Saccheri's Postulate

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0. Introduction

Three jewels have been contributed to logical theory by Saccheri in the Logica Demonstrativa: his postulate stipulating the existence of various types of predicates, his proposed "nobler" method for achieving, without the postulate, the same results as obtained with the postulate, and, as a bonus, a brilliant use of the consequentia mirabilis in at least some of the applications of the nobler method.

It took almost two centuries for the scholarly world to discover the third of these gems, and there is by now a certain literature on it. The other themes have been neglected, perhaps overshadowed by the "admirable consequence", although they alone would suffice to secure for our author an outstanding place in logic. The present paper is intended to correct this situation by focussing on the postulate.

The postulate as stated by Saccheri is examined in section 1. The postulate as used by Saccheri is examined in section 2. In section 3 I describe the significance of the postulate not only relative to Saccheri's Aristotelian logic but also relative to our modern logic. In section 4 I propose a definition of Saccheri's "nobler way" (i.e. postulate-free logic) in terms of a restriction to "internal" interpretations in the use

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1 The research involved in this paper was partly done under a FRA grant from the University Research Institute of The University of Texas at Austin, 1990-1991.
2 As scholars seem to unanimously report, Girolamo Saccheri (or Hieronymus Saccherius) was born in San Remo, Italy, 1667 and died in Milano, 1733. He entered the Society of Jesus in Milano, 1685; after ordination in 1694, he taught philosophy and theology in Turin, and mathematics in Pavia. One little embarrassing textual issue is that the 1697 edition (reprinted by Olms, and consequently the only one really available today) does not show Saccheri's name anywhere.
3 It is very revealing that a rather extensive logical bibliography published in Turin (cf. Pavesio), a few decades after Saccheri's death, i.e. very close to him both chronologically and geographically, does not even hint at the Logica Demonstrativa or at its author.

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of the "method of interpretations". Finally, in section 5 I introduce the phrase via nobilissima (not in our author) to refer to a via nobilior that is even more "admirable."

1. The Literal Postulate

The following terminology is standard. Term (terminus) is an expression that can be subject or predicate of a sentence (in the traditional, pre-Fregean sense of "subject" and "predicate"). Examples: i) Petrus est homo (Peter is (a) man), where terms are "Petrus", "homo" ("est" is not a term); ii) homo est animal (man is a living being), where the terms are "homo" and "animal" (here we notice the difference between pre-Fregean and Fregean predication: for Frege homo is not subject of the predicate animal but is a predicate "subordinated" to animal). Terms are singular (singularis: "Peter") or common (communis: "homo", "animal").

Less known today is the following distinction of four relations between two terms: 1) irrelevance, 2) relevance by repugnance, 3) relevance by mutual consequence, 4) relevance by non-mutual consequence. Two common terms are irrelevant (impertinentes) if neither term infers or excludes the other (white, warm); relevant (pertinentes) by repugnance (repugnantia) if one excludes the other (white, black); by mutual consequence (sequela mutua) if both terms imply each other (animal, sensitive); by non-mutual implication (sequela non mutua) if one implies the other but not viceversa (animal, man). In the non-mutual implication, the implying term is called inferior (man), the implied term is called superior (animal).

Saccheri states his postulate as follows: "It is postulated that not all terms are relevant by mutual consequence, or by repugnance, but that some are superior terms and some inferior, and some irrelevant" (Postulatur non omnes terminos esse pertinentes mutua sequela, aut repugnantia, sed quosdam esse terminos superiores, et inferiores, quosdam etiam impertinentes, Logica, I, ch. 4).

Strictly, the postulate is neither true nor false ("let there be such and such terms..."'); its propositional content ("there are such and such terms...") is true or false. In talking about the postulate this distinction should be kept in mind, although for brevity it can be often ignored.

The postulate makes four claims: 1) not all couples of terms are in the relation of relevance by mutual consequence, 2) not all couples of
terms are in the relation of relevance by repugnance, 3) some couples of terms are in the relation of relevance by non-mutual consequence, 4) some couples of terms are in the relation of irrelevance.

It seems reasonable to read claims (1) and (2) as saying that, of course there are terms that are mutually repugnant and terms that are relevant by mutual consequence, but also that there are terms that are not mutually repugnant and that there are terms that are not relevant by mutual consequence.

Thus, the postulate says that there is at least one couple instantiating each of the four relations.

For the satisfaction of the postulate at least four different predicates are needed (for instance, T₁ relevant by repugnance to T₂, T₁ inferior of T₃, T₃ irrelevant to T₂, T₁ relevant by mutual consequence to T₄), or three if we content ourselves with the relevance by mutual consequence of any term to itself.

After stating the postulate, Saccheri makes the following very important remark: “That this is true is clear; however, since it cannot be proved, at least in logic, it must be postulated in order that we may proceed scientifically.” (Hoc verum esse manifeste constat; quia tamen probari non potest, saltem a logica, debet postulari, ut scienti
cere procedamus).

Saccheri affirms that the non-emptiness of the four relations between terms cannot be established by logic, and this is for him the reason why it must be postulated.

2. The Postulate as Used

In this section I will examine Saccheri’s use of the postulate in the proofs of several propositions that occur between the statement of the postulate and chapter 11 (of the first part of the Logica).

1) Chapter 4, Proposition 3: “Two contrary propositions cannot be at the same time true, but can be at the same time false”. For the proof of the second part of this proposition Saccheri uses the postulate. He writes:

Let A and E be two contrary propositions, one of which, say A, is the negation of C, whereas the other, E, affirms “CÆF”. They will be contraries, because one of them namely E, says more than is needed to falsify the other, A. [...]. If it happens that C is true and F is false, the two propositions A and E will be false. [...] If it cannot happen that C be [quod C sit] without F being at the same time [quin simul sit F], the terms C and F will be relevant by implication; but the terms C and F represent any arbitrary terms; hence all terms will be relevant by implication, which is absurd, and against the postulate. Hence it can happen that C be [quod sit C], while at the same time F be not [et simul non sit F].
The letters A and E obviously refer to the classical universal affirmative and the universal negative categorical sentences respectively. The modern reader may be confused by the use of the variables C and F: they seem to range both over sentences and terms. Sententially, C may be thought of as the particular negative O, and F as the negation of I. In fact, E implies not only O but also the negation of I, i.e. F, while A implies the negation of O, that is C.

In order to show that A and E can be both false it is enough to suppose that C is true while F is false, or according to our reading, that both O and I are true.

If it was not possible to have both C true and F false, i.e. not both O and I true, then O would imply not-I, that is O would imply E. E however is convertible, and the converse of E implies (by subalternation) the converse of our initial O. Thus, if A and E cannot be both false, O is convertible. But the convertibility of O entails the convertibility of A, which makes all terms pertinentes sequela, relevant by consequence, and indeed by mutual consequence. Saccheri does not add this last point perhaps because he realized it was redundant; in fact, if for every t and t’, t and t’ are relevant by consequence, then for every t and t’, t and t’ are relevant by mutual consequence.

2) Chapter 7, Proposition 3: “The universal affirmative is convertible in part (per accedens), not simply.” Here the postulate is used by Saccheri for the second part: “all B are A” does not imply “all A are B” since there are terms t1 and t2 such that the interpretation of B as t1, of A as t2 verifies “all B are A” and falsifies “all A are B”.

3) Chapter 9, Proposition 3: “No term can be posited in the conclusion which was not posited in the premises.”

The point of this proposition is not to show that implication claims such as: \{S \rho M, M \sigma P\} \implies S \tau X, where \rho, \sigma, and \tau are any of the a, e, i, o relations of the four categorical sentences, are not valid syllogisms (they are not even syllogisms from a syntactical point of view) but simply to show that they are not implications. Saccheri proves his proposition as follows:

For if an affirmative conclusion is sought, the newly assumed term may be relevant by repugnance with respect to the other term of the conclusion. If, on the other hand, a negative conclusion is sought, the newly assumed term may be relevant by consequence, with respect to the other term of the conclusion (si enim velis conclusionem affirmatissam, poterit terminus de novo assumptus esse pertinentes repugnantia relate ad reliquam terminum conclusionis. Si autem velis conclusionem negativam, poterit terminus de novo assumptus esse pertinentes sequela relate ad alium terminum conclusionis).

This shows that what Saccheri has in mind goes beyond the strict,
literal formulation of the postulate in chapter 4. Here Saccheri is assuming that for every term t there is a t’ such that t and t’ are relevant by repugnance, and there is a term t” such that t and t” are relevant by consequence.

4) Chapter 9, Proposition 4: ‘‘From premisses including four terms no conclusion follows.’’

Here, as in the preceding proposition, Saccheri’s purpose is certainly not to show that sets like {ΣpP, ΣM} are not conclusive syllogistic premisses (they are not even syllogistic premisses) but to show that from them no conclusion of the form Y⇒Z can be derived. Saccheri restricts himself to the case where the two terms of the conclusion are in the premisses, and, moreover, each term occurs in a different premiss:

If however, in the conclusion, one term of the minor premiss be compared with either one of the terms of the major premiss, we proceed as follows. If an affirmative conclusion is sought, either term of the minor premiss may be relevant by repugnance with respect to either term of the major premiss; if a negative conclusion is sought, then conversely either term of the minor premiss may be relevant by consequence with respect to either term of the major premiss (si autem comparetur in conclusione unus terminus minoris praemissae cum alterutro ex terminis maioris praemissae, sta proceditur. Si enim velis conclusionem affirmativam, poterit uterque terminus minoris praemissae esse pertinens repugnantia relate ad utrumque maioris praemissae; si velis negativam, poterit e converso uterque terminus minoris praemissae esse pertinens sequela relate ad utrumque maioris praemissae.)

If τ is a or i: interpret S into a term relevant by repugnance with respect to the terms assigned by our interpretation to X and to M; interpret P into a term relevant by repugnance with respect to the terms assigned by our interpretation to X and M; thus SeX, PeX and PeM are all true, and our candidate for conclusion is false.

If τ is e or o: same interpretation with ‘‘by consequence’’ instead of ‘‘by repugnance’’.

Here we see the application of an even stronger understanding of the postulate: for each term t there are two terms t’ and t” such that t is relevant by repugnance with respect to each of them, and there are also two terms t’ and t” such that t is relevant by consequence with respect to each of them.

3. The Spirit of Saccheri’s Postulate

The discovery that the letter of the postulate, the literal postulate, is not definitive naturally leads one to wonder what is the spirit, or significance, of the postulate—not only relative to Saccheri’s
Aristotelian logic but also to the more complex modern logical theory.

It has been always customary among logicians to refute certain logical properties (such as logical implication, and logical truth) or to establish others (such as logical consistency) by means of suitable interpretations, or models, of the variables involved. I will refer to this procedure as the *method of interpretations*.

Aristotle, for instance, shows that the particular negative "A does not belong to some B" is not convertible, i.e. refutes the implication from "A does not belong to some B" to "B does not belong to some A", by interpreting the variable A into "man" and B into "living being" (*Prior Analytics I*, 2). It is well known that the non-valid syllogistic implications are rejected by the father of logic in the same way.

In modern logic the method of interpretations is more diversified. On the one hand, there are not only variables for general terms or predicates, as in the Aristotelian syllogistic, but also for singular terms, for sentences, etc. On the other hand, while in Aristotle interpretations are mainly used to disprove logical implication, in modern logic they are equally employed to disprove logical truth as well as to prove logical consistency.

The use of the method of interpretations presupposes, of course, the availability of the chosen expressions (and their corresponding denoted entities) in each case, or at any rate the possibility that such expressions occur in language. This possibility—if not secured by logic—should be postulated: such is, in my view, the general significance, the spirit, of Saccheri’s postulate, applicable to modern as well as to traditional logic. Modern Fregean logic was motivated by the need of making explicit the logical rules and laws taken for granted by Euclid—Saccheri’s postulate tells us to move one step further in the same direction.

One point remains undecided: are there interpretations whose availability is secured by logic, and that therefore do not have to be postulated?

The reader of the *Logica*, before reaching chapter 11 of the first part, is likely to favor a negative answer. But in chapter 11 (cf. our next section), Saccheri shows, without saying it, that the answer is affirmative. Before taking up this issue, a few further remarks on the postulate are in order.

A) First, with the understanding of the postulate as concerning every interpretation possibly used by the method of interpretations—
but not guaranteed by pure logic—and not just those listed by the literal postulate, Saccheri’s project becomes transparent. We do not have to worry any more about whether the postulate posits the existence, for each given term t, of one, or rather of two or perhaps of even a greater number of terms with which t stands in one, two, three or all of the four relations in question. All these complications are unnecessary, and miss the point.

Now the search for uses of the postulate in the Logica Demonstrativa becomes a trivial matter: the postulate is used any time the method of interpretations is used, and Saccheri need not state explicitly in each case an ad hoc postulate.

Without aiming at completeness, we may add the following to the four applications of the postulate mentioned in section 2:

5) Chapter 9, Proposition 5: “In the first figure the major cannot be particular; the minor cannot be negative; nor can a particular negative become a premiss”.

6) Chapter 9, Proposition 6: “In the second figure the major [premiss] cannot be particular, nor can both premisses agree in quality.”

7) Chapter 9, Proposition 7: “In the third figure the minor cannot be negative”.

8) Chapter 9, Proposition 8: “In the fourth figure a particular negative cannot be a premiss.”

9) Chapter 9, Proposition 9: “From purely negative premisses nothing follows.”

10) Chapter 9, Proposition 10: “From purely particular premisses nothing follows.”

The ten examples of the postulate’s application in the Logica concern proofs of non-implication, except the first (ch. 4, prop. 3: “two contrary propositions can be at the same time false”) where the satisfiability of \{O, I\}, or the falsifiability of \{A, E\} are established. Once we have seen the connection between the postulate and the method of interpretations it is not surprising to find the postulate used not only to refute implications but also to prove satisfiability (or falsifiability): wherever one interpretation is enough to establish or to refute a logical property the method of interpretations can be correctly employed, and, if the existence of the interpretation cannot be secured by logic alone, the postulate is activated. I have not found in Saccheri applications of the method of interpretations to refute logical truths.

B) As a second remark, once we consider the spirit of the postulate
instead of its narrow literal formulation, the pseudo-problem of whether the postulate applies to negative rather than to affirmative propositions—apparently suggested by Saccheri’s text—vanishes. In chapter 11 we read:

The negative propositions (beginning from the sixth chapter up to this point) have not been established without the help of a postulate; for e.g. that in the first figure the minor premiss should be affirmative, or that it cannot be negative, has been demonstrated by us from the fact that, from the negation of the subject, even of a universal proposition, there is no inference to the predicate. The fact that from the negation of the subject, even of a universal proposition there is no inference to the predicate has been established by this fact: that, otherwise, inference would be valid from the consequent to the antecedent; and hence there would be no inferior and superior term, but all relevant terms would be mutually convertible, or relevant by mutual consequence, which is contrary to the postulate. Wherefore, in short, the whole argument is based on the aforesaid postulate. Something similar occurs in the demonstration of all negative propositions. But, in truth, affirmative propositions e.g., all the moods having a major universal and a minor affirmative conclude rightly in the first figure, needed no postulate. [...] I note only that the propositions demonstrated in chapter four which require this postulate were affirmative, e.g. that two contraries may both be false. In the other chapters, however, these propositions are negative. The first part (till “[...]”) suggests that only negative propositions may require help from the postulate. This is corrected by the second part. Still, Saccheri fails to explain the nature of the linkage between negative or affirmative propositions and the postulate, and why some propositions depending on the postulate are negative while others are affirmative.

All this becomes perfectly clear once the proposed understanding of the postulate is adopted, namely that to use the postulate is the same as to explicitly recognize the possibility of the method of interpretations. In fact, statements of falsifiability or satisfiability (such as the third proposition of chapter 4) are, or, at any rate tend to be, of affirmative form, whereas statements of non-implication will normally be of negative form.

In the second part of the quoted text (after “[...]”), Saccheri refers in the plural to affirmative propositions in chapter four that have been proved by the postulate. In our list of Saccheri’s uses there is only one example from chapter four, namely item (1), i.e. proposition 3; to this the second part of the proposition 4 of the same chapter (“two subcontraries can be true at the same time”) should be added, since it is proved on the basis of proposition 3, and thus depends on the postulate.
C) Thirdly, with regard to the originality of Saccheri’s postulate all I can say is the following. As mentioned in my (1975), a similar postulate occurs in Leibniz, and may have occurred in other mathematically oriented postmedieval authors who were related to Euclid. Aside from this, however, the postulate seems original with respect to what may be roughly called the Aristotelian-scholastic tradition, and it may turn out to be original even with respect to logical theory after Saccheri, through to today.

D) Finally, Saccheri’s view is not to be confused with such critiques as Ross’s complaint that Aristotle’s refutation of invalid forms, involving empirical elements (‘‘horse’’, ‘‘stone’’...), in the end is not really satisfactory:

But in his treatment of the invalid moods he [Aristotle] does not point out the formal error involved in drawing a conclusion, e.g. that of reasoning from knowledge about part of a class to a conclusion about the whole. He relies instead on empirical knowledge (or supposed knowledge) to show that, major and middle term being related in a certain way, and middle and minor term being related in a certain way, sometimes the major is in fact true of the minor and sometimes it is not. He thus shows that certain forms of premiss cannot warrant a conclusion, but he does not show why they cannot do so (1949, p. 33).

More interesting and intriguing is the comparison of Saccheri with Lukasiewicz. The latter writes in a way very similar to Saccheri:

Aristotle rejects some forms by means of concrete terms, as ‘man’, ‘animal’, ‘stone’. This procedure is correct, but it introduces into logic terms and propositions not germane to it. ‘Man’ and ‘animal’ are not logical terms, and the proposition ‘All men are animals’ is not a logical thesis. Logic cannot depend on concrete terms and statements (1957 p. 72).

The sentence: ‘‘This procedure is correct, but it introduces into logic terms and propositions not germane to it’’ is particularly reminiscent of Saccheri’s remark following the postulate (cf. above, section 1). There are two differences though; the first will be stated here, the second will be presented at the end of section 4.

It is not just the empirical flavor of horses, swans, humans or stones that disturbs Saccheri; the issue is for him a deeper one. For instance, to take Aristotle’s refutation of the convertibility of O, Saccheri is not troubled by the concrete predicates ‘man’, ‘animal’ as such but primarily by the general, abstract assumption that there are two predicates related to each other as ‘‘in inferior’’ and ‘‘superior’’.

4. The Nobler Way: Restriction to ‘‘Internal’’ Interpretations

It is obvious that Saccheri does not like the use of interpretations extraneous to logic, even after these interpretations have been blessed

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by the official statement of the postulate. In fact, he is very happy to announce, at the beginning of chapter 11 of the first part of the Logica, the discovery of a way of doing logic without having to state the postulate. This he calls the via nobilior, the nobler way. Let us examine one of the examples of such a nobler procedure.

In the second proposition of chapter 11 Saccheri proves, without the postulate, the proposition 6 proved in chapter 9 with the postulate. The proposition says: "In the second figure the premisses cannot have the same quality." The proof goes as follows.

First, we have the obvious observation that if a syllogism with premisses agreeing in quality were conclusive in the second figure, then the premisses AA or EE would be conclusive in the second figure (for instance, if AI is conclusive, a fortiori AA is conclusive, if EO is conclusive, a fortiori EE is conclusive, since A implies I and E implies O in Aristotelian logic).

The case EE is regarded as already dismissed by a previous postulate-free result, so that only the possible conclusions of II-AA ("II" referring to second figure) are to be considered. Let us state the two premisses of II-AA as follows: "All B are A, all C are A".

Let us recall that in the Aristotelian approach the question is not whether there is any conclusion from the premisses II-AA but whether there is a conclusion of one the four categorical forms A, E, I or O, linking, moreover, the two terms that occur only once in the premisses. Thus, Saccheri's task is to show that II-AA does not yield any of the following four relations: A, E, I, O, between the terms represented by B and C.

For the rejection of A and I, Saccheri uses the following interpretation. The predicate variable B is interpreted into "being a syllogism that has a universal major and has premisses different in quality", the predicate variable A into "being conclusive in the second figure", the predicate variable C into "being a syllogism with premisses AA".

Under this interpretation, the two premisses become: "Every syllogism having a universal major, and having premisses differing in quality, concludes in the second figure" and "Every syllogism AA concludes in the second figure".

Suppose the conclusion is A. Then the two stated premisses should yield the conclusion: "every syllogism AA has premisses not agreeing in quality". Thus, however, the allegedly good implication "II-AA implies A", with premisses that are true independently of the postulate, yields an absurdity.

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Suppose the conclusion is I. Then the two stated premisses should yield the conclusion: "some syllogism AA has premisses not agreeing in quality". Thus, however, the allegedly good implication "II-AA implies I", with premisses that are true independently of the postulate, yields an absurdity again.

For the rejection of the E and O candidates Saccheri constructs the following interpretation: A = "having one of the premisses affirmative", B = "being conclusive in the second figure", C = "being a syllogism AA". Thus, the two premisses become: "Every syllogism that concludes in the second figure has one of its premisses affirmative", "Every syllogism AA has one of its premisses affirmative".

Suppose the conclusion is E. Then the two premisses should yield: every syllogism AA does not conclude in the second figure. The premisses are true independently of the postulate. The conclusion says that the form II-AA is not good.

Suppose the conclusion is O. Then the two premisses should yield: some syllogism AA does not conclude in the second figure. The premisses are true independently of the postulate. The conclusion says that the form II-AA is not good, because if an instance of AA is not conclusive, then no AA is conclusive (this is an application of Saccheri's interesting lemma, chapter 11 of part I of the Logica). Q.e.d.

After considering this example of the via nobilior, one may object that, given that the allegedly postulate-free proofs still depend on interpretations of the variables, the availability of the latter ought to be postulated, if, as Saccheri says, one is to "proceed scientifically."

The interpretations employed in the via nobilior are, however, very special, and different from the Aristotelian and usual ones: they are linguistic expressions, terms already introduced as part of the logical theory itself. These interpretations, which we may call internal, do not have to be postulated "in order to proceed scientifically": their existence is secured by logic.

I propose to define the via nobilior as the restriction of the method of interpretations to internal interpretations.

A comparison between Saccheri and Lukasiewicz is unavoidable as soon as we consider the immediate continuation of the Lukasiewicz’ text quoted above in section 3:

If we want to avoid this difficulty, we must reject some forms axiomatically. I have found that if we reject the two following forms of the second figure axiomatically:

(7) If A belongs to all B and A belongs to all C, then B belongs to some C, and

(8) If A belongs to no B and A belongs to no C, then B belongs to some C,
all the other forms may be rejected by the rules (c) and (d) (Aristotle’s syllogistic, p. 72)

where Lukasiewicz’ rules (c) and (d) are respectively the following: if an implication is asserted, but its consequent is rejected, then its antecedent must be rejected too; if a substitution instance of a formula is rejected, then the formula must be rejected too.

The question arises: is Lukasiewicz’ rejection method similar to Saccheri’s via nobilior? My tentative answer is no. Let us ask on which grounds does Lukasiewicz select his two rejection axioms. Presumably because they can be falsified under some interpretation—but here is where the crucial issue for Saccheri begins. Saccheri would demand, for the via nobilior, that the falsifying interpretations be internal, and there is no hint of this in Lukasiewicz.

5. The Noblest Way. Admirable Update

Saccheri does not even hint at a definition of his via nobilior in terms of a restriction to internal interpretations. His account—at the beginning of Logica, I, ch. 11—is quite different:

Now in fact we may conceive another way of proceeding, and as I think a beautiful way, by which I demonstrate the same truths without the assistance of the postulate. I shall proceed thus: I shall assume the contradictory of the proposition to be demonstrated and from it I shall elicit ostensively [ostensive] and directly [directe] the proposition to be proved. This method of proof was employed by Euclid, prop. 12, book 9.

Thus, all Saccheri sees in his postulate-free, nobler project is the beauty of the consequentia mirabilis (not his phrase). As opposed to the ordinary indirect argument or via negativa (this expression is found in the scholium of chapter 11), in the reasoning envisaged by Saccheri the supposition of the negation of the desired result leads directly to the desired result, without the intermediate step of bumping into some absurdity (that negates the negation of the desired result and consequently affirms the latter).

We can appreciate this peculiar argument in two cases of the postulate-free proof presented in the preceding section: when the assumed conclusion is O and E. But then anyone perceives that in the other two cases (when the assumed conclusion is A and I) the via negativa seems to prevail. This happens several times in the postulate-free proofs of chapter 11. Saccheri anticipates the reader’s objections, and defends himself from them in the scholium of that chapter. Even if his defense is correct, as I have suggested in (1975) and Hoorman
in (1976), the conceptual distinction between postulate-free reconstruction of logic and postulate-free-admirable reconstruction of logic ought to be respected, unless a deeper connection between the two becomes apparent. This is why I would suggest that Saccheri’s own phrase “via nobilior” (“nobler method”) be reserved for the sheer postulate-free procedure, while the new designation “via nobilissima” (“noblest method”), not in Saccheri) be applied to those cases that are both postulate-free and admirable.

As a final remark, I would like to point out that the admirable consequence continues to attract the interest of scholars and logicians. In the bibliography I mention two important recent contributions, by Nuchelmans and Thiel, and I list a few Saccherian titles that appeared after my (1975) or not mentioned therein. From earlier admirers of the admirable reasoning I will quote here an impressive letter from the mathematician Halsted, the translator into English of Saccheri’s *Euclides Vindicatus*. The letter, addressed to the topologist R.L. Moore, refers to the same paradigmatic passage in Euclid mentioned by Saccheri in the above quoted text.1

June 21, 1919

Dear Professor Moore,

Your letter of June 17 gives the ordinary views and therefore is not you. Now let your genius wake, and give me just the opposite side of the debate. Show that Eu.[clid] IX, 12 is in sharpest opposition to the Reductio ad absurdum.

The R.a.a. proves by reaching a contradiction of a known theorem. Thus the first Reductio, Eu. [?] 5, proves the part equal to the whole, then accepted as an absurd proposition. Eu. IX. 12 proves no absurd theorem. On the contrary, it gives a direct demonstration of the very theorem we wish to prove.

It is a wholly new method of proof, which never occurs before (or after) in Eu. It is this new method that Saccheri is trying, expecting it to give a direct demonstration of the parallel-postulate.

I am grateful to Albert C. Lewis for drawing my attention to this letter, kept in the R.L. Moore archives, The University of Texas at Austin, and published here with the kind permission from the Barker Center, The University of Texas at Austin.
The world had always supposed he was engaged in giving or trying to give a Reductio ad absurdum. Far otherwise.

And he maintains that he has justified Euclid in making Euclid ['s] parallel assumption by showing that the parallel postulate is indemonstrable (as we all now know since Bolzai).

Sleep on it, and give me a spark from your genius. Yours always.

George Bruce Halsted

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