Macroeconomics Comprehensive Examination
University of Texas at Austin
June 2004

Instructions. There are two parts to this exam. Part A consists of a series of six short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 40 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Try to answer each question, so budget your time carefully.

A. Short Answer Questions (One hour: 60 points)

Instructions. Each of the following problems contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and defend your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each question is worth 10 points for a total of 60 points.

1. In the model economy outlined in Lucas (JET, 1972), output, employment, productivity and prices are all increasing in the stochastic money transfer.

2. The calibration of the stochastic growth model sets the discount factor equal to the ex post marginal product of capital.

3. Solving dynamic programming models through value function iteration works only if the initial guess is sufficiently close to the true value function.

4. According to the standard overlapping generations model, if there is a finite horizon, then the monetary equilibrium is always social optimal.

5. In the stochastic growth model, shocks to the marginal utility of consumption will lead to a negative correlation between consumption and investment.

6. The result that consumption follows a random walk holds for any specification of the utility function but this result does require that income follow a random walk.
B. Multipart Questions (Two hours: 120 points)

Instructions. Do all of the multi-part questions below. Each question is worth 40 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your name and question # on each page.

1. Consider an overlapping-generations model with a single perishable good. The representative agent of generation \( t \) has preferences over consumption in old age \( c_t^q \) and employment in youth \( n_t \) given by: \( u(c_t^q) = g(n_t) \). Assume:

   - \( u(c) = c \) and \( g(n) = \frac{3}{2} n^2 \)
   - the size of each generation is equal to \( N \)
   - output \( y_t \) equals labor input \( n_t \)
   - there is a fixed stock of fiat money endowed only to the old in the initial period.

   In this economy, there are no firms: output is produced by the young. In each period, there is a market in which goods are exchanged for money.

   There are two types of agents. In each generation, a fraction \( \lambda \in (0,1) \) of agents have rational expectations about future variables. Call these the rational agents. The remaining agents are not rational. Instead, they forecast that all period \( t+1 \) variables will take the same value that these variables took on in period \( t \). Call these the irrational agents. Take \( \lambda \) as an exogenous constant.

   Given this economy, answer the following questions. Credit is indicated for each part.

   (a) (5 points). Write down the first-order conditions for two optimization problems: one for a young rational agent in generation \( t \) and the second for a young irrational agent of generation \( t \). Explain how this problem captures the interaction across sellers within a generation.

   (b) (10 points) What are the conditions for market clearing in this economy? Using the conditions for individual optimization and market clearing, characterize the set of perfect foresight equilibria in terms of a path for employment. Explain your findings.

   (c) (5 points) Find the steady states for this economy. How many steady states are there? Explain

   (d) (10 points) What can you say about the stability properties of the steady states as a function of \( \lambda \)? In particular, are there values of \( \lambda \) such that the monetary steady state is locally stable?

   (e) (10 points) Add stochastic proportional transfers \( x \) to this economy. Is money neutral in this economy with \( \lambda < 1 \)? Does the level of economic activity depend on realized values of \( x \), the mean of \( x \), the variance of \( x \)? Explain your findings.
2. Consider the following cash-in-advance model with competitive markets. A representative household (composed of a cooperative worker-shopper pair) has preferences given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t^n)] \]

where \( n_t^n \in [0,1] \) is labor supply, \( c_t \geq 0 \) is consumption, \( \beta \in (0,1) \), \( u \) is strictly increasing, concave, and differentiable, \( v \) is strictly increasing, convex, and differentiable with \( v'(0) = 0 \) and \( v'(1) = \infty \). A representative firm has production function \( y_t = \theta n_t^n \). The government budget constraint is given by \( P_t \tau_t = M_t^s - M_t^f \) where \( M_t^s \) is the nominal money supply, \( P_t \) is the price level, and \( \tau_t \) are real lump-sum taxes or transfers. The money supply evolves according to \( M_{t+1}^s = (1 + \mu) M_t^s \) where \( \mu \) is the growth rate of the money supply. Households can also buy or sell private nominal discount bonds. The timing is as follows: (i) the household enters with \( M_t \) units of money and \( B_t \) units of money from its prior bond activity (if \( B_t < 0 \), then the household owes that number of units of money); (ii) lump sum taxes/transfers are paid; (iii) the bond market opens so that the household purchases \( B_{t+1} \) at price \( Q_t \) (where \( Q_t \) is the number of units of money that buys one bond); (iv) the shopper purchases consumption goods at price \( P_t \) with available money holdings while the worker supplies labor to the firm (receiving \( w_t n_t^n \) units of money for her services); and (v) the worker and shopper meet to eat and count their money.

(a) (5 points) Write the household’s decision problem including the cash-in-advance constraint that money must be used to purchase goods and bonds according to the above timing.

(b) (2.5 points) Write down the firm’s static decision problem.

(c) (5 points) Define an equilibrium.

(d) (2.5 points) What is the equilibrium real wage?

(e) (15 points) Characterize the household’s decision problem. Under what conditions is the c-i-a constraint binding? How does inflation affect the household’s work decision?

(f) (10 points) With commitment, what is the optimal money growth rate in this environment? Explain in terms of a planner’s solution.

3. Consider the following environment in which an employer does not know the quality of her match with a worker and must decide whether to fire the worker. Any period in which an employer is not already matched with a worker, she can post a vacancy (which costs \( \kappa \)) that assures the employer she will be matched with a worker next period. When an employer and worker are matched, they produce \( y_t \) units of nonstorable goods. While employers and workers observe output \( y_t \in \{y^H, y^L\} \) from the match (where \( y^H > y^L \)), they do not observe the quality \( q \in \{g, b\} \) of the match. Good quality matches are more likely to yield high output and bad quality matches are more likely to yield low output. Specifically, the conditional probabilities are given by

\[ \pi^g \equiv \Pr(y_t = y^H | q = g) > \Pr(y_t = y^L | q = g) \]

\[ \pi^b \equiv \Pr(y_t = y^L | q = b) > \Pr(y_t = y^H | q = b) \]
The unconditional probability of a good quality match is given by $\bar{\gamma}$ (i.e. $\gamma = \Pr(q = g)$). Both employers and workers are risk neutral and discount the future at rate $\beta < 1$. Each period that an employer and a worker are matched, the employer makes a take-it-or-leave-it wage offer $w_t$ to the worker. If the worker rejects the offer, the match is broken and the worker must permanently leave the workforce but enjoys leisure yielding utility $c > 0$ each period. In a match, the timing of events is given by: (i) employers and workers decide whether to stay in the match; (ii) if the match remains, the employer pays the worker his wage; (iii) the output realization is observed and agents consume.

(a) (5 points) At the beginning of the $T$th ($> 1$) period of being in a match, denote the probability that the employer and the worker assign to the match quality being $g$ having observed a history $h_T = (y_0, y_1, \ldots, y_{T-1})$ as $\gamma(h_T)$. Assume $\gamma(h_0) = \bar{\gamma}$. Let $V(\gamma(h_T))$ denote the value function of an employer in a match which has lasted $T$ periods before the observation of $y_T$. Write the value function of the employer explicitly.

(b) (5 points) Let $J(\gamma(h_T))$ denote the value function of a worker in a match which has lasted $T$ periods before the observation of $y_T$. Write the value function of the worker explicitly.

(c) (5 points) Given that the employer makes a take-it-or-leave-it wage offer to her worker, what is the wage in any given match history $h_T$? What does this imply about $J(\gamma(h_T))$?

(d) (5 points) Assume that output realizations are perfectly correlated with quality. In particular, agents are able to infer quality of the match from the first observation (i.e. $\pi^g = 1 = \pi^b$). What is $V(\gamma(h_0))$? Under what conditions will employers enter a match? Under what conditions will an employer fire the worker when the output realization is low?

(e) (5 points) When output realizations are not perfectly correlated with quality (i.e. $\pi^g \neq 1 \neq \pi^b$), assume that agents use Bayes’ rule to try to infer the quality of a match. According to Bayes’ rule, for any history of output realizations $h_{T+1}$, the probability that $h_{T+1}$ is consistent with the match being good quality is given by

$$
\gamma(h_{T+1}) = \frac{\Pr(q = g | h_T \cup y_{T+1})}{\Pr(q = g | h_T)} = \Pr(y_{T+1} | q = g)\Pr(q = g | h_T) = \frac{\Pr(y_{T+1} | q = g)\gamma(h_T)}{\Pr(y_{T+1} | q = g)\gamma(h_T) + \Pr(y_{T+1} | q = b)(1 - \gamma(h_T))}
$$

where $\gamma(h_0) = \bar{\gamma}$. Notice that this implies a law of motion for the posterior $\gamma(h_{T+1})$ as a function of the prior $\gamma(h_T)$ which depends on which output realization occurs. Draw these posteriors in $[0,1] \times [0,1]$. Are they everywhere increasing?

(f) (7.5 points) Suppose that the operator $T$ in $(TV)(\gamma)$ implicitly defined in part (a) is a contraction in the space of continuous, bounded functions. How would you establish that $V$ is weakly increasing in $\gamma$?

(g) (7.5 points) Is it optimal for employers to fire workers when their belief about the quality of a match is sufficiently low? In particular, is there a $\gamma^*$ such that for all $\gamma(h_T) < \gamma^*$, the employer fires the worker and for all $\gamma(h_T) \geq \gamma^*$ the employer retains the worker? Is $\gamma^*$ bigger or smaller than $\bar{\gamma}$?