PHILOSOPHY OF ARITHMETIC

PSYCHOLOGICAL AND LOGICAL INVESTIGATIONS

BY DR E.G. HUSSERL

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TO MY TEACHER

FRANZ BRENTANO

IN [INNIGER] GRATITUDE
PREFACE

The "Philosophy of arithmetic" that I publish here does not aim at building a [regelrechtes] system of this borderline discipline, so important for the mathematician as well as for the philosopher, but rather to prepare, in a series of "psychological and logical investigations", the scientific foundations for a future construction of it. More than such a preparation, in the present situation of knowledge, could not be attempted. I would not be able to name one single important question, for whose answer even a tolerable agreement exists among the relevant researchers — which is a sufficient proof that in our domain we cannot yet talk of a merely architectonic organization of a knowledge already acquired. The given task is rather the following: in patient individual research to look for the defensible foundations, in meticulous critique to examine the theories worth of consideration, to separate what is right from what is erroneous in order to put, once we have learned, something new and possibly better in their place. Such is the plan of this book.

I have not strived for critical completeness. I took it not to be [required] [in the interest of the subject matter] to consider the innumerable /vi/ attempts that concern the basic questions of our field, and that I know of; it was enough to choose those that seemed to deserve being preferred either because of their peculiar nature, or because of their far reaching diffusion, or because of their internal importance.

I hope that a critical method that I have often followed will not be the source of objections. [...]
CONTENTS OF VOLUME I

Preface

Part I

The genuine concepts of plurality, unity and (cardinal) number

Introduction

Chapter 1. The origin [generation] of the concept of plurality through that of collective union
The analysis of the concept of number presupposes the analysis of the concept of plurality
The concrete foundations of abstraction
The independence of abstraction from the nature of the collected contents
The origin [generation] of the concept of plurality through reflection on the collective union

Chapter II. Critical developments
The collective unification and the unification of the partial phenomena in the total consciousness at each moment
The collective togetherness and the temporal simultaneity
Collection and succession
The collective and the spatial synthesis
A. The theory of Lange
B. The theory of Baumann
Collecting, counting, and distinguishing
Critical addition

Chapter III. The psychological nature of the collective union
Retrospect
The collection: a special kind of union
For a theory of relation
Psychological characterization of the collective union

Chapter IV. Analysis of the concept of number in its origin and content

Completion of the analysis of concept of plurality
The concept something
The numbers and the generic concept of number
Relationship of the concepts cardinal number and plurality
One and something
Critical addition

Chapter V. The relations of greater and less
The psychological origin of these relations
Comparison of arbitrary pluralities as well as of numbers according to greater and less
The distinction of the species of numbers is conditioned by the knowledge of greater and less

Chapter VI. The definition of equinumerosity through the concept of the one-to-one correspondence
Leibniz's definition of the general concept of equality
The definition of equinumerosity
On special definitions of equality
Application to the equality of arbitrary pluralities
Comparison of pluralities of a genus
Comparison of pluralities with regard to their numbers
The true sense of the considered definition of identity
Mutual correspondence and collective union
Independence of the equinumerosity from the mode of connection

Chapter VII Definitions of number by equivalence
Construction of the theory of equivalence
[Belege]
Critique
Frege's attempt
Kerry's attempt
Chapter VIII Discussions on unity and plurality
The definition of number as plurality of units
One as abstract, positive partial content. One as mere sign.
One and zero as numbers
The concept of unity and the concept of number one
Further distinctions concerning one and unit
Identity and difference of the units
Further misunderstandings
The various meanings of the name "unit(y)"
The arbitrariness of the distinction between unit and plurality. The conception of plurality as plurality, as a counted unity, as a whole.
Herbart's arguments

Chapter IX Sense of the statement of number
Conflict of views
Rejection and decision

Appendix to the first part
The nominalist attempts of von Helmholtz and Kronecker

II PART
The symbolic concepts of number and the logical sources of the arithmetic of cardinal numbers.

Chapter X The numerical operations and the genuine concepts of number
Numbers in arithmetic are no abstracta
The basic [operations] on numbers
Addition
Division
Arithmetic does not operate with the "genuine" concepts of number

Chapter XI the symbolic ideas of plurality
Genuine and symbolic representations
The sensible sets
Attempts to explain [temporary] conceptions of sets
Symbolizations by [mediation] of the whole process of the individual conception
New attempts to explain [temporary] conceptions of sets
Hypotheses
The [figuralen] moments
Resolution
The psychological function of fixing individual set members
Where lies the guarantee for the completeness of the [durchlaufenden] individual conception of a set?
Conception of genuinely representable sets by [figuralen] moments
The elementary operations and relations of plurality transferred to pluralities symbolically represented
Infinite sets

Chapter XII. The symbolic representations of number
The symbolic concepts of number and their infinite variety
The non-systematic symbolizations of number
The natural series of numbers
The system of numbers
Relationship of the system of numbers to the natural series of numbers
The election of the basic number of the system
The systematics of the concepts of number and the systematics of the symbols for number
The sensible- symbolic procedure of counting
Extension of the domain of symbolic numbers by means of the sensible symbolization
The differences of the sensible ways of designating
The natural generation of the system of numbers
Estimates of number by means of [figuralen] moments

Chapter XIII. The logical sources of arithmetic
Computing, computational techniques, and arithmetic
The arithmetical computing methods and the concepts of number
The systematic numbers as representatives of the numbers in themselves
The symbolic formations of number outside the system, as arithmetical problem
The first basic task of arithmetic
The elementary arithmetical operations
Addition
Multiplication
Subtraction and division
Computational methods with abacus and in columns. The natural generation of the Indian numerical system
Influence of the manners of designation on the formation of the computational methods
The higher operations
Mixing operations
Indirect characterization of numbers by equations
Conclusion. The logical sources of general arithmetic

Corrections
FIRST PART

THE GENUINE CONCEPTS OF PLURALITY, UNITY AND CARDINAL NUMBER
INTRODUCTION

The concept of number [Zahl] is multiple [divided into many species, vielfach]. This is already indicated to us by the amount of different numerical words that occur in the language of ordinary life and that are usually listed by grammarians under the following rubrics: cardinal numbers [Anzahlen] or basic numbers [Grundzahlen] (numeralia cardinalia), the ordinal numbers (numeralia ordinalia), the generic numbers (numeralia specialia), the repetition numbers (numeralia iterativa), the multiplication numbers (numeralia multiplicativa) and the fractions (numeralia partitiva).

That the cardinal numbers [Anzahlen] occur first in this list, just as the characteristic designations they receive ("basic" or "cardinal numbers"), does not rest on mere convention. They obtain linguistically a privileged position by the fact that all the other numerical words result from the cardinal words [Anzahlwörtern] by just minor modifications. (For instance, two, second [zweiter: does not work in English], twofold [zweierlei], double, twice, half [zweitel: does not work in English]. The cardinal words [Anzahlwörter] are therefore truly basic numerical words.

Language thus leads us to the thought that all the corresponding concepts also stand in a similar relation of dependence to those of the cardinals [Anzahlen], and represent certain notions with a richer content, of which the cardinals are only a part. The simplest reflection appears to confirm this.

Thus, the generic numbers (of one kind [einerlei], twofold?? [zweierlei], etc.) have to do with a cardinal number of differences within a genus; in the case of the repetition numbers /4/ (einmal, zweimal, etc.) there is a cardinal number of repetitions. In the case of the multiplication and fractional numbers the cardinal number serves the purpose of determining more exactly the relation of a whole, divided in equal parts, to one part, or of a part to the whole. If the whole is divided into n parts, then it is called the n-[fach] of each part and each part is an nth [n-tel] (n-part) of the whole. In similar fashion, the words zweiteilig, dreiteilig etc. express the cardinal number of the parts of a whole, as an external mark of it. All these and similar concepts have obviously a subordinate nature; even if they are not logical specializations of the concept of number, they are more
restricted notions, which presuppose the concept of the cardinal number in that they link the latter in some way to other elementary concepts.

This view is disputed only in the case of the ordinal numbers. The linguistic dependence of their names on the names of the cardinals is certainly not less obvious, and also the comparison of the concepts seems to point, as in the other cases, to a relation of dependence. In fact, as the cardinal numbers are related to sets, the ordinal numbers are related to sequences. But sequences are ordered sets. Thus, one would be a priori inclined to deny to the ordinal numbers a more autonomous position.

Nevertheless, outstanding researchers -and no less than W. Rowan Hamilton, v. Helmholtz and Kronecker- hold the opposite view; they go as far as to vindicate for the ordinal numbers the superiority over the cardinal numbers, in such a way that the latter should emerge just from special applications of the former. The paradox of this view can be however dispelled by the provable fact that what these researchers call cardinal and ordinal number does not correspond to the concept that one otherwise associates with those names. If one pays attention to the common and usual meanings of those names, then it becomes clear that the concept of ordinal number includes, hence presupposes, the concept of cardinal number, as the formation of the names rightly expresses.

Aside from the kinds of number of practical life there is still a long series of other types of number, which are peculiar to the science of arithmetic. The latter talks about positive and negative, rational and irrational, real and imaginary numbers, about quaternions, alternating, ideal numbers, etc.

However, different as the arithmetical expressions of all these numbers might be, they always include as parts the cardinal-number symbols 1, 2, 3..., and thus the cardinal numbers seem to somehow play also in arithmetic the role of basic numbers, to the extent that the inference from the dependence of the notations to that of the concepts is not entirely deceptive. In fact, many and among them very important mathematicians, as Weierstrass and Kronecker [the text has "Dedekind", corrected by

\[\text{Cf. [Husserl adds in Corrections: "in connection with the the two last mentioned scholars"] the appendix to the first part.}\]

\[\text{Weierstrass used to open his epochmaking lectures on the theory of analytic functions with the statements: pure arithmetic (or pure analysis) is a science which is based uniquely and only on the concept of number. It needs otherwise no other assumption, no postulates or premisses [Vordersätze]. (Thus, almost literally,}\]
Husserl on p. 324], are even convinced that the cardinal numbers are the genuine and unique fundamental concepts of arithmetic. Other researchers have, to be sure, a different opinion. For those who view the cardinals as specializations of the ordinals, the latter are usually the fundamental concepts of arithmetic. Still other authors reject both views and take the concept of linear magnitude as the fundamental concept of arithmetic. Etc.

Now, in order to adopt provisionally one point of view at all, we want to subscribe to the first considered view, as the one that is first [available] and accordingly to which we want to begin with an analysis as careful as possible of the concept of cardinal number.

Thereby no definitive judgement should be anticipated at all. It might even happen that the progress of our work in the second volume shows the unfeasibility of the presupposed point of view. At any rate, this would not deprive the following analyses from any of their value, for they are independent of any arithmetical theory and useful for all of them. Regardless of how the various parties prefer to understand the proper and original conceptual field of arithmetic, they all agree, if carefully observed, that the concept of cardinal number plays an absolutely important role in all arithmetical matters.

Who, for example, does not take that concept but the linear magnitude as the truly fundamental arithmetical notion, will not however because of that deny that the presupposed measurements of magnitudes rest everywhere on counting, i.e. on the specification of cardinal numbers; moreover, that the cardinal numbers, in the form of multiplicators and divisors, of power and radicals, etc, are indispensable auxiliary tools for the formation of arithmetical concepts.

Therefore, a highly significant position would still be secured for the cardinal numbers in arithmetic even if arithmetic was not defined as science of the cardinal numbers but as science of the linear magnitudes or in any other way.

Thus, in any case an analysis of the concept of cardinal number is an important prerequisite for a philosophy of arithmetic; and it is its first requisite, if the logical priority is not assigned, say, to the concept of ordinal number, as it has been affirmed from another standpoint.

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in the summer semester 1878, and in the winter semester 1880/81. To this the analysis of the concept of number, in the sense of cardinal number, was adjoined.
The possibility of an analysis of the concept of cardinal number completely independent of the concept of ordinal number will be the best proof against that view.

Besides, such an analysis does not just serve arithmetical purposes. The inter-connected concepts of unity, plurality and /7/ and cardinal number are fundamental concepts of human knowledge, without qualifications, and claim, as such, to have a particular philosophical interest, especially since the difficulties associated with their understanding have always caused dangerous errors and subtle disputes. The difficulties are in close relationship with certain peculiarities of the psychological construction [Constitution] of the mentioned concepts, for the clarification of which also psychology is especially interested [interesting?]. I think that the task of the following analyses is to satisfy not just those arithmetical interests, but above all these logical and psychological interests.
CHAPTER 1

THE GENESIS OF THE CONCEPT OF PLURALITY BY MEANS OF THE CONCEPT OF COLLECTIVE UNION

THE ANALYSIS OF THE CONCEPT OF NUMBER PRESUPPOSES THE ANALYSIS OF THE CONCEPT OF PLURALITY

The well-known definition of the concept of number \([\text{Zahl}]\) - thus we may say briefly for cardinal number \([\text{Anzahl}]\) - reads: number is a plurality \([\text{Vielheit}]\) of units \([\text{Einheit}]\). Since Euclid\(^3\) used it, it is always there [back].

Instead of plurality people also say manifold \([\text{Mehrheit}]\), aggregate \([\text{Inbegriff}, \text{Aggregat}]\), collection \([\text{Sammlung}]\), set \([\text{Menge}]\), etc., just names which are synonymous or almost synonymous, although not without noticeable differences \([\text{Nuancen}]\)\(^4\).

To be sure, with this definition not much has been accomplished. What is plurality and what is unit? As soon as we raise these questions we find ourselves in the midst of controversies. Some authors objected to the definition that plurality means more or less the same as number. Right about this is only that the word number can be taken in a broader sense, according to which it is really \(7\) synonymous with the word plurality. But in the narrower and proper sense it stands only for\(^5\) any individual number: two, three, four, and expresses then a thought richer than the name plurality.

Nevertheless even then a close relationship between the two respective\(^6\) concepts remains. Wherever a particular number is in question, one can also talk of a plurality, and

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\(^3\) At the beginning of the book vii of the Elements

\(^4\) In order to exclude these nuances [sic?], for the time being we avoid using just one of these terms. The reasons why we prefer in different contexts differen terms (sometimes "aggregate", sometimes "plurality" or "set") will be explained later on [p. 102 explains "Inbegriff" translated "aggregate"].

\(^5\) [translator's note: the name "Anzahl", number, "supponi[e]rt für" the individual numbers: this is an amazing example of the verb supponieren used in the strict sense of the medieval semantic theory of supposition, according to which general terms supponunt pro individuis. In contrast, on p. 165 we find a normal use: "...Frege fälschlich supponirt..."; on p. 178 a still different use: "Herbart supponirt der Einheit von vornherein die Einfachheit...".]

\(^6\) [translator's note: "respective" is for "beiderseitig", cf. Slaby-Grossmann. I suppose this use of "beiderseitig" can be exemplified as follows: John has a car, Peter has a car; John is taller than Peter; do the
wherever a plurality is considered, one can also talk of a particular number; only the numbers that thus mediate [sic? between the two concepts number and plurality] are different from case to case. The concept of number therefore covers, although through the extensions of its specific concepts, the numbers two, three, four... , the same concrete phenomena as the concept of plurality.

Also the close connection between the corresponding conceptual contents is clear from the very beginning, if one only observes that the individual numbers must be regarded as determinations of the somehow indeterminate concept of plurality. Whenever a plurality is given, the question "how many?" is in order, and the answer to that question is precisely given by the corresponding number.

It will be therefore natural to first aim at the analysis of the more general and in the indicated sense more indeterminate concept of plurality, and only subsequently to characterize the determinations by which the sequence of individual numbers and the general concept of number that presupposes that sequence emerge [emerges?].

We will put aside now, however, the above given definition of number, since it cannot be of any use for our present purposes.

THE CONCRETE FOUNDATIONS OF ABSTRACTION

With regard to the concrete phenomena which constitute the foundation for the abstraction of the concepts in question, there is no uncertainty. They are aggregates, pluralities of determinate objects. What is meant /10/ by this expression anyone knows. Nobody will doubt whether in a given case one has to do with a plurality or not, which is a proof that the corresponding concept in spite of the difficulty of its analysis is entirely sharp, and the scope of its validity is exactly defined. Hence we may regard this scope as something given, even if we are still unclear about the nature and the origin of the concept itself. The same holds, for similar reasons, for the concepts of number.
We limit ourselves at first to pluralities genuinely considered, we exclude pluralities that are considered symbolically - the correlative expression ["symbolically"] perhaps appears to some readers as the clearer one.

Let our field therefore be that of the aggregates of objects individually given by themselves [für sich\(^7\)] and collectively united. Accordingly, our analyses will first concern only the genesis and the content of the genuine concepts of plurality and number, while the analysis of the symbolic notions that presuppose the former will be considered in detail only much later.

**INDEPENDENCE OF ABSTRACTION FROM THE NATURE OF THE CONTENTS BROUGHT TOGETHER**

Now we begin with the psychological characterization of the abstraction which leads to the (genuine) concept of plurality and subsequently to the concepts of number.

The concreta to which the abstracting activity is referred we have already mentioned: they are aggregates of determinate objects; now we add also: entirely arbitrary. In fact, for the formation of concrete aggregates there are no limits with regard to the individual contents to be collected. Any object of an idea, whether physical or psychological, abstract or concrete, whether given through sensation or phantasy, can be put together with any other and with arbitrarily many others into an aggregate, and /11/ accordingly, counted as well. For example some individual trees; the sun, the moon, the Earth, and Mars; a feeling, an angel, the moon, and Italy; etc. We may always talk in these examples of an aggregate, a plurality and a determinate number. The nature of the individual contents is therefore of no relevance at all.

This fact, as simple as uncontroversial, immediately rules out a class of views concerning the origin of the concepts of number, namely those that restrict the concepts to particular domains of contents, for example to the domain of physical objects.

Already Leibniz found it necessary to fight against such errors. He says\(^8\): "the scholastics erroneously thought that number proceeds from the mere division of a continuum and

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\(^7\) [translator's note: someone proposed "of themselves"]

\(^8\) *De arte combinatoria* 1666, Opp. phil. Erdm. p. 8
cannot be applied to incorporeal things". But number is, for Leibniz, somehow "an incorporeal figure, originated from the putting together of any entities whatsoever, for instance God, an angel, a man, motion, which are, together, four". Hence (for Leibniz) number is an "universalissimum". In the same sense also Locke\textsuperscript{9} calls "number the most general of our ideas, applicable to men, angels, actions, thoughts, briefly to each thing that can be or can be thought".

The old error has found a new endorsement in the school of J. St. Mill, otherwise so meritorious for psychology. "The state of affairs expressed in the definition of a number", thinks this philosopher\textsuperscript{10} "is a physical fact". "Each of the numbers two, three, four etc. denotes physical phenomena and connotes a physical property /12/ of these phenomena. For instance two denotes all pairs of things and twelve all dozens and connotes what makes them pairs or dozens, and this is something physical; for one cannot deny that two apples are physically distinguishable from three apples; two horses from one and so on, that they are a different visual and tangible phenomenon".

This view is so evidently false that one can only wonder how a thinker of Mill's stature could be satisfied with it. Two apples may without doubt be physically distinguishable from three apples; but not two judgements from three judgements, two impossibilities from three, etc. Therefore the numerical difference as such cannot be physical, visual or tangible either. The mere reference to psychological acts or states, which can be counted equally well as the physical contents, defeats Mill's theory\textsuperscript{11}

**THE GENESIS OF THE CONCEPT OF PLURALITY THROUGH REFLECTION ON THE COLLECTIVE UNION**

Now, if the general concepts of plurality and of determinate number do not stand to the corresponding concreta, from which they are abstracted -the aggregates of determinate

\textsuperscript{9} Essay book II, chap xvi, sect 1

\textsuperscript{10} J. St. Mill's Logik, booc xxiv, cap. §5, Gomperz' translation II, 342. Cf. also book II, iv cap §7 (I, 237), where the numerical property is seen as parallel with the physical properties of colour, weight and extension.

\textsuperscript{11} Also among mathematicians we find analogous views. Examples are given by Frege in his "Grundlagen der Arithmetik", Breslau, W. Koebner, 1884, p. 27.
but arbitrary contents- in the relation of physical properties to physical things, how is that
relation to be understood?

The particularities of the contents put together and to be counted do not matter at all, as we saw. But if this is the case, how should we reach the desired general concepts? How should we conceive the abstraction process that yields those concepts? What is retained in the abstraction as the content of the concept, and what is abstracted from? /13/

Since by a previous remark [p. 10] the extensions of our concepts are available to us as something given, it should be possible for us to approximate the contents of the intended concepts through the comparison and distinction of appropriately chosen examples of the respective extensions. Disregarding the differentiating marks, we retain those that are common to all cases as those that should belong to the content of the concept in question.

Let us try to follow this plan.

It is obvious that the comparison of the individual contents that we find in the given aggregates should not yield the concept of plurality or determinate number, and it was (for this has also occurred) nonsensical to expect anything of that sort.  

[this sound mysterious but somehow explained by the following:]

In fact, it is not those individual contents which constitute the basis for the abstraction, but the concrete aggregates as wholes, in which they are brought together. However, it seems that the desired result does not happen even through the comparison of the aggregates as wholes [?]. The aggregates consist in fact only of their individual contents. How then could any common marks of the wholes be available, if the parts that constitute those wholes may be fully heterogeneous?

Also this difficulty is only apparent. It is misleading to say that the aggregates just consist of their individual contents. Even if it is easily overlooked, there is something beyond the individual contents, which can be noticed and which is necessarily there in all cases where we speak of an aggregate or of a plurality: the union of the individual elements into a whole.

The situation is here the same as in some other classes of relations; it is possible that in spite of the greatest diversity of the related contents, there be similarity, nevertheless, with respect to the uniting relations.
Thus there are equalities /14/, increases?? [Steigerungen], continuous unions over entirely heterogeneous domains, they may occur among sensible contents as well as among psychological acts. Therefore it is presumably quite possible that two wholes are as such similar [gleichartig] although their constitutive parts are mutually [beiderseits] totally heterogeneous.

Now, whenever a plurality is in question, those similar unions are the foundations for the formation of the general concept of plurality.

With regard to the nature of the abstractive process which yields our concept, we will be able to characterize it in the best way by referring to the way in which other concepts of composition (wholes) are originated. If for instance we pay attention to the connections of the points on a line, of the instants in a period of time, of the nuances of colour of a continuous series of colours, of the [sound qualities] of a [Tonbewegung] etc., we obtain the concept of the continuous union, and through it the concept of the continuum.

This concept is not presumably included in the representation of each concretely given continuum as a particular, in itself recognizable partial content.

What we notice in the concrete case are on the one hand the points, or the extended parts, on the other hand their peculiar combinations. Now, the latter are the similar which is always present whenever we speak of continua, regardless of how different the absolute contents (places, times, colours, sounds, etc.) related by them might be.

Now, the concept continuum as of a whole, whose parts are united precisely in the manner of a continuous union emerges by reflection on this characteristic union of contents.

Or let us consider, to take another example, the entirely peculiar way in which, in the case of arbitrary visual objects, the spatial extension is combined, in mutual penetration, with the colour and this in turn with the intensity. /15/ With reference to this type of
union\textsuperscript{12}, we may form again the concept of a whole, whose parts are precisely united in such a way.

We may say quite generally: wherever a special class of wholes appears before us, the concept of that class can have originated only through reflection on a well characterized type of union of the parts, similar in each whole of the class.

Now, the same happens in the case with which we are dealing here. Also of an aggregate we may say that it constitutes a whole. The idea of the aggregate of given objects is a unity, in which the ideas of the individual objects are included as partial ideas. Certainly, this union of parts, as we find it in each arbitrary aggregate, a union to be called loose and external in comparison with other cases of union, indeed so much so that one might hesitate \[?] to continue to speak here of a union at all.

Nevertheless, no matter how, there is a particular unification in this case, and it must have been noticed as such, since otherwise the concept of an aggregate (of a plurality) could have never emerged.

Therefore, if our view is right, the concept of plurality is obtained through reflection on the particular and in its peculiarity well recognizable manner of unification of contents, as shown by each aggregate, analogously to the way in which the concept of any other kind of wholes is obtained through reflection on their peculiar ways of union.

From now on, we want to use, as a designation of the union which characterizes the aggregates, the name "collective union".

/16/ Before we continue our considerations, it will be good to defeat a likely objection. If the plurality is defined as a whole whose parts are unified by collective union, then this definition is, so one could object, circular. For if we speak of "parts", then we think of a plurality, and since the parts are not individually determined, we conceive this plurality in general. Thus, we explain plurality by itself.

\textsuperscript{12}Brentano speaks here of a "metaphysical" union; Stumpf (\textit{Über den psychologischen Ursprung der Raumvorstellung}, 1873, p. 9) of the relation "of psychological parts".
In spite of all the apparent force that this objection may have, we cannot accept that it is right. First let it be observed, that we have not aimed at a definition of the concept plurality but at a psychological characterization of the phenomena on which the abstraction of that concept rests. Hence, everything that helps to this purpose must be welcome.

The plural "parts" (aside from its correlation to the concept of whole) certainly implies the general idea of a plurality; but it does not express what is it that particularly characterizes this plurality as plurality. When we added that the parts must be collectively united, we hinted at the point to which our special interest is directed, and by which the plurality is precisely characterized as plurality in contrast with other wholes.