The hierarchy of classes of Johann Caspar Sulzer (1755)

Abstract. In our contemporary set theory we are inclined to arrange sets in a hierarchy that depends on the membership relation (sets, sets of sets, etc.). Sulzer arranges sets according to the relation of inclusion, so that his second order sets are not (as we would expect) sets of sets but more inclusive sets. This is viewed as a typical consequence of the traditional theory of predication.

The history of the theory of classes is brief because, before the advent of mathematical logic, logicians preferred to work with predicates rather than with sets of objects which satisfy a predicate\(^1\). Even the term classis is late, and not properly scholastic.

In this note I will refer to an author of the eighteenth century, Johann Caspar Sulzer, who not only works with the notion of class but also uses the term classis and, what is more interesting, constructs a hierarchy of classes of the first, second, etc. order. Let us first look at the text\(^2\).

Chapter 1. Concerning concepts and in what way they are ordered in species and genera.
1. whatever we observe by means of our senses in reality is singular.
2. every singular differs from another singular, by the very fact that it is another, and is called an individual.
3. Thus all things in reality are diverse, and although they are innumerable, nevertheless, one is never another one individual is not another individual: Jacobus is not Johannes.
4. But although they are diverse, in many respects however we discover that they agree. In virtue of that respect in which they agree or are similar, they are referred to the same

\(^{1}\) In my article Leibniz's misunderstanding of Nizolius' notion of multitudo , Notre Dame Journal of Formal Logic, VI, 1965, 319- 322, I refer to the extraordinary case of Nizolius, a figure of the Italian Renaissance who wanted to view everything in terms of sets or "multitudines".

\(^{2}\) Sulzer, Johann Caspar: Facies nova doctrinae syllogisticae, qua multo plures modi figurarum syllogisticarum facillimis et certissimis regulis proponuntur quam hactenus exhibiti sunt, et quonames syllogismos, cujuscunque sint conditionis, sine ulla inmutatione conclusionis in quavis figura exhiberi posse, demonstratur, Tiguri, ex officina Heideggeri et Soc., 1755, 133 p. Original at the Staatsbibliothek Bamberg, microfilm at the University of Texas at Austin library. As noted in the preface, Sulzer was a professor at Winterthur: rector scholarum vitoduranarum. The preface was signed in that Swiss city, in Kalendis majis 1754.
class; in virtue of that respect in which they do not agree, they are referred to a distinct class.

5. Those things which pertain to the same class, have the same name; a different class obtains a different name and the name of one class is not suitable for another. For example, Jacobus and Henricus, since they are similar, belong to one and the same class, of which the name is "man". Hence each is man. But Anna and Maria, since they differ greatly from Jacobus and Henricus, constitute a diverse class, which enjoys the name woman. Therefore both Anna is woman and Maria is woman, but man is not woman nor woman man.

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6. Thus from the collation of all things many classes of things arise, and they are always diverse, since class is not another.

7. Classes which are formed from the similitude of individuals, will be classes of the first order, and are called species, for example: males, females.

8. Classes of the first order, although diverse, have a certain similitude, which similitude yields classes of the second order, or classes of species, and such classes are called genera. Thus males and females, derived from Jacobus, Henricus, Anna, and Maria, are named humans, hence males and females are species, but humans is a genus.

9. Classes, which are derived from the similitude of second order classes produce classes of the third order, and go by the name of higher genera. Such a superior genus we have when humans and beasts are compared and from the fact that both live and sense, they are ordered in one class, which class has the name "animal"; therefore animal will be a higher genus.

10. Proceeding further, thus, classes of the fourth, fifth, and the other orders are reached, and thus even higher genera are established for us, until at last we may arrive at the one ultima class, which is called the highest genus, namely being [...].

13. Whatever name then the genus has, and whatever is said of the genus, that name have also all the things that are comprehended under the genus, all the way down to the individuals, and is said of these absolutely as well. Let animal be an example; because animal comprehends humans, beasts, males, females, etc., it will be said 'human is
"animal", "beast, male, female, Jacobus, Maria, etc. is animal". And if it is said, "animal senses", then also human, beast, male, female, Jacobus, Maria, senses.

Starting from a set of things (res) or individuals, Sulzer constructs classes of things (classes rerum) which he calls first order classes. An individual belongs to a first order class if it satisfies a certain predicate. The individuals Anna and Maria belong to the class woman because Anna is a woman and Maria is a woman. The truth of the predication is the basis for the truth of the classification. The relation of similitude between the individuals Anna and Maria is a way of saying that they satisfy the same predicate. Analogously for the individuals Jacobus and Johannes. If we were asked to transcribe those examples given into the modern language of sets, we could certainly do so in the following way: Anna ∈ woman, Maria ∈ woman, Jacobus ∈ male, Johannes ∈ male, where we write the names of elements to the left of the ∈ and the names of classes to the right.

Next, Sulzer wants to compare the classes of the first order among themselves and discover among them relations of similitude which permit the introduction of new second order classes. Now then, this does not consist in finding a predicate M or a class M such that man ∈ M, woman ∈ M, where 'man' and 'woman' are names of classes and M is a class which contains classes as elements. The second order classes of our author are not classes of classes but the union of first order classes, that is, they still contain individuals as elements. It is easy to see that this holds in general for the formation of higher orders, as he indicates in 9, where the third order class animal is formed by uniting the class of humans and the class of beasts. Translating this into our language of sets, we will say that if a is a class of an order inferior to that of the class b, then a ⊂ b holds. The hierarchy of the orders of classes of our author is based exclusively, apart from the first-order classes, on the relation of inclusion.

This is not something accidental: in the Aristotelian tradition it has been customary to order the concepts (classes in the extensionalist Sulzer) according to relations of inclusion. A predicate occupies a higher or lower place in the hierarchy according to its greater or lesser universality. The tree of Porphyry is the image of this. Going up the scale of the Porphyryan tree one does not encounter concepts of concepts, predicates of predicates, or classes of classes, but always concepts, predicates, or classes of individuals. Nowadays, we tend to conceive a different hierarchy of classes; we think
rather in terms of classes of classes than of inclusion of classes. The paradigm of Prior Analytics I, 27 or of the arbor porphyriana has been replaced by a new hierarchy. The latter is clearly presented in the following text from Tarski (in fact, it is hard to imagine a better example to be contrasted with Sulzer than this passage):³

Apart from separate individual things, which we shall also, for short, call individuals, logic is concerned with classes of things; in everyday life as well as in mathematics, classes are more often referred to as sets. Arithmetic, for instance, frequently deals with sets of numbers, and in geometry our interest attaches itself not so much to single points as to point sets (namely, to geometrical configurations). Classes of individuals are called classes of the first order. Comparatively more rarely we also meet in our investigations with classes of the second order, that is, with classes which consist, not of individuals, but of classes of the first order. Sometimes even classes of the third, fourth,...orders have to be dealt with.

I wish to mention now two problems the interpretation of our text presents to us. The first of them consists in observing that there is nothing appearing in Sulzer which justifies our distinction of the two relations $\in$ and $\subset$. This distinction is introduced by us on the basis of our intuition and interpretation of the concrete examples which our author gives. But Sulzer utilizes only one term, the copula est, in order to express the relation between his individuals and the classes as well as in order to designate the relation between classes of an inferior order and superior classes. This is seen with exceptional clarity in 13. "Jacobus est vir" and "vir est animal" are two propositions in which we distinguish the relation of an individual to a class (in the first) and the relation of subclass to a class (in the second), but Sulzer uses a copula which is imperturbably unique. Just in case that were not sufficiently clear, Sulzer also utilizes a single term in order to cover the converse relation: "comprehendere". On the basis of the first phrase of 13 the two examples cited can be transformed into "vir comprehendit Jacobum" and "animal comprehendit virum".

The second problem arises à propos the phrase "classes specierum", in 8. The author presents that expression as a synonym for "classes secundi ordinis" and "species" had

been defined in 7 as equivalent to "classis primi ordinis". Consequently, we may say that classes of the second order are classes of classes of the first order. But then it seems that we lose control of the text: it had been clearly established that the classes, of whatever order, were always classes of individuals and only of individuals, never classes of classes.

In order to confront the first difficulty, there is an exegetical trick: to interpret the "individuals" of Sulzer as that which we call "unit classes". Then, for example, instead of "Jacobus ∈ vir" we should put "Jacobus ⊂ vir". In this way, we would have a single symbol for the unique symbol or copula "est" of our author. As I have pointed out in another place⁴, there is no lack of philosophical attraction or historical justification for this new focus.

I do not believe that the second difficulty can be overcome by means of interpretive tricks. The simplest is to say that it involves an error in expression, a wrong phrase. It is not difficult to imagine the origin of such an error. When we say, for example, that the second order class human is a class of males and females (homo a class of vir and femina) even though the sense is that human is a class of individuals, our phrase "class of males and females" (or the Latin equivalent) can give rise to the error of thinking or saying that human is a "class of species", that is, a class whose elements are the class male and the class female. Although this error would not be tolerated today, it is understandable that it occurs accidentally in authors who, like Sulzer, dared to work with classes in a tradition where this was not customary.

The error of expression to which we have just referred seems to be reduced to a mere consequence of ordinary language (Latin, English) which impedes the development of new technical terms. But within the same logical tradition, that is, within the same technical language of the authors previous to mathematical logic, there was a very serious difficulty for those who would attempt to work with a logic of classes. I refer to the traditional theory of predication, in force till the advent of the new logic and perhaps precisely until Frege. According to that theory "human" (homo, to follow Sulzer's examples) is predicated of male (vir) and female (femina), which are sub concepts, subpredicates, as well as of Jacobus and Anna, which are individuals. Thus the "things" which satisfy the predicate "human" are not only the individuals but also the intermediate predicates or concepts, so that the class (set, collection, etc.) determined by a predicate

becomes a heterogeneous mixture of individuals and subaltern predicates. This has already been observed in the notion of "extension" of the Port Royal Logic. In Sulzer the confusion is obvious when he says that the higher classes "comprehend", without qualifications, both subclasses and individuals, cf. 13. Surely an easy way out is to construe Sulzer's individuals as unit-classes. Whether this is historically appropriate, should be left to a further examination of Sulzer's interesting work.

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5 In my book (cf. previous note) section 4.52, I refer to the ambiguity of the "extension" of Port-Royal as a consequence of the traditional theory of predication. Kneale and Kneale ("The development of logic", Oxford, 1962, p. 318-9) point out the ambiguity but restrict themselves to denouncing it as a mere confusion owed to the "metaphorical and unclear" use of the term "inferior". The Kneales fail to show that the ambiguity of the notion of extension is not an isolated accident; it is not just the "unclear" use of the term "inferior" individually taken, it is the entire conception of predication that leads to conceive the extension of a predicate as made up of both individuals and subordinate concepts. Besides, the adjective "metaphorical" in Kneale's criticism is quite inappropriate: metaphors occur everywhere.