Instructions. There are two parts to this exam. Part A consists of a series of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 50 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following statements contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each statement is worth 10 points for a total of 30 points.

1. If capital markets are perfect, the level of government debt has no effect on the capital stock.

2. If income effects dominate substitution effects, then the inflation tax generates high output and high employment along with high inflation.

3. In an overlapping generations model with stochastic money transfers, if prices in period $t$ are independent of money transfers in periods $t$ and $t-1$, then output in period $t$ will depend on money transfers in periods $t$ and $t-1$.

B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is worth 50 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your name and question # on each page.

Question 1.

Suppose that you are a farmer. You can produce output in one of two ways. One way to produce output is from your cow. If your cow is alive your production function is given by $y = \overline{A} l$, where $l$ is labor input. If your cow is dead, your production function is given by $y = \overline{A} l$. Assume $\overline{A} > A$.

Output is split between consumption $c$ and feed $f$ for the cow. Assume that utility over consumption and labor time is given by $u(c - \frac{v^2}{2})$, where $u(\cdot)$ is strictly increasing and strictly concave and $v$ is a parameter.

Let $V^a$ be the household's value function if the cow is alive and $V^d$ the value function if the cow is dead. The probability the cow survives is given by $\delta(f)$, where $\delta(\cdot)$ is increasing and concave in $f$. Assume $\delta(0) = 0$.

(i). Write down the dynamic programming problem of this farmer. What are the states? What are the control variables? (10)

(ii). Write down the first order condition for the choice of $l$ for both a live and a dead cow. Is $l$ bigger or smaller when the cow is alive? Why is the level of $l$ for a live cow independent of $\delta(\cdot)$? Explain. What is the first order condition for the choice of $f$? Explain it.(15)

(iii). Prove that $V^a > V^d$. (10)
(iv). As $A$ decreases, what happens to $l$ and to $f$ when the cow is alive? (5)

(v). If there is a subsistence consumption constraint that $c \geq c$, would it ever be the case that households with a dead cow would work more than households with a live cow? (10)

**Question 2.**

Consider a three-period economy with uncertainty. There is one (type of) consumer and this consumer derives utility from consumption and leisure at all dates and states. She has a utility function of the expected-utility kind:

$$u(c_1, l_1) + \beta E_1[u(c_2, l_2) + u(c_3, l_3)]$$

where $c_t$ and $l_t$ are (nonrandom) consumption and leisure at time $t$ and $c_t$ and $l_t$ for $t = 2, 3$ are the random variables for consumption and leisure in periods 2 and 3. Each period, the production of consumption goods takes place using a production function $y = zn$, where $n$ is the total amount of labor supplied by the consumer and $z$ is a productivity shock. The total amount of labor supplied equals the consumer's total time endowment, which is $1$, less her chosen leisure time. The productivity shock takes on one of two possible values in each period, $z_1$ or $z_2$, each with probability $1/2$: the $z$ process is iid over time ($z_2$ and $z_3$ are independent random variables). The first-period productivity, $z_1$, equals $z_0$.

Production takes place in a representative firm which rents labor and sells the output, taking prices as given. The consumer, on the other hand, supplies labor and buys consumption goods, taking prices as given. Suppose, finally, that consumers and firms buy and sell goods at time-zero (Arrow-Debreu) markets. 

(i) Carefully define a sequential competitive equilibrium for this economy. How many commodities are there, and how many prices? What is an Arrow-Debreu security? (15)

(ii) Suppose that you have solved for the prices in this Arrow-Debreu economy (you do not need to do it at this point). Express, as a function of these prices, the value of the riskless real interest rate at time 2 (that is, the interest rate between dates 2 and 3) conditional on the realization of $z_2$. (15)

(iii) Simplify the setup in two ways. First, assume that there are only two periods in the model (that is, the setup is the same as before but ignore the third period). Second, suppose that $u(c, l) = \log c + \log l$. Solve for all the equilibrium prices and quantities for this economy. (20)

**Question 3.**

Consider an overlapping-generations model with a single perishable good. The representative agent of generation $t$ has preferences over consumption in youth and old age given by $W(c_t^y, c_t^o) = u(c_t^y) + c_t^o$. Assume $u(·)$ is strictly increasing and strictly concave. Each generation is of size $N$.

Agents are endowed with $\omega$ units of the single good in youth and nothing in old age. Agents differ in the amount of endowment in youth: the cdf of the distribution of endowments is given by $G(\omega)$.

There are two stores of value, money and storage. The return on storage is exogenous and is given by $R$. If you put a unit in storage, you get $R$ units next period. Access to the storage technology requires the payment of a fixed cost $F$ units of the endowment good. The return on money depends on intertemporal prices and is determined in equilibrium.

(i). Suppose that an agent with endowment $\omega$ holds money as a store of value. Show that the demand for money is increasing in the endowment level of the agent and also increasing in the return on holding money. (10)

(ii). Suppose that an agent with endowment $\omega$ uses the storage technology as a store of value. Show that the amount stored is increasing in the endowment level of the agent and also, under some conditions, increasing in the return on storage, $R$. (10)
(iii). Assume the stock of money is constant. **Construct and characterize** a steady state monetary equilibrium with some agents holding money and others using the storage technology. In this steady state, when will there exist a critical level of \( \omega \), call it \( \omega^* \), such that only agents with \( \omega \leq \omega^* \) hold money? (20)

(iv). Suppose that the money supply grows at rate \( \sigma > 0 \). New money is introduced into the economy as a lump-sum transfer to agents storing goods but not to money holders. Is money neutral in this economy? What real variables depend on \( \sigma \)? Explain (10)