Macroeconomics Comprehensive Examination
University of Texas at Austin
June 2006

Instructions. There are two parts to this exam. Part A consists of a series of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 50 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully (we give suggested points in each subquestion on part B).

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following problems contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and defend your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each question is worth 10 points for a total of 30 points.

1. Because of consumption smoothing, labor supply is not sufficiently responsive to productivity shocks in the standard RBC model.

2. In the Overlapping Generations Model with money, if households can store goods, then money will not have value.

3. A well-specified dynamic programming problem must have: (i) more state than control variables, (ii) discounting and (iii) a bound on the rate of technological progress.
B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is worth 50 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your name and question # on each page.

Question 1.

Consider an infinite horizon overlapping generations model where agents live for two periods, consuming in each. In each generation, there are two types of agents: borrowers and lenders. The endowment of the single good over the two periods of life for lenders is \((y, 0)\) while the endowment of the borrowers is \((0, y)\). A fraction \(\pi\) of the agents born each period are borrowers.

There is no money in this economy and no storage. There is no population growth.

Let \(c_t^j\) be the consumption of an agent of age \(j\) in period \(t\) where \(j = o\) denotes an old agent and \(j = y\) is a young agent. Assume that utility from consumption in each period is given by \(u(c_t^j) = ln(c_t^j)\) so that lifetime utility of a generation \(t\) agent is \(ln(c_t^o) + ln(c_{t+1}^y)\). Note there is no discounting.

Use this model to answer the following questions. If you need to add additional pieces to the environment, you can do so but be sure to make clear any assumptions you are making.

a. Write down the planner’s problem for this economy. Explain the objective function and the constraints. Find the first order conditions for an optimal allocation. Interpret them. (15 points)

b. Now consider a competitive equilibrium. Set up the optimization problems for a representative lender and a representative borrower of a given generation. Find the first order conditions for these optimization problems and find the decision rules for borrowers and lenders. (10 points)

c. What is a competitive equilibrium in this economy? Find the rate of interest which clears the loan market. (10 points)

d. Is the competitive equilibrium Pareto Optimal? If not explain how adding a social security system could be welfare improving. (15 points)
Question 2.

Consider an economy in which households live for two periods. There is a single consumption good. In each period of life they work and consume. They have an endowment of \( e \) units of the consumption good in the first period and nothing in the second. In each period they have a unit of time which can be split between leisure and production. If a household works \( n \) units of time, they produce \( wn \) units of output. Households can save through a storage technology: each unit saved yields \( R \) units in the next period. Negative storage is not feasible.

There is a government which provides a public good to the citizens of this economy. Let \( G \) denote the amount of public good provided in the first period only. The revenue to finance the public good is provided through the taxation of labor income in each of the two periods. The government can run a deficit in the first period financed by issuing government debt.

Preferences of a household are given by \( u(c_1 - h(n_1)) + v(G) + u(c_2 - h(n_2)) \). Here the subscript is the time period. Assume \( u(\cdot) \) is strictly increasing and strictly concave and \( h(\cdot) \) is strictly increasing and strictly convex.

a. Write down the optimization problem of a representative individual in this economy. What are the first order conditions? Interpret them. (10 points)

b. Suppose that the government can commit at the start of time to a sequence of proportional labor taxes \( (\tau_1, \tau_2) \) and finances all of its public expenditures \( G \) that way. What are the optimal labor tax rates for the two periods? Show that the tax rates are equal across periods. Does the government need to issue debt? If so, who does the government borrow from? (15 points)

c. Suppose that the government faces a balanced budget requirement. This means it must finance \( G \) through labor taxes in period 1 only. Characterize the resulting equilibrium levels of labor supply. Compare your answers to part (b). Is there a social cost to a balanced budget requirement and, if so, what is the nature of the cost? (10 points)

d. Suppose that the government can tax labor income in both periods. But, in contrast to part (b), the government cannot commit to a sequence of tax rates but can commit to repay debt. In particular, in each of the two periods, the government sets the tax rate for that period and then households make a labor supply choice. Is the labor tax policy of the government characterized in the answer to part (b) an equilibrium here as well? Or is there a time consistency problem in this economy? (15 points)
Question 3.
Consider the following variant of a divisible good, indivisible money search model. The key difference from the standard model is that households can hold not only one but two units of indivisible money. Specifically, time is discrete and there is a continuum of agents with population normalized to 1. Any particular agent specializes in the production of a nonstorable good but has strictly increasing and concave preferences \( u(q) \) from consuming \( q \in \mathbb{R}_+ \) in a different set of goods than he produces. An agent discounts the future at rate \( \beta = (1 + r)^{-1} \) with \( r > 0 \). There is a constant disutility \(-q\) to producing \( q \) units of the good. Production and consumption occur at the end of the period (and hence should be appropriately discounted). The per capita amount of money in the economy is \( M \in (0, 1) \). Let \( m_n \) denote the fraction of the population who hold \( n = 0, 1 \) or 2 units of currency. Obviously \( M = m_1 + 2m_2 \) and \( 1 = m_0 + m_1 + m_2 \). Agents are randomly matched each period. Note that an agent with 1 unit of money can be either a potential buyer or a seller, while an agent with 0 units of money is only a potential seller and an agent with 2 units of money is only a potential buyer. In the case where the agent has 1 unit of money and he is randomly matched with another person with one unit of money (which occurs with probability \( m_1 \)) assume he is a buyer with probability \( \frac{1}{2} \) and a seller with probability \( \frac{1}{2} \). Finally, assume that buyers submit take-it-or-leave-it offers. Depending on the money balances of the buyer \( m_b \in \{1, 2\} \) and seller \( m^s \in \{0, 1\} \), this could take the form of \( Q_1(m^b, m^s) \) units of goods for 1 unit of money or \( Q_2(m^b, m^s) \) units of goods for 2 units of money.

a. Taking the quantities \( Q_d(m^b, m^s) \) as given, write down the steady-state value functions \( V_n \), \( n = 0, 1, 2 \) under the assumption that in all meetings where trade is possible, buyers give up \( d = 1 \) unit of money and sellers produce. (12.5 points)

b. Taking the value functions \( V_n \) as given, what equations characterize the values of \( Q_1(m^b, m^s) \) from the buyer's take-it-or-leave-it offer? (5 points)

c. Under the assumption that all exchange involves one unit of money for the quantities determined in (b), what equations describe the steady-state equilibrium distribution of money? (10 points)

d. Define a steady state monetary equilibrium where one unit of currency is traded in each buyer-seller match in terms of necessary conditions. In particular, what incentive conditions must you include? (7.5 points)

c. Let \( u(q) = q^{1-\gamma} \) with \( \gamma \in (0, 1) \) and assume a steady state monetary equilibrium defined in (d) exists. Do sellers with no money offer better prices than sellers with one unit of money? Show and explain why or why not. Hint: Let \( V_{n+1} = A_n V_n \) and verify under what conditions your answer applies. (15 points)