Two attempts to disenchant the "consequentia mirabilis"

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Nem hic maxime videtur esse cyanic primume veritatis veluti character, ut non nisi exquisita aliqua redargutione, ex suo ipsius contradictorio, assumpto ut vero, illa ipsa sibi tandem restitutu possess

Abstract. After a preliminary section (1) in which some of the personal references included in the original presentation of the paper are kept, the nature of the consequentia mirabilis is explained and some alleged examples thereof are offered (2, 3). The admirable type of reasoning has been both admired (4) and criticized (5). The main objective of the essay is to examine those criticisms which, while granting the correctness of the admirable reasoning, say that this type of inference is, under close examination, quite trivial: nihil mirum (nothing admirable)! I refer to such criticisms as attempts to "disenchant" the consequentia mirabilis. Two examples are considered: Aldrich's and Couturat's; both are found unsuccessful (6, 7). In (8) it is observed that there is nothing wrong with the same thesis having an admirable as well as a non-admirable proof. A brief concluding comment on current "admirable" research is offered in (9).

1 Personal introductory remarks

It is for me a great pleasure, both personal and academic, to participate in this meeting in honor of Christian Thiel, and I am deeply grateful to the Leibniz-Förderein Altdorf-Nürnberg for having invited me to come – from very far indeed.

1 For chiefly this seems to be as it were the character of every primal verity, that only by a certain exquisite argumentation starting from its contradictory, assumed as true, it can be at length brought back to its own self. Saccheri, Euclides Vindicatus, last scholium, p. 237.
One special theme in the *Briefwechsel* between Christian Thiel and myself, extended over forty-five years, has been our common interest in Gerolamo Saccheri as well as in a topic loved by Saccheri: the *consequentia mirabilis*.

Professor Thiel has contributed one of the best essays on the issue, shedding first-class scholarly and conceptual light on it: *Clavius und die consequentia mirabilis*. But there are other expressions – less academic, more personal – of Christian Thiel’s interest in the admirable consequence and its champion Saccheri. I hope you don’t mind, dear colleague, if I refer to a postcard you sent to me, in the Summer of 1983, from San Remo – Saccheri’s birth place in 1667. Not the Italian sunshine was the important fact to report from your holidays – rather, you preferred to tell me: “Ich bin durch die Via Padre Gerolamo Saccheri gegangen”; and you regretted that the Italian postal service had overlooked the great man from San Remo: “eine Briefmarke von Saccheri gibt es noch nicht”.

2 The admirable reasoning: its nature. Terminology

We have learned from Frege to sharply distinguish between concepts and objects. No problem with describing the concept of admirable consequence, just like no problem with describing the concept of round square, or of an extra-terrestrial intelligent being. But, are there objects falling under such concepts? It is in this Fregean spirit that I will begin by harmlessly describing the concept of *consequentia mirabilis*.

You want to prove $p$ and you start, as in any indirect proof, with the assumption that $p$ is false, i.e. not-$p$. Then, if the proof succeeds – admirably – you reach $p$ but not by stumbling, anywhere in the course of the inference, upon an absurdity or contradiction which leads to not-$p$ hence to $p$. As Saccheri puts it, one reaches $p$ *ostensive ac directe* (ostensively and directly, *Logica*, I, ch. 11, at the beginning). This is indeed a peculiar sort of indirect reasoning – a *classe remarquable* thereof, to use Vailati’s French phrase (cf. references); indirect only because the starting point is the negation of the thesis to be proved, otherwise quite direct.

With regard to the terminology, the first statement of admiration was made by Cardano in the 16th century (cf. Thiel, Bellissima and Pagli). Thiel has traced the combined phrase “consequentia mirabilis” back to the early Jesuit Antonius Pérez. Alas, Pérez (1559–1649) indicates that others before him have used the phrase: “hanc consequentiam, quam Authores merito vocant mirabilem” (“this consequence, which some authors rightly call admirable”, Bellissima and Pagli, p. 85). The admiration can be expressed, of course, with words other than “admirable”. For example, Saccheri, one of the greatest fans, has the adjective “exquisita” (cf. quotation at the beginning of this essay). The word “dilemma”, used by Guelinx (*Logica*, pars IV, sectio 2, cap. 14.4) seems inadequate in the sense of too broad, since in the set of premises of a dilemma: \[ p \lor \neg p, p \rightarrow p, \neg p \rightarrow p \] the only “admirable” ingredient is the (inference reflected by the) conditional \( p \rightarrow p \). In this paper the “admirable” terminology will be kept, but if one prefers an emotionally neutral descriptive phrase that avoids the reference to “admiration”, the best is found also in Guelinx. Quite unemotionally, this author points out that *demonstratio negativa duum generum est* (the negative demonstration is of two sorts), the one negative per indirectum (which first infers an *absurdum*) and the other *negativa directa* (*Disputationes Metaphysicae*, Isagoge Pars Prima, VIII).

3 Examples of admirable reasoning

Having described the concept of the *consequentia mirabilis*, the question arises: are there actual inferences that fall under such a concept? (I will not discuss the issue of whether for the existence of such inferences it is sufficient to prove the possibility of the concept, similarly to Leibniz in connection with the notion of God).

Although, as noted by Guelinx, it is *rarum* that a proposition follows from its own negation (*Logica*, pars IV, sectio 2, cap. 14.4), instances of the admirable consequence have been alleged, rightly or wrongly, from various fields: logic, mathematics, theology, general philosophy. Probably the best logical examples are in Saccheri’s *Logica* (cf. my *On Saccheri* and my *Saccheri’s postulate*). General philosophical examples have been adduced from Plato’s *Theetetus* to Descartes’ justification of the claim “I exist” (cf. Vailati, my *Descartes*). Brunschvicg thought the Cartesian argument for “I exist” was “irresistible” precisely because of being of the admirable type; Brunschvicg regards Saccheri’s arguments as rigorous because the truth of their conclusions, “comme la vérité du cogito cartésien, […] s’affirme dans sa négation même” (as the truth of Descartes’ cogito, it affirms itself through its own negation, p. 316). Hintikka’s “performative” interpretation of Descartes, intended to be opposed to a deductive interpretation, is perhaps nothing else but an instance of the admirable type of deduction. A number of mathematicians have been mentioned in the secondary literature (cf. Vailati, Thiel, Bellissima and Pagli) as contributors to the admirable gallery. The phantasy of the collectors of admirable inferences is likely to be excited by Fermat’s statement (in connection with his famous theorem) on the narrow margin of his copy of Diophantus’ treatise: “cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadrautos, et generaliter nullam in infinitum ultra quadratum potestatem duos eisdem nominis fas est di-

Euclid considers a sequence of natural numbers, starting with 1, then having as second term a number A, then as third term A.A, and so on. He asks us to suppose that the prime number E divides the n-th term A^n. Given these hypotheses, Euclid claims that E divides A too. Euclid’s proof starts in the indirect style: he assumes that E does not divide A. Without losing anything that is relevant for our purposes, we may restrict our consideration to the shortest case: just three terms: 1, A, A.A (Euclid considers four terms besides the initial unit). We have assumed that A.M = A.A. Hence A/E = M/A. At this point Euclid applies earlier results, according to which the fraction A/E is irreducible, and, because of E being prime, its numerator divides the numerator of any other, equivalent fraction, and its denominator divides the denominator of any other, equivalent fraction, i.e., in this case, the denominator E divides the denominator A. The desired result: E divides A has been attained but is still dependent on the supposition (E divides A). Logical readers of Euclid know that the supposition (E divides A) can be discharged by conditionalization. Thus the conditional “¬E divides A → E divides A” emerges as a new line in the derivation, no longer dependent on the supposition that E does not divide A. The tautology “¬E divides A → E divides A” helps us then to detach the desired conclusion: E divides A. To be sure, this proposition, while independent now of the supposition ¬E divides A, still may depend, aside from the particular hypotheses of Theorem IX 12 (E prime, E divides A.A), on the previously constructed Euclidean theory. The striking fact remains that in the path from “¬E divides A” to “E divides A” no absurdity has emerged as a reason to deny the supposition “¬E divides A”.

Two remarks are in order. First, it must be acknowledged that Euclid’s text offers only a virtually admirable proof. Surely, it is well known that logic in general, not just the admirable reasoning, in Euclid remains tacit; Frege rightly views his own achievement as “going beyond Euclid” in the precise sense of demanding that the inferential rules be explicitly stated. Secondly, the demonstration is wrapped up with a superfluous extension of it into a regular indirect proof, which easily misleads the reader into thinking that IX 12 is proved ac-

according to the usual reductio ad absurdum. Obviously, once the proposition “E divides A^n” has been reached admirably, one can create the conjunction “E divides A and ¬E divides A”, by using the supposition “¬E divides A”, and then proceed to the negation of the same supposition “¬E divides A”, i.e. to “E divides A” again but no longer depending on “¬E divides A”. This unnecessary addition, as pointed out by Vailati, does not occur in “many among the best interpreters of Euclid”, and may well be an “interpolation” (A proposito, in Scritti, p. 231; cf. also Bellissima and Pagli, p. 30).

Also it is conceivable that the ordinary reductio ad absurdum was adjoined for pedagogical reasons (Vailati, in Sur une classe remarquable, mentions something similar involving Gauss and Legendre). Besides, it is awkward that the text, from the contradiction “E divides A and ¬E divides A”, does not move straightforwardly to the negation of the negation of the thesis to be established (¬¬E divides A), but to the negation of the proposition that E and A are relative primes (this proposition occurs in the derivation as established on the basis of other assumptions, not as a supposition).

4 The “consequentia mirabilis” admired

Some authors have thought that the argument is fortissimum, and that any proposition established by it is necessarily true – indeed summe necessaria (Galenus, Logica, pars IV, sectio 2, cap. 14.4; cf. Saccheri’s text quoted at the beginning of this essay).

With particular reference to Euclid’s theorem IX 12, important men, in the past five hundred years, have expressed their enthusiasm towards Euclid’s proof of it. Clavius writes in a scholium in his commentary on the Elements:

“Est autem admirabilis prima huius propositionis demonstratio. Nam in ea Euclides ex eo, quod E, dicatur non metiri ipsum A, ostendit demonstratione affirmativa E, ipsum A, metiri: quod videtur fieri non posse. Nam si quis demonstrare instituat, Socrates esse album, ex eo, quod non est albui, paradoxum aliquid, et inopinatum in medium

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1 It is not possible to decompose a cube into two cubes, or a fourth power into two fourth powers, or in general any other power, apart from the square, into two powers of the same name. Of which I have found a demonstration, admirable indeed. The narrowness of this margin does not hold it.
2 Grundzüge, Vorwort, p. vi: “Ferner, und darin gehe ich über Euclid hinaus, verlange ich, dass alle Schluss- und Folgerungswisen, die zur Anwendung kommen, vorher angeführt werden“ (Moreover, and in this I go beyond Euclid, I require that all the used types of inferences and deductions be previously listed).

5 It is however awkward that Vailati says, ibid., that the “true demonstration” ends, after having obtained “E divides A”, with the assertion: “but also E does not divide A, which is absurd”. This is not part of the admirable proof, and consequently one wonders if Vailati was clear about the nature of the latter.
6 Ian Mueller, who has produced the most comprehensive modern commentary on Euclid, without however seeming to join the group of admirers, writes that Euclid “proceeds in a confused, indirect way“. It is not clear to me whether he refers to the superfluous reductio ad absurdum or to the entire demonstration.
videatur affere: Cui tamen non absimile quid factum hic est in numeris ab Euclide, et in aliis nonnullis propositionibus, quae sequuntur."

I suppose that by "first demonstration" Clavius means the proof without the above mentioned final, indirect part. Further, I have not been able to find the other propositions in Euclid that, according to Clavius, are proved in an equally admirable way.

A second example is Saccheri, who is a great admirer even though he does not seem to use the word "admirable".

A third example is from the early 20th c.: George B. Halsted, one of the first professors of mathematics at the University of Texas at Austin. I have published elsewhere his letter to the mathematician R. L. Moore from June 21, 1919. The following portion deserves being quoted here:

"Show that Euclid IX, 12 is in sharpest opposition to the Reductio ad absurdum. The R.a.a. proves by reaching a contradiction of a known theorem. Thus the first Reductio, Euclid IX, 5, proves the part equal to the whole, then accepted as an absurd proposition. Euclid IX, 12 proves no absurd theorem. On the contrary, it gives a direct demonstration of the very theorem we wish to prove. It is a wholly new method of proof, which never occurs before (or after) in Euclid. It is this new method that Saccheri is trying, expecting it to give a direct demonstration of the parallel postulate."

Halsted hopes that Moore will grasp the new deductive style, especially through the easy case of IX 12, and ends his letter as follows: "Sleep on it, and give me a spark from your genius."

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5 The "consequentia mirabilis" rejected

To close the list of admirers, I will mention Christian Thiel, who in one of his frequent visits to Austin, gave us a beautiful diagram meticulously displaying the structure of the demonstration of IX 12. Showing Thiel's diagram is the most effective way, pedagogically speaking, of introducing students and colleagues to the peculiarity of the consequentia mirabilis.

5 The "consequentia mirabilis" rejected

There has been opposition to the admirable reasoning, in different ways. For example, Huygens, as reported by Bellissima and Pagli (p. 10), claims that la consequentia mirabilis è una forma imperfetta, non conclusa, di dimostrazione per l'assurdo" ("the consequentia mirabilis is an imperfect, unfinished form of demonstration through the absurd"), whereas Lambert, according to the same authors, denies the very possibility of admirable inferences (p. 123ff.). C. I. Lewis claims that the "reaffirmation through denial" is circular (The structure of logic, pp. 507–8) as well as that it is fallacious (Mind and the world order, p. 205). Similarly, A. Emch views Saccheri's "admirable" proofs of some metatheorems of the syllogistic as "paralogisms" (The logica demonstrativa, p. 230), and says that "a reaffirmation through denial is invariably open to criticism at least on the grounds that it is ambiguous, condones vicious circle fallacies, and is really not a test of truth at all" (p. 227). Finally, some authors observe that the reasoning involved in the consequentia mirabilis is logically correct but that, under closer inspection, it is quite trivial, not admirable at all; such authors may be described as planning the disenchantment of the admirable consequence.

In this essay I will just focus on the disenchantment type of criticism, without trying to properly interpret and classify all the various objections.

6 Aldrich's attempt to disenchant the "consequentia mirabilis"

Aldrich, in his Compendium Artis Logicae argues against those who claim that in certain demonstrations "conclusionem ex sui contradictoria, per legitimas necessariasque consequentias directe inferri".

"Dico igitur, quod nulla hujusmodi demonstratio supponit solam suae conclusionis contradictoriam, sed quaelibet cum contradictoria ponit aliquod quod eam evertit, et evertere demonstrando ostendit. Quare conclusionem non infert ex ejus contradictoria, sed ex contradictoria cum

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7 The first demonstration of this proposition is admirable. For in it Euclid from the fact that E is assumed not to measure A, shows by an affirmative demonstration that E does measure A, which seems to be unfeasible, because if someone plans to prove that Socrates is white from the fact that he is not white, he appears to generate a paradox and something unexpected; but this is not different from what has been done by Euclid with regard to numbers, here as well as in some propositions that follow.

8 Bellissima and Pagli (p. 32, footnote) say that, if not Clavius, Tacquet cites one further example.

9 Cf. my Saccheri's postulate. The original is kept in the R. L. Moore archive, University of Texas at Austin. Halsted taught at the University of Texas till 1901, when he was fired by the Board of Regents (I owe this information to A. C. Lewis). Interestingly, it was thanks to scholars like Halsted that the University of Texas deserved Vailati's congratulations for the inclusion of courses in the history of science, and in particular of mathematics; cf. Vailati's note "The Catalogue of the University of Texas for 1893–94", in Scritti, vol. III, originally published in Peano's Rivista di Matematica, v. 4, 1894.

10 The conclusion is directly inferred, by legitimate and necessary consequences, from its contradictory.
contradictoriae eversiva, quod si faciat nihil mirum” (Artis logicae, 133–35, emphasis mine; this text is cited by Hoormann in the unpublished essay listed in the references).

Aldrich has a point. For example, he leads us to view the above quoted Clavius’ description of Euclid’s proof (“Because in it Euclid shows, by an affirmative demonstration, that E divides A from the supposition that E does not divide A, which appears to be impossible”) as inaccurate. The same criticism applies to Saccheri’s statement quoted at the beginning of this essay. Thanks to Aldrich’s observation, a clause should be added: “from the supposition that E does not divide A in conjunction with some other assumptions”. There are other suppositions, in addition on the sheer negation of the desired conclusion.

On the other hand, once it is acknowledged (as Aldrich does) that the supposed negation of the desired conclusion, albeit together with other assumptions, does play a role in the proof, and the latter is not of the usual indirect type, then it becomes a subjective matter whether one wants to call such a proof “admirable” with Clavius, “remarquable” with Vailati, or rather say that there is nothing admirable (nihil mirum) to it, with Aldrich. The sheer truth remains that the reasoning starts indirectly but reaches its goal, as Saccheri says, ostensive ac directe and, in this sense, Aldrich’s attempt to disenchant the consequentia mirabilis is not successful.

7 Couturat’s attempt to disenchant the “consequentia mirabilis”

A second, but more threatening attempt to disturb the peace of mind of the lovers of the consequentia mirabilis is found in Couturat, who rightly says (p. 36) that the reasoning in Euclid IX 12 can be described by the formula (¬p → p) → p, and regards the thereby expressed inference as a “mode de raisonnement tout à fait paradoxal” (“an entirely paradoxical type of reasoning”). He observes, however, that the paradoxical air is quickly dispelled by modern logic — indeed “seule la Logistique explique et justifie” (only the Logistique explains and justifies) the apparent paradox. The dispelling occurs, according to Couturat, by observing that the antecedent ¬p → p of Clavius’ law is equivalent to p ∨ p, and replacing the former by the latter. What is presented as an admirable derivation of p from ¬p is really a trivial move from p ∨ p to p!

The same trivialization appears to be recommended by Frege in his comment on theorem 43 of the Begriffsschrift, which presents the law (¬p → p) → p in bi-dimensional notation and with “a” as sentential variable instead of “p”. Frege paraphrases his theorem as follows: “if the choice is only between a and a, then a occurs”. The trivialization has found its way to the heart of the late 20th century “logic for the millions” (to use the title of Gilbart’s 1851 manual): when Copi’s manual asks, as an exercise, to prove that (¬p → p) → p is a tautology, the hint given to the student is to replace the antecedent ¬p → p by p ∨ p (9.2, exercise IV I).

Now, it is easy to say that ¬p → p is to be replaced by its equivalent p ∨ p after having introduced by conditionalization the conditional ¬p → p. The real issue for Couturat (or for Frege, assuming that Frege was even aware of Euclid’s proof or that he would be interested in its disenchantment) is how to replace the process leading from ¬p to p by a sequence of reasoning that does not go from ¬p to p. As long as this is not done, the Couturat trivialization of Euclid’s demonstration of IX 12 remains a ludicrous failure.

Now, Couturat might reply that the earlier theorem VII 30 in Euclid helps his disenchantment project. VII 30 says that if a prime number E divides a product AB, then E divides A or E divides B. Euclid proves VII 30 in the same way he proves IX 12: The product AB = EX, hence D/A = B/E, but D/A is “minimal”, hence D must divide B.

From the point of view of the earlier theorem VII 30, IX 12 is just the special case A = B. With the previous knowledge of VII 30, IX 12 says that if E divides the product AB, then E also divides A or E divides B, which generates the conclusion “E divides A” without any admirable procedure but only applying Frege’s remark: “if the choice is only between a and a, then a occurs”, and IX 12 hardly deserves the status of a theorem.

Fans of the admirable consequence should gather to determine whether this possible Couturat—reply achieves its goal. Cautiously, I would argue for the negative: in proving IX 12 it is legitimate to proceed as if VII 30 had not been established, so that the proof of IX 12 continues to be what it actually is: admirable, without any trivial choice between two equal disjuncts. That IX 12 can be proved as a trivial corollary of VII 30 does not entail that a genuinely admirable proof cannot be given as well.

8 The same proposition can be proved both admirably and non-admirably

There is nothing wrong with having two proofs, one admirable, the other trivial or at any rate non-admirable of the same thesis (this seems to be contrary to...
Saccheri’s view, expressed in the passage quoted at the beginning of this paper, as well as to the opinion of the first admiral, Cardanus, as reported by Bellissima and Pagli, p. 12: l’ineluidibilità dell’ intervento della Consequentia Mirabilis, the unavoidability of the use of the admirable consequence).

Saccheri, after having established several theorems on syllogistic theory in a non-admirable style, proves them, again, according to the via nobiliar, in a “more noble” manner, which is the way he refers to admirable proofs. Saccheri’s examples are for cases with two premises (syllogisms). Let us consider a shorter example, involving just one premise, not in Saccheri but quite within his spirit.

That the categorical negative “Some P is not Q” is not convertible is proved by Aristotle by the method of interpretations. Let P = animal, Q = human. The given formula becomes a true sentence but its converse: Some Q is not P, becomes a false sentence. Saccheri does not like this Aristotelian reliance on such interpretations (humans, animals ...). He prefers the via nobiliar, which would be as follows. One begins, as in any indirect proof, with the supposition that the desired conclusion is false: not-not-O is convertible, i.e. O is convertible. Consider then the following interpretation (no longer extra-logical but using terms already constructed within the logical theory): P = being convertible, Q = being a categorical sentence of type O. Under this interpretation, the formula “Some P is not Q” comes out true. As we are assuming that O is convertible, its converse “Some Q is not P” must be true as well. However, this converse is telling us that some O sentences are not convertible which, as Saccheri knows, entails that the categorical O is not convertible simpliciter.

9 Concluding remarks

The consequentia mirabilis is alive and well. The volume published by Bellissima and Pagli in 1996 is an indication of the increasing interest in this topic (one detail to be added to the otherwise wonderful book: a reference to Thiels’s essay). New or unexplored primary sources are being examined, making up for unfinished projects (such as Hoormann’s). For example, already in 1983 Thiels observed that two Polish authors (cited by Łukasiewicz) refer to an earlier, third source, the 16th century theologian Antonius Perez, as the source of the phrase. Next, Bellissima and Pagli (p. 86) moved further, and gave concrete textual and biographical references to the Jesuit Perez. The search must focus now on Perez and his sources, his “aureos”. Perez was called Pontecereginesis, from the town Puente la Reina, in Navarra. Schmutz provides good information for anyone who wishes to pursue the admirable story from Perez backwards. Hopefully, Perez’s works become available in a foreseeable future within the editorial project of the Pensamiento clásico español de los siglos XIV al XVII, at the University of Navarra. Another sign of scholarly activity on our topic is the finding (reported to me by Bellissima and Pagli) of an unpublished Leibniz manuscript on the consequentia mirabilis – an event most appropriate to be mentioned in this Leibniz-Forum. Finally, the consequentia mirabilis is not only a historiographical theme: it is also an important part of recent logical research (Sanz).

I think it is healthy for logic that attention is paid, again, to a historical topic like the admirable consequence, moving away from the excessive ordinary language linguistics or phenomenism that has prevailed in recent decades. The subsequentia mirabilis is a marvellous lieu de rencontre of philosophy, mathematics, and logic, in a framework of serious interest towards history – and its discussion is an excellent intellectual exercise for our retirement years (ultim sine litteris mors est).

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