1. Introduction

The German word 'Bedeutung' has two meanings—on the one hand we may say, for example, that the Bedeutung of the singular term 'Bariloche' is this town; on the other hand we may say that the Bedeutung of Bariloche, the town Bariloche, is great for the Argentinian touristic development. Briefly, we may speak of the semantic meaning and the importance meaning of the word 'Bedeutung'.

Standard English translations such as 'reference', 'denotation', convey only the semantic side, and hide entirely the importance aspect. The word 'meaning' might do better, but then the translator should explicitly remind the reader that both the semantic and the importance sense are to be considered; 'significance' expresses very well the importance sense, but may fail to highlight the semantic aspect, unless the reader is, again, explicitly advised. In Spanish ‘significado' is indeed the translation to be recommended, since it expresses with equal force both aspects.

Is the ambiguity of 'Bedeutung' relevant at all for the understanding of Frege's notion? This is the question I want to raise in this paper. My answer is affirmative, but not based on any Fregean text— it is an interpretation.

The convenience of reading 'Bedeutung' as importance rather than semantically was first proposed in (Angelelli, 1967), where I insisted on the ambiguity of the German word as a key for the understanding of Frege's doctrine, especially with regard to sentences¹. This issue was also raised, later on, by Tugendhat.²

¹ This point was already emphasized in my Fribourg dissertation, submitted in 1965.
² I have some comments on Tugendhat's approach in (Angelelli, 1982).
Frege's three principal categories of expressions are singular terms, sentences, and predicates (I use a terminology different from his). In each of these expressions Frege distinguishes Sinn and Bedeutung. The case of singular terms is deceivingly simple. For the case of predicates there are not too many explanations. The interesting case is that of sentences. In fact, students of Frege should go directly to sentences, and forget about the Morning Star and the Evening Star.

2. The complex obscurity of the Fregean presentation of the "Bedeutung" of sentences

The discussion in this paper will be referred to the portion of Frege's essay Über Sinn und Bedeutung (Frege, 1892) that begins (on p. 32 of the original pagination) with the remark that "So far we have considered the sense and Bedeutung only of such expressions...as we have called proper names. We now inquire concerning the sense and Bedeutung of an entire assertoric sentence"\(^3\), and ends (on p. 36 of the original pagination) with the following statement: "Now if our view is correct, the truth-value of a sentence containing another as part must remain unchanged when the part is replaced by another sentence having the same truth-value"\(^4\).

This text, approximately two or three pages long, is devoted to a general consideration of what is the Bedeutung of sentences, and has four parts. First, Frege rules out the "thought" (Gedanke, proposition) as a candidate for playing the role of Bedeutung of sentences. The second, central part includes an argument whose conclusion plays the role, for Frege, of being a good reason to propose a certain conjecture concerning the nature of the Bedeutung of sentences. The third part describes, and successfully performs (Frege believes), a test of the conjecture. The fourth part describes, without performing, a second test of the conjecture (the actual carrying out of this second test fills the remaining twelve or so pages of the paper)\(^5\).

\(^3\) Bisher sind Sinn und Bedeutung nur von solchen Ausdrücken...betrachtet worden, welche wir Eigennamen genannt haben. Wir fragen nun nach Sinn und Bedeutung eines ganzen Behauptungssatzes.

\(^4\) Wenn nun unsere Ansicht richtig ist, so muss der Wahrheitswert eines Satzes, der einen andern als Teil enthält, unverändert bleiben, wenn wir für den Teilsatz einen andern einsetzen, dessen Wahrheitswert derselbe ist.

\(^5\) In (Angelelli, 1982) I have analyzed each of these four parts into many shorter fragments.
Any reader of Frege should be shocked by the obscurity of Frege's presentation of the notion of Bedeutung of sentences. The obscurity is multiple: I count three main obstacles or problems, and as we shall see, one of these three difficulties is itself a bundle of at least two sub-problems.

First, there is a general thesis on Bedeutung revealed by the following statement: If we now replace one word of the sentence by another having the same Bedeutung, but a different sense, this can have no effect upon the Bedeutung of the sentence. In general formulation: if Bed(E) = Bed (E'), then substituting E by E’ does not change the Bedeutung of any compound expression A(E) in which E occurs. I refer to this as "the first principle of Bedeutung", briefly BP1. This principle is not any version of the familiar Leibniz's law; the latter claims that the substitution is salva veritate. Frege's principle that it is, so to speak, salva Bedeutung. There are several occurrences of BP1 in Frege's writings (Angelelli, 1982).

Frege states his principle BP1 in an exceedingly casual way, as if the reader should already be very well acquainted with it. But this is not at all the case. On the contrary, even if the reader is a good Frege-scholar, he or she will wonder: What is the secret notion of Bedeutung that Frege has here in mind, and which allows him to assert BP1 as an obvious matter? No such notion is explained by Frege in any of his writings.

The second surprise is another general thesis on Bedeutung that emerges from the following passage: The fact that we concern ourselves at all about the Bedeutung of a part of the sentence indicates that we generally recognize and expect a Bedeutung for the sentence itself. Also: We have seen that the Bedeutung of a sentence may always be sought, whenever the Bedeutung of its components is involved. Other occurrences of the principle, briefly BP2, that seems to be presupposed here, allow us to take it as a biconditional (Angelelli, 1982, footnote 6): a compound expression has Bedeutung iff its components have Bedeutung. This may be called the principle of "existence" of Bedeutung.

As in the case of BP1, one cannot help wondering what is the mysterious notion of Bedeutung that Frege takes for granted in order to assert, so confidently, BP2 as a matter of course. Again, such a notion is never explained by Frege in any of his writings.
The third obstacle encountered by a reader of Frege's famous paper is the conjecture (German: Vermutung), that Frege advances as an answer to the question of what is the Bedeutung of sentences. This conjecture — briefly V — says that the Bedeutung of a sentence is identical to the truth-value of the sentence.

The conjecture V is presented by Frege as strongly suggested by the conclusion of a certain argument. This argument starts with the question: "What is the situation now with regard to the Bedeutung? Have we a right even to inquire about it?" and ends with the statement: "We have seen that the Bedeutung of a sentence may always be sought, whenever the Bedeutung of its components is involved; and that this is the case when and only when we are inquiring after the truth-value."

The argument is conveniently read as involving a story about three characters, all of them associated with an arbitrary sentence s: 1) the Bedeutung of parts of s (mainly understood as of singular terms in s), 2) the Bedeutung of s, 3) the truth-value of s. The first and the third characters are known, the second is the one that has to be discovered. There are three basic statements: p) all the singular terms in s have Bedeutung, q) s has a truth-value, and r) s has a Bedeutung. With these statements Frege builds three biconditionals: p ↔ r, p ↔ q, and r ↔ q. The biconditional p ↔ r is backed up, or expresses BP2. It seems that Frege's plan is to regard both biconditionals as premisses leading to the conclusion r ↔ q: a sentence has Bedeutung iff it has a truth-value.

On the basis of that conclusion Frege feels justified to submit his conjecture: "We are therefore driven into accepting the truth-value of a sentence as constituting its Bedeutung." (The word "conjecture", "Vermutung", does not occur here but shortly afterwards.)

The obscurity surrounding V is twofold: both its content is very awkward (for anyone who takes Bedeutung in the semantic sense it is hard to accept that the Bedeutung of a

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6 Wie ist es nun aber mit der Bedeutung? Dürfen wir überhaupt danach fragen?
7 Wir haben gesehen, dass zu einem Satze immer dann eine Bedeutung zu suchen ist, wenn es auf die Bedeutung der Bestandteile ankommt, und das ist immer dann und nur dann der Fall, wenn wir nach dem Wahrheitswerte fragen.
8 So werden wir dahin gedrängt, den Wahrheitswert eines Satzes als seine Bedeutung anzuerkennen.
9 In (Angelelli, 1982) I distinguished three difficulties related to V.
sentence is merely its truth-value) and the justification, given by Frege, of the plausibility of V, appears to be amazingly fallacious. In fact, suppose we view the story about the three characters as a police matter. The police is trying to identify the unknown character — the Bedeutung of a sentence. Within this context, the discovery of the biconditional \( r \leftrightarrow q \) can be celebrated as a helpful step forward in the investigation. It amounts to saying that the unknown person is always "seen together" with a known person, that the Bedeutung of a sentence is always "seen together" with the truth-value of the sentence. But it would be a surprising blunder to take, as Frege does, the "being always together" of the two items as a reason suggesting their identity, or suggesting that it is very plausible to conjecture their identity.

In sum, BP1, BP2 and V indicate that Frege is presupposing some general notion of Bedeutung of which he has not said absolutely anything in his famous paper or indeed in any of his other writings. Furthermore, one fails to understand both V and Frege's claim concerning its plausibility. Such is the complex mystery of Frege's notion of Bedeutung. (An associated mystery is that so few of the Frege scholars have worried about this.)

Notwithstanding the mystery, Frege moves on, undisturbed by the readers' troubles, and puts V to a double test. The idea of both tests is to show that something implied by V is not false. Of course, this is useful only to the extent that, if we discovered that V implies something false, then V would have to be rejected as false too.

Let \( T_1 \) be the thesis that singular terms of same Bedeutung can be interchanged salva veritate, and let \( T_2 \) be the thesis that sentences of same Bedeutung can be interchanged salva veritate. Frege claims that \( V \rightarrow T_1 \) as well as \( V \rightarrow T_2 \). In the first case \( T_1 \) is regarded by Frege as well established: Leibniz's law. But Frege is not entirely sure that Leibniz's law secures the truth of \( T_2 \). In fact, most of the famous paper Über Sinn und Bedeutung is devoted to a discussion of apparent counterexamples to the truth of \( T_2 \). The result reached by Frege at the end of the paper is that the truth of \( T_2 \) has been established with "sufficient probability".

For the poor reader who only knows about Bedeutung the trivial part on singular terms, the urgent problem is not the truth of \( T_1 \) or \( T_2 \) but the truth of the conditionals \( V \rightarrow T_1 \) and \( V \rightarrow T_2 \), which Frege assumes are true. Why should the conjecture \( V \), i.e. the identity of the Bedeutung of a sentence \( s \) and the truth-value of \( s \) entail that singular terms or sentences of equal Bedeutung are interchangeable salva veritate? This is not, however, a
new problem in the list of difficulties encountered by a reader of Frege: the assumption of BP₁ makes the two conditionals \( V \rightarrow T_1 \) and \( V \rightarrow T_2 \) even logically true. Thus, it is BP₁ what underlies the Fregean twofold test of his conjecture. Given that the second test is the most important part of Frege's paper Über Sinn und Bedeutung, we see that BP₁ is really at the center of the classic paper—consequently, not to make sense of it is certainly shameful.

3. Interpretation and clarification

The disastrous Fregean presentation of his Bedeutung concept for sentences leaves us with no choice but to look for some interpretation. As anticipated, my interpretation uses the importance meaning of the German word Bedeutung. The idea is that both meanings play a role. Of course, one has to say: importance for what. Frege was a logician and said that logic is interested in truth. The importance of expressions is, in Frege, for truth. The Bedeutung of an expression \( E \), briefly \( \text{Bed}(E) \), in the importance sense of 'Bedeutung', is for truth. But what is it? Let us imitate Frege's behavior in his plan of defining number in Grundlagen or courses of value in Grundgesetze. That is, rather than beginning with an attempt to describe the nature of \( \text{Bed}(E) \), let us begin by establishing a necessary and sufficient condition for the identity of two Bedeutungen, i.e. for statements of the form \( '\text{Bed}(E) = \text{Bed}(E')' \). Finding this necessary and sufficient condition is not difficult: The importance for truth of \( E = \) the importance for truth of \( E' \) iff \( E \) and \( E' \) are interchangeable salva veritate. This leads to the formulation of the following principle, that I call the principle B (principle of Bedeutung): \( \text{Bed} (E) = \text{Bed} (E') \) iff \( E \) and \( E' \) are interchangeable salva veritate.

This principle has the same form of two most important Fregean statements: one for numbers, the other for Wertverlauf. This abstract form may be represented as follows: \( \sim a = \sim b \) iff \( a \sim b \). If \( a, b \) are concepts, and \( \sim \) is the relation of bijection among the individuals of \( a \) and the individuals of \( b \), then \( \sim a \) is the number of \( a \), \( \sim b \) the number of \( b \). When again \( a, b \) are concepts, but \( \sim \) is the relation of applying to the same objects, then \( \sim a \) is the Wertverlauf of \( a \), or the set of a's. The former is, by current fashion, referred to as Hume's principle, the latter is the famous Axiom V. Thus, although principle B does not occur at all in Frege's writings, its "soul" is entirely Fregean.

With principle B, the problem of making sense of BP₁ disappears: Frege's first principle of Bedeutung becomes trivially true. Suppose \( E \) and \( E' \) have the same Bedeutung. \( E \) may
occur in arbitrary compound expressions $A(E)$ which do not have to be necessarily sentences. To show that $\text{Bed}(A(E)) = B(A(E'))$, let us view $A(E)$ as part of an arbitrary sentence $C$. Now the sentence $C(A(E))$ will have the same truth-value as the sentence $C(A(E'))$ by principle B. But this means that $A(E)$ and $A(E')$ are interchangeable \textit{salva veritate} in any context, since $C$ is arbitrary. Hence they have the same \textit{Bedeutung} by principle B.

The problem of making sense of BP$_2$ is greatly reduced thanks to the acceptance of principle B\textsuperscript{10}. By principle B, any expression $E$ having a \textit{Bedeutung} is interchangeable \textit{salva veritate} with some expression $E'$ — for example with itself — and any expression that is interchangeable \textit{salva veritate} with some expression has a \textit{Bedeutung} identical to the \textit{Bedeutung} of some expression. Thus, in the formulation of principle BP$_2$ the phrase "having a \textit{Bedeutung}" can be replaced by the phrase "being in the field of the relation of interchangeability \textit{salva veritate}". In the equivalent version, BP$_2$ is far less mysterious: "all expressions $E$ that are components of an expression $A$ are interchangeable \textit{salva veritate} (each of them with some expression) iff $A$ is interchangeable \textit{salva veritate}". Or: "all the expressions that are parts of a compound expression belong to the field of the relation of interchangeability \textit{salva veritate} iff the compound expression belongs to that field."

In order to solve the problems surrounding the conjecture V, we have to remember the structure of Frege's fundamental method, as applied by him to the definition of number as well as of \textit{Wertverlauf}. In the paradigmatic case of number Frege has reached, after a very long discussion, a point where he has plenty of singular terms of the form "the number of the concept $F$", briefly $N'F$, but feels unable to explain what is exactly the nature of the objects denoted by such singular terms — he does not know what a number is. So, he decides to start with determining necessary and sufficient conditions for the identity of two numbers: $N'F = N'G$ iff $F$ and $G$ are equinumerous (i.e. there is a bijection among the individuals falling under $F$ and those falling under $G$). Now he feels free to assign to the singular terms of the form $N'F$ \textit{any} denotation — as long as this assignment is compatible with the former stipulation. More generally, one has a domain of objects $\{a, b, c\ldots\}$ and an equivalence relation $x \sim y$ defined over it. One has, in addition, singular terms "$a"$, "$b"\ldots$ (for example, "the number of the concept $F$") whose meaning needs to be rigorously determined. Then one begins by stating the condition that, whatever $\sim a$, $\sim b$

\textsuperscript{10} For more details on how to handle BP$_2$ cf. (Angelelli, 1982).
are, \( \sim a \) and \( \sim b \) are identical iff \( a \) and \( b \) stand in the relation \( \sim \). The second part of the method consists in picking appropriate denotations for the singular terms of the form "\( \sim a \)", where all "appropriate" means is that the choice of entities must be compatible with the identity condition stated at the outset.

I cannot go here into a detailed criticism of this philosophically terrible procedure, to which I have referred as the "looking around" or (with a more dignified, quasi-Latin terminology) "circunspection" method. Amazingly, this terrible method has been praised by many scholars, and continues to be praised\(^{11} \). Here I just want to point out that assuming that Frege tacitly applied to his \textit{Bedeutung} the same method he applied to number and \textit{Wertverlauf}, the problems surrounding the conjecture \textit{V} vanish. First, with \textit{Bedeutung} importance it is no longer awkward to have truth-values as \textit{Bedeutung} of sentences. It is quite reasonable to say that what is important about a sentence is its truth-value. Secondly, Frege's taking the biconditional \( r \leftrightarrow q \) as a reason "driving" him to conjecture that the \textit{Bedeutung} of a sentence can be identified with the truth-value of the sentence makes perfect sense now. Frege has singular terms of the form "the \textit{Bedeutung} of sentence \( s \)", temporarily lacking a denotation, for which he only knows what principle \textit{B} stipulates, namely that "the \textit{Bedeutung} of sentence \( s = \textit{Bedeutung} \) of sentence \( s^* \) iff \( s \) and \( s^* \) are interchangeable \textit{salva veritate}. Having learned, from the biconditional \( r \leftrightarrow q \), that a sentence has a \textit{Bedeutung} iff it has a truth-value, Frege thinks that it is plausible to conjecture that the assignment, to each singular term of the form "the \textit{Bedeutung} of sentence \( s \)", of the truth-value of \( s \) as its denotation is compatible with principle \textit{B}. Of course, as long as that compatibility is not fully demonstrated, conjecture \textit{V} will remain only a conjecture. This is exactly what happened. As said, at the end of \textit{Über Sinn und Bedeutung}, Frege regards \textit{V} as established "with sufficient probability" — not as demonstrated.

\(^{11} \) The looking-around method can be overhauled into the method of modern abstraction (Angelelli, 1979). There is indeed a secret desire of doing genuine abstraction behind the looking-around method, shown in the fact that the choices made in the second stage do not matter, as long as they comply with the condition put forward in the first stage. This is no justification yet, however, to refer to the looking-around method, without a previous overhauling, as "logical abstraction" (Dummett) or "definition by abstraction" as so often done in the 20th century, following a curious misunderstanding of Peano's original phrase (\textit{definizione per astrazione}).
By way of final remarks, I would like to point out, first, that the proposed interpretation generates some new problems of its own\textsuperscript{12}. Secondly, it is tempting to extend to \textit{Sinn} a similar interpretation. Frege's leading idea appears to have been the following: \textit{Bedeutung} has to do with \textit{Wahrheitswert} (truth-value) just as \textit{Sinn} has to do with \textit{Erkenntniswert} (cognitive value). Thus, the first step in an analysis of \textit{Sinn} would be to introduce a "principle of sense", principle S, which would be like principle B except for the insertion of the notion of a "person" relative to whom the interchangeability of two expressions \textit{salva veritate} takes place (Angelelli, 1978). Perhaps: "two expressions have the same sense iff for every person \textit{p} the two expressions are interchangeable \textit{salva veritate} within any sentence accepted as true by \textit{p}". In order to imitate Frege's method, once this stipulation is made concerning the necessary and sufficient conditions for identity of senses, one should "look around" and find suitable entities to be assigned to the singular terms of the form "the sense of 'E'" as their denotation. Considering that only "sufficiently educated" persons should be admitted in the range of the quantifier "for all persons...", one easily anticipates the difficulties of such an approach.

References


\textsuperscript{12} Some of them have been considered in (Angelelli, 1982).