *dictum de omni*, one cannot deny that some of his texts are among the sharpest statements of RS for ordinary English ever made, and in this sense Jevons' texts, if not Jevons himself, are to be retained in (1a).

4. Critical friends

It is not difficult to argue that RS is fully accepted by Leibniz in some of his texts. Consider for example: „Idem autem esse A ipse B significat aliorum alteri substitutum posse in propositione quacunque salva veritate“ [1] (Leibniz [1] p. 362). The range of *A* and *B* (*a* and *b* in our rule) includes, for Leibniz, proper names (Alexander Magnus), descriptions (rex Macedonie victor David) predicates (triangulum, trilaterum) and even sentences (enunciationes, ibid. p. 363) as Frege will propose in *Uber Sinn und Bedeutung*. For our purposes, however, the essential point is not so much that *a* and *b* range over so many sorts of things, but that they range at least over singular terms (proper names and descriptions) and that they are interchangeable in all contexts: *in propositione quacunque*.

As I have shown elsewhere (Angelelli [1]), in other texts Leibniz restricts RS in a way reminiscent of Aristotle and Aquinas, so that the real Leibniz's *rule is the following*: a = b → A(a) = A(b) provided A(x) is not reduplicative, which means that it is not of the form B(x qua M). I would like to add here that there is some possibility of finding still a further restriction of RS more or less potential in Leibniz's writings. To this end we must consider the Leibnizian thesis that external differences must be accompanied by internal differences. Let us abbreviate by *E* a predicate representing an external property and by *I* a predicate representing an internal one. One reading of the thesis (suggested by John D. Stone) says that if (a is E and b is not E) then (a = b and for some I a is I and b is not I). Another reading is: if (a is E and b is not E) then (a = b then (a is I and b is not I for some I). This second reading allows that a = b while a is E but b is not E for some I. In other terms, using a Fregean example for *a* and *b*: the Morning Star = the Evening Star, yet the Morning Star = E while the Evening Star is not E. The new restriction of Leibniz's law would be: if a = b, then *a* and *b* are interchangeable in any A(x) except for those A(x) that represent external properties of *x*. As soon as one sees that in the scholastic tradition the most typical examples of *denominations extrinsecas* or external properties were the cognitive contexts, the new restriction to RS becomes very familiar. Further, one might view the first restriction (ruling out reduplicative contexts) as a special case of this new restriction, by assuming that reduplicative contexts A(a) represent an external property of *a*. The latter seems to be the only choice, if reduplicative contexts A(a) represent properties of *a* all, because otherwise *a = b* would be compatible with a and b differing in some internal property, a rather awkward result. Of course, one should still investigate whether the new restriction is affected by Leibniz's assertion: „nullas esse denominiones pure extrinsecas“ ([1] p. 8). If there are no *E*'s, the new restriction is void.

17 „That *A* and *B* are identical means that one can be substituted for the other in any proposition salve veritate.“

18 In the case of Aristotle I have in mind *Sophistici Elenchi* 168a 34—168b 3, where reduplication seems to be related to failures of substitutivity; for Aquinas cf.
Let us turn now to Frege. In his Begriffsschrift (BS), 1879, § 8, Frege provides two reasons why his language requires identity. One of them is rather practical and superficial; we want to abbreviate long expressions and we use the identity sign to say that a new, shorter, or more convenient expression means the same as an old, less perspicuous group of symbols. This use of the identity sign is primarily a metalinguistic one. The statement that the old and the new symbols have the same meaning is not in the language, in the Begriffsschrift, but is about the language. Only afterwards may the definition become a sentence in the language, one which is, according to Frege, analytically true in Kant's sense (BS § 23). The more profound reason for the use of an identity sign in our language is that one and the same content (Inhalt) can be fully determined in different ways (Bestimmungswesen). Frege gives as example a geometrical point, given as the point A or as the point B that fulfills such and such conditions. The difference of the various proper names and descriptions that originate from the need of pointing to the same object in one way or another, is not merely notational: "die verschiedenen Namen für denselben Inhalt nicht immer bloß eine gleichgültige Form sich, sondern [...] sie das Wesen der Sache betreffen, wenn sie mit verschiedenen Bestimmungswesen zusammenhängen" (BS § 8).

The distinction Bestimmungswesen — Inhalt will later become the famous couple of terms Sinn and Bedeutung. Thus the fact that names or descriptions, say 'a' and 'b', that have the same Bedeutung may have different Sinn, makes the statement 'a = b' non-trivial and not merely reflecting an arbitrary, playful tendency to multiply many names or nicknames for the same object. Paradoxically and in a way that Frege probably would not like (because he liked to insist on the völages Zusammenfallen, full coincidence, meant by the sign '='), we may interpret the author of Begriffsschrift as saying that sentences of the form 'a = b' are interesting to the extent that 'a' and 'b' differ in their meaning.

For anyone acquainted with the logical tradition, no new ideas are contained in Frege's argument for the justification of the identity sign. Frege uses very familiar and old insights. He talks of Bestimmungswesen or Sinne (opposed to Bedeutung) where the earlier authors talked of rationes (opposed to res). Still, here as in many other cases, one is pleased to observe how certain topics are re-discovered again and again, and this is even more interesting in the case of a young mathematician who had not been trained in the history of logic.

But the great surprise waits for us at the very end of the § 8 of BS, where Frege suddenly affirms that if 'a = b', then 'a' and 'b' are interchangeable salva veritate iberallay everywhere.

After having been persuaded that we need in our new language sentences of the form 'a = b' mainly and primarily because 'a' and 'b' connote a diversity (even when 'a = b'), we are now urged to believe that there is no context A(x) in ordinary German where that difference of Bestimmungswesen plays a role. Even if the symbols 'a' and 'b' differed only qua figures in their shape and not qua signs in their Bestimmungswesen or Sinne (to use Frege's distinction at the beginning of SB), one might hesitate to affirm that 'a' and 'b' are universally interchangeable salva veritate, but this is definitely unreasonable if one has emphasized the diversity of rationes or Sinne expressed by 'a' and 'b'. The reasonable move is to abstain from the assertion of a wild, unrestricted RS, recalling Aristotle's warning "οὔτε ἄγρα" or Husserl's profound remark "Solange noch ein Rest von Verschiedenheit vorhanden ist, wird es Urteile geben, in welchen man die betreffenden Dinge nicht vertauschen darf" or De Morgan's statement: "All the fallacies which attempt the substitution of a thing in one form for the same thing (as it is called) — in another, belong to this head [= fallacy of accident]."

Frege's opposite move is amazingly incoherent. The error could be explained by assuming that Frege was influenced by the Leibnizian "eadem sunt..." while ignoring Leibniz's restrictions on "his" law or by recalling that after all Frege was a mathematician used to working with substitution. Alternatively one might argue that there was no error; to this end one should show that Frege did not have in mind a full RS for ordinary German but only a rule for the mathematical contexts related to his foundational program. Under this assumption the "iberallay" in Frege's rule at the end of BS § 8 should be replaced by "in all mathematical contexts" and we would have to do with a pseudo-occurrence of RS. Against this solution one objection is that already in 1884 Frege reiterates his principle in a way that indicates that he has in mind all contexts of ordinary German:

Welches sind diese [...] die Erklärung Grundsätze nennen, der das Wesen der Gleichheitsbeziehung zum Ausdruck bringt, und als solcher ist sie von grundlegender Wichtigkeit. (GLA § 65)

Consider also the following text from the early 1890's:

Man könnte jene Leibnizische Erklärung Grundsatz nennen, der das Wesen der Gleichheitsbeziehung zum Ausdruck bringt, und als solcher ist sie von grundlegender Wichtigkeit. (RH p. 184 [320])
But more than in these passages, in SB (1891) it becomes definitely clear that Freges RS has to do with the whole ordinary language. Thus, from the standpoint of these later writings, it is not plausible to read BS § 8 as offering a pseudo-occurrence of RS. Hence we must come back to the view that BS § 8 is incoherent. Was this inconsistency ever removed from Freges theory? The answer is negative.

In SB Frege confidently reaffirms RS: „Wir haben gefunden, dass der Wahrheitswert eines Satzes unberührt bleibt, wenn wir darin einen Ausdruck durch einen gleichbedeutenden ersetzen“ (SB p. 151 [36]). The „wir haben gefunden“ is quite misleading and in fact false if taken in the strong sense of „we have shown“ or „we have established“. So far in the paper RS has occurred exactly once, and in this occurrence RS is merely stated, not proved or justified in any way, except ex auctoritate Leibnizii:

Wenn unsere Vermutung richtig ist, dass die Bedeutung eines Satzes sein Wahrheitswert ist, so muss dieser unverändert bleiben, wenn ein Satzteil durch einen Ausdruck von derselben Bedeutung, aber anderem Sinn ersetzt wird. Und das ist in der Tat der Fall. Leibniz erklärt geradezu: „Eadem sunt, quae sibi mutuo substitui possunt, salva veritate“. (SB p. 150 [35])

Note that the statement of RS properly occurs in the second part of this text: „Und das [= RS] ist in der Tat der Fall“. Frege’s confident reaffirmation of RS in his famous paper is not to inform the reader of such an obvious, undisputed truth, already stated by Leibniz. Frege mentions RS simply because he wants to investigate the possibility of extending even further the range of application of RS, by allowing „a, b‘ to be sentences and not only singular terms (Eigennamen in his sense: proper names and descriptions), in the understanding that the Bedeutung of a sentence is its truth-value and the Sinn its Gedanke. This is the central topic of Frege’s paper.

Still the paper provides a significant piece of information concerning RS for „a, b“ singular terms. Frege affirms that in reported speech, singular terms denote their sense, i.e. their Bedeutung is their Sinn.

It must be said, in the first place, that this claim is not justified in any acceptable way by Frege. Let us quote his unconvincing text:

In der ungeraden Rede spricht man von dem Sinne z. B. der Rede eines anderen. Es ist daraus klar, dass auch in dieser Redeweise die Worte nicht ihre gewöhnliche Bedeutung haben, sondern das bedeuten, was gewöhnlich ihr Sinn ist. (SB p. 145 [28])

What is the Sinn of somebody else’s Rede? This depends on what somebody else’s Rede is. Surely not a singular term, which is unfortunate, because we have defined „Sinn“ so far only for singular terms. If Rede is something like a sentence, the answer has not yet been given by Frege at this point of SB. But even if we anticipate and assume that the sense of a sentence is the Gedanke, it is not obvious (pace Frege’s „daraus klar“ in the second sentence) that the singular terms occurring in reported speech should denote their sense rather than their reference.

Leaving aside the weaknesses of Frege’s claim, let us observe that if the claim is accepted, RS cannot be applied to A(a) contexts with „a“ occurring in reported speech, which does not mean that one must refrain from applying RS, but that there is no reason to apply it, just as one would not think of applying RS to contexts A(a) where one talks of the symbol „a“ or in general to any context A where one does not talk of the object a. The intended effect of Frege’s thesis is that the apparent failures of RS in contexts of reported speech are explained away and are shown to be only apparent; RS does not fail, we fail to see that in a = b and A(a) the term „a“ stands for different things in each occurrence and hence there is no reason to apply RS.

This is the same approach: Frege defends in connection with RS extended to sentences; the apparent failures are shown to be only apparent (cf. SB in fine). The fact of not using RS in reported speech contexts does not mean that one grudgingly restricts RS. This would be the defeatist attitude of group 1(ba). Frege belongs to, and in fact seems to inaugurate, the more ambitious group 1(bb). Even if we accept Frege’s construal of reported speech, it remains for him to make sure that RS is truth-preserving in all other contexts of ordinary German. He knows that „es ist schwer, alle in der Sprache gegebenen Möglichkeiten zu erschöpfen“ (SB p. 161 [49]), but unfortunately such is the task to which he committed himself when he asserted RS for ordinary German.

In SB there is a classification of all occurrences of a singular term „a“ into (1) those where we want to talk of the Bedeutung of „a“, that is, (2) those where we want to talk of the sense of „a“, and (3) those where we want to talk of the symbol „a“. Group (2) is subdivided into a trivial case (2i) contexts of the form „the sense of „a““ and (2ii) reported speech contexts (SB p. 145 [28]). At least one serious question arises concerning this classification. Is it exhaustive? Not so, if one considers the possibility of talking of the object a qua or insofar as „illuminated“ by the special sense of „a“ (cf. Frege’s „einzigt beleuchtet . . .“ in SB p. 144 [27]), or sub quadam ratione, as the scholastics would say.
Disregarding these difficulties, the problem is whether Frege proves that RS is truth-preserving in all contexts of type (1). As we may expect, there is no justification of this in Frege's writings and one wonders how could there be. Perhaps Frege's confidence in the success of RS for contexts of type (1) depends on the concealed, fallacious move of construing such contexts as contexts where all that matters is the Bedeutung of ‘a’, which is different from, yet sounds similar to the definition of contexts (1) as those where one talks of the Bedeutung of ‘a’ (= a). But then the assertion of the truth-preserving nature of RS amounts to the assertion of a truth-functional tautology. If the difference of Sinne is irrelevant, why not use just one designation, say ‘a’ for the object o, instead of ‘a’ and ‘b’? Thus, the conclusion of RS is really A(o) → A(o). Is this the grand principle that explicates the concept or the essence of identity (GLA § 65, RH)?

One may suspect that Frege went too fast. He probably at some point in the 1880's realized that the RS of BS § 8 seems to fail. Unfortunately such apparent failures must have appeared to Frege (who thought of RS as revealing the essence of identity: GLA § 65, RH) as an intolerable scandal. Under such a pressure, his genius hastily devised an elegant theory, in which it is assumed that the apparent failures are only apparent and occur in reported speech and where one sweepingly decides that all words in reported speech denote their customary sense. It is very significant that later in his career he indicated that he felt it was wiser to abandon the ambitious (1bb) program and to content himself with the (1ba) approach. The following text shows this shift. ‘Gedankengefüge erster Art’ means conjunctions; the ‘andere sechs Arten’ refers to various truth-functional combinations of propositions.


 einen wahren Gedanken ausdrückt. Nachdem dies festgesetzt ist, kann unser Satz so ausgesprochen werden:

„Wird in einem mathematischen Gedankengefüge ein Gedanke durch einen Gedanken von denselben Wahrheitswert ersetzt, so hat das so gewonnene Gesächengefüge denselben Wahrheitswert wie das ursprüngliche.“ (Gef p. 394 [51])

In other words, if the 'a' and 'b' of our RS are sentences, we have to restrict the context A(x) in the conclusion of the rule to those that are called today „truth-functional“. For the rest, Frege prefers to leave the issue unentschieden. It is important to observe that this affects the validity of RS for 'a' and 'b' singular terms as well, namely when they occur in the „undecided“ contexts.

It is very easy to collect examples of „critical friendship“ (1b) of RS in the post-Fregean literature, especially if we do not commit ourselves to distinguish in each case between 1ba and 1bb. Carnap [1] reiterates the assertion that RS is „quite plausible“ (§ 71 and Summary of chapter III). I have already referred to Quine in section 1 of this essay. Future historians of logic will be certainly intrigued by Quine's simultaneous assertions that RS is a „fundamental law“ and that it is easy to find counterexamples[8] to it (Quine [1], first page). They will be less intrigued by Quine's inclination toward assimilating the failures of interchangeability of two singular terms 'a' and 'b', to their being used in material supposition („intermediate occurrences“ assimilated to „accidental occurrences“ as Kaplan [1] has it), inasmuch as the same move was already made, for example, by Birudan and Leibniz[9]. To add a few more names, I would like to mention Wilson [1], Sellars [1] („if we keep Leibniz's law — and, like many others, I see no viable alternative …“), Davidson [1], where we find a splendid statement of (15b): „... a singular term [...] can be replaced, so far as the truth or

19 For Buridan see Buridan [1], p. 51 and for Leibniz my [1]. Buridan: „... iter accusativi quodammodo videntur participare suppositionem materialem, quia appellation conceptus suos.“ Leibniz says: „est in eo aliquid materiale.“
falsity of the containing sentence is concerned, by any other singular term that refers to the same thing. The notorious apparent invalidity of this rule can only be apparent, for the rule no more than spells out what is involved in the idea of a (logically) singular term. Perhaps also Kripke may be added to our list, cf. his [1] p. 136—137, especially his use of formula (1), where the 'F' ranges over "any property at all" (p. 137).

5. Concluding remarks

We have found examples for most of the conceivable approaches to RS. Pseudo-occurrences of RS (0) are certainly in Jevons, although the force of his formulations is such that he must be kept also in the group of naive friends (1a). Frege, Carnap, Quine and others, are examples of critical friendship (1b). At least in his main writings, Frege exemplifies (1bb). Although certainly many of the members of (1b) are in (1ba), and in the first place Frege himself, qua author of his 1923 paper on Gedankenfabrik, I have not tried to provide clear-cut examples for (1ba). Nor have I attempted at producing examples for (2b), which may be a superfluous or irrelevant category for our purposes.

From a historical point of view, we obviously have to do with a difficult case: what is regarded as a fallacy by the Aristotelian-scholastic tradition, is for Frege a profound insight into the "essence of identity" and has become in the recent logico-philosophical tradition a highly respected principle of logic.

How to evaluate this discrepancy in the history of Western logic of course depends on how one feels about RS. In White [1], p. 181, we see that the author is himself in group (1b), so that he is inclined to view Aristotle's lack of appreciation of RS as a fault. Being a good friend of Aristotle too, the author aims at "excusing" Aristotle from that fault (showing how his notion of identity was not the one leading to "Leibniz's law" p. 192). On the contrary, I think that (2) is the right approach. My reason for joining (2) is, briefly, that I believe that Frege's argument for the justification of an identity sign in RS § 8 is right: I think the difference between various singular terms 'a' and 'b' designating the same object is not merely notational. What makes 'a = b' interesting is the diversity of rationes, Bestimmungsweisen, or Simes of 'a' and 'b'. Now, and at this point I depart from Frege, it seems to me that even if the difference were merely notational one might hesitate to assert RS, but the difference being at the level of the senses (rationes, etc.), RS becomes definitely implausible and should be expected to fail, as it in fact does.

Naturally, from the standpoint of (2) I do not feel that it is necessary to explain why the Aristotelian-Scholastic tradition rejects RS as a fallacy, but rather why Frege and the post-Fregean tradition come to think of RS as a great and plausible principle. My impression is that accidental circumstances may have helped to inflate RS's prestige. In the case of Frege, his enthusiasm for RS may have been favoured if not fully conditioned by (1) the influence of Leibniz's authority, (2) his unawareness of Leibniz's restrictions and his knowledge of Leibniz's endorsements of RS only, (3) the prevailing mathematical orientation of Frege (the ordinary language considerations came in later). Among post-Fregean authors or other pioneers of mathematical logic, some of these conditions seem to have been at work too. In the English-speaking world Leibniz's RS without Leibniz's restrictions (1 and 2) became popular through C. I. Lewis' translation of "two fragments from Leibniz" appended to his Symbolic Logic. Quine, in his early O Sentido, refers to that translation. Carnap quotes Leibniz's formula in his Aufbau (§ 51) and we know of course his close relationship to Frege. Peano also refers to Leibniz (Formulae I, § 2, 5.2) and in the case of Russell (RS occurs for example in Principe I, p. 23) his early acquaintance with Leibniz is well-known. When the connection with Leibniz is not apparent, we still have that the authors in question have been primarily preoccupied with mathematical contexts. This may be applied to Hilbert ("Axiome der Gleichheit") and to Jevons, insofar as we read his "substitution of similars" as RS. Jevons emphasizes that mathematics (and not Leibniz) led him to the discovery of his principle (Jevons [2], preface to second edition, p. XXII).
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22. „The idea of Sameness challenges reflection”

Twentieth-century discussions of referring in analytic philosophy began with Freges 1891—1892 discussion of identity statements. For they began no later than Bertrand Russell’s „On Denoting” of 1905, which explicitly discusses this treatment of Freges, mentioning that Russell had formerly adhered to Freges’s doctrines and offering what purports to be a superior semantics, which, among other things, is required to and supposedly does solve Freges’s Problem of Identity. The 1892 discussion, however, itself takes as its point of departure Freges’s earlier views on identity and on the problem, given in his Begriffsschrift of 1879. Thus it is reasonable to regard this latter discussion as the starting-point for the twentieth-century discussions of referring that continue to the present time, often inspired and illuminated by these original papers of Frege’s.

The 1892 discussion of identity (in SB) takes as its point of departure the 1879 account of the nature of the relation; Frege says, beginning the essay:

Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In my Begriffsschrift 1 assumed the latter. (SB, 143 [26].)

and he goes on to criticize this conception of the relation as one upon which the Problem of Identity could not be solved; on it,

2 The discussion is primarily in „On Sense and Reference” of 1892, but the same doctrines are put forward in „On Function and Concept” of 1891 (where reference is made to the former paper); these papers will be cited as SB and FB, respectively. I use unless otherwise noted the translations in Translations from the Philosophical Writings of Gottlob Frege (Oxford: Blackwell, 2nd ed., 1960), ed. P. T. Geach and Max Black.
3 Mind, 14 (1905), 479—493.
4 I use `Begriffsschrift’ without italics to refer to the ideography itself.