Microeconomics Comprehensive Exam

August 2011

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) Please write only on one side of a sheet of paper
(3) When finished, please arrange your answers alphabetically
   (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.).
1. (25 points) Let $\mathcal{Z}$ denote a (finite) set of monetary prizes. Let $\mathcal{L}(\mathcal{Z})$ denote the (infinite) set of all lotteries on $\mathcal{Z}$, and $\succeq$ denote a rational preference relation on $\mathcal{L}(\mathcal{Z})$.

(a) (5 points) Spell out the axioms which ensure that $\succeq$ admits the von Neuman-Morgenstern utility representation?
(b) (10 points) For any $p \in \mathcal{L}(\mathcal{Z})$, define

$$U(p) = E(p) - \frac{1}{4} \text{var}(p),$$

where $E(p)$ is the expected value of the prize of lottery $p$, and $\text{var}(p)$ is the variance of the prize $p$. Show that $U$ defined above induces a preference relation which is not consistent with the assumptions of the von Neuman-Morgenstern theorem.

*Hint:* Consider, for example, lottery $p$, which gives a deterministic payoff of $1$ and "coin" lottery $q$, which pays 0 in the event of "heads" and $4$ in the event of "tails", and their convex combinations with coin lottery $r$, which pays 0 in the event of "heads" and $2$ in the event of "tails".

(c) (10 points) Define

$$U(p) = E(p) - (E(p))^2 - \text{var}(p).$$

Show that $U$ defined above induces a preference relation which is consistent with the assumptions of the von Neuman-Morgenstern theorem.
2. (25 points) Consider an asset structure given by the following payoff matrix:

\[
D = \begin{pmatrix}
\delta^1 & \delta^2 & \delta^3 \\
\alpha & 0 & \beta \\
1 + r & 1 + r & 1 + r
\end{pmatrix},
\]

where \(\delta^1 > \delta^2 > \delta^3 > 0\), \(\alpha > \beta > 0\), and \(r > 0\). Let \(S^1, S^2, S^3\) be asset prices.

(a) Critically analyze the following propositions:

(1) (10 points) If \(S^1, S^2, S^3 > 0\), then the asset structure \(D\) permits no arbitrage.

(2) (10 points) If the asset structure \(D\) permits no arbitrage, then \(D\) exhibits no redundancy. Is this true for a general asset structure specified by an \(n \times m\) payoff matrix \(D\)?

(b) (5 points) Let \(\delta^3 < (1 + r)S^1 < \delta^2\), and \(S^3 = 1\). Derive the no-arbitrage boundaries for \(S^2\).
3. (25 points) Consider the following extensive-form game.

The dotted information set says that C cannot distinguish between X and Y.

(a) (9 points) Find all (pure or mixed-strategy) Nash equilibria for the normal-form expression.
(b) (7 points) Find all (pure or mixed-strategy) subgame-perfect Nash equilibria.
(c) (9 points) Find all (pure or mixed-strategy) weak perfect Bayesian equilibria.
4. (25 points) There are two individuals. Producing a public good costs 1. Assume that the individuals have quasi-linear preference over the project decision and transfers. Also assume that they are risk neutral. Denote individual 1’s valuation of the public good by $v_1$, and denote 2’s by $v_2$.

(a) Consider a game-form as follows. Let them submit bids, $b_1$ and $b_2$ respectively. If $b_1 + b_2 \geq 1$, do the project and make 1 pay $b_1$ and 2 pay $b_2$. Otherwise, do nothing (no payment either).

(a-1) (7 points) Consider a complete information case with $0 < v_1, v_2 < 1$ and $1 < v_1 + v_2$. Find all the pure-strategy Nash equilibria.

(a-2) (6 points) Consider an incomplete information case in which $v_1$ and $v_2$ are drawn from $\{1/3, 2/3\}$ with even chance independently. Show that the following bidding function

$$
\beta(v) = \begin{cases} 
0 & \text{if } v = 1/3 \\
1/2 & \text{if } v = 2/3
\end{cases}
$$

forms a symmetric Bayesian Nash equilibrium.

(b) (6 points) Assume that $v_1$ and $v_2$ are drawn from $[0, 1]$ according to the uniform distribution independently. What is the prescription by the expected externality mechanism?

(c) (6 points) What is the prescription by the Clarke mechanism to this problem? Hint: For (b) and (c), you may formulate the net welfare terms first by assuming equal division as the default point for cost sharing.