Stereopsis at isoluminance in the absence of chromatic aberrations

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Stereo (front–back) discrimination thresholds were measured in a two-interval forced-choice paradigm for chromatic (red–green) random-dot stereograms that had all detectable longitudinal and transverse aberrations removed by low-pass filtering. The thresholds were measured as a function of the luminance ratio of the red and the green stereo elements. Although individual differences were apparent, three subjects were able to fuse all the stimuli, including those at isoluminance. A quantitative, ideal-observer analysis was used to determine the neural efficiency with which color and luminance information was used in this stereo task. For two subjects, efficiency was constant as a function of the red-to-green ratio; for the third subject, efficiency was less near isoluminance.

During the past two decades there have been numerous attempts to determine the effectiveness of chromatic cues in stereopsis. Most studies compared stereo performance for patterns that contain luminance variations with stereo performance for patterns that contain only chromatic variations (i.e., isoluminant patterns). The conclusions drawn in these studies are not in complete agreement, and hence the role of chromatic cues in stereopsis remains uncertain. In an early study, Lu and Fender found that the fusion of random-dot stereograms was not possible at isoluminance for any wavelength paired with red, green, or blue. Comerford found that stereopsis was possible at isoluminance for a spoke pattern, using several combinations of target and background spectral composition. In agreement with these previous studies, Gregory and De Weert reported that stereopsis was not possible for isoluminant random-dot stereograms but was possible for isoluminant contour stereograms. Similarly, Osuobeni and O'Leary found fusion possible for isoluminant contour stereograms. On the other hand, De Weert and Sadza showed that fusion was possible in their random-dot stereograms for all luminance ratios, including isoluminance.

A major hindrance to evaluating studies of stereopsis at isoluminance is that chromatic aberrations may have affected the results. Chromatic aberrations can create luminance artifacts at the boundaries between regions of different spectral composition; these artifacts might be sufficient to prevent the fusion of isoluminant stereograms. An unconfounded test for the effectiveness of chromatic cues in stereopsis requires the elimination of both longitudinal and transverse (lateral) chromatic aberrations. Of the above studies, only two Attempted to correct for chromatic aberration, using an achromatizing lens. However, the authors did not describe the achromatizing lens or how its effectiveness was evaluated, and they did not measure stereopsis for random-dot stereograms that isolate the cyclopean mechanisms.

The first aim of this study was to measure stereo sensitivity for red–green, random-dot stereograms in which all chromatic aberrations have been eliminated. Because aberration artifacts contain mostly high spatial frequencies, it proved possible to remove both types of aberration artifact by blurring the stimuli (as described in the Method section).

The second aim of this study was to determine the relative efficiency with which luminance and chromatic cues are used in stereopsis. To do this we used an ideal-observer analysis.

METHOD

Subjects
The two authors and one naïve, hired subject participated in the experiment. All the subjects had normal color vision (Dvorine Color Plates, 2nd ed.) and uncorrected 20/20 vision. In addition to these three subjects, several other potential subjects were pretested, but they were unable to perform the experimental task. One subject could not fuse any disparate stereograms. It is possible that this person represents one of the stereoanomalous types described by Richards. Two others could not fuse moderate- and low-contrast luminance stereograms. These two subjects were not used in our experiments because it would have been impossible to calculate their relative efficiencies for chromatic and luminance cues (see the Discussion section).

Apparatus
The stereo images were computed on a VAX workstation and then transferred to a PDP-11/73 computer and an ADAGE graphics processor, which presented them on a Tektronics 690SR color monitor. A stereoscope apparatus with front surface mirrors was attached to the front of the display monitor. This apparatus permitted the left- and the right-hand images of the stereo pair to be projected to the left and the right eyes, respectively, resulting in a central stereo image. Head position was stabilized with a chin rest.

Calibration
The luminances produced by the red, the green, and the blue guns of the cathode-ray tube (CRT) were controlled...
by look-up tables in the ADAGE processor. Each gun was calibrated independently in the following fashion. First, a photometer was used to measure the maximum luminance of the gun. Next, the relative luminance was measured as a function of the output value in the look-up tables by reading the response of a United Detector Technologies (PIN 10AP) photodiode with a 12-bit analog-to-digital converter. The 1024 luminance measures from each gun were then fitted with a smooth function, and this function was used to create linear look-up tables with 256 entries. The contrast in the stimulus patterns was controlled by adjusting the luminance ranges of the look-up tables.

Before each experimental session, the luminances of the red and the green guns were checked by reading the response of the photodiode at a fixed look-up table value. The day-to-day variations were negligible for both guns.

**Stimuli**

For this experiment, the stimuli were blurred random-dot stereograms with either crossed or uncrossed disparity. The left- and the right-hand halves of the stereograms were each 128 × 128 pixels, one pixel equaling 4 arcmin at the viewing distance of 45 cm. Before the blurring, the elements of the stereogram were 16 arcmin on a side, the shifted square was 12 elements on a side, and the horizontal disparity was 1 element (16 arcmin). A black fixation box with a linewidth of one pixel surrounded each half of the stereogram and served as a fixation target when the stereograms were not on the screen. The direction of the disparity shift was always to the right. An uncrossed disparity stereogram was created by displaying the shifted patterns to the right eye, and a crossed disparity stereogram was created by displaying the shifted patterns to the left eye.

The 16-arcmin disparity of the stereogram was within the range (3–22 arcmin) investigated by Frisby and Mayhew. They found that the effect of disparity on contrast sensitivity was slight; only their highest disparity caused a small reduction in sensitivity. Therefore it is likely that we would have obtained similar results for a wide range of disparities.

The red-green stereograms each consisted of four components, one random-dot pattern in each color for each eye. The random-dot patterns in each color were identical, except that they were reversed in contrast. Specifically, the luminance distributions, \( I_r(x, y) \) and \( I_g(x, y) \), of the red and the green primaries, for one eye, are described by the following equations:

\[
I_r(x, y) = I_r[C_0(x, y) + 1], \\
I_g(x, y) = I_g[-C_0(x, y) + 1],
\]

where \( I_r \) and \( I_g \) are the mean red and green luminances, \( C_0(x, y) \) is the relative amplitude function (ranging from -1 to 1), and \( C \) is the contrast. The luminance distributions for the other eye were identical, except for the relative amplitude function, \( a(x, y) \), which differed because of the shifted central square. In the background region, \( a(x, y) \) is zero; thus the background luminance was the sum of the red \( (I_r) \) and the green \( (I_g) \) luminances (the appearance was yellow). Throughout the study the mean luminance \( (I_r + I_g) \) was held constant at 10 cd/m². Thresholds were obtained by varying the contrast \( C \).

To allow for individual differences in the point of isoluminance and to compensate for the possible 5% error range of the photometer used to make the maximum luminance measurement, we varied the ratio of the red to the red-plus-green luminance, \( r \), where \( r = I_r/(I_r + I_g) \). The following ratios were used: 0.05, 0.20, 0.35, 0.45, 0.50, 0.55, 0.65, 0.80, and 0.95. A ratio of 0.50 represented isoluminance as measured by our photometer; ratios increasingly distant from 0.50 contained a greater luminance component. Thresholds were also measured for a monochrome yellow (pure luminance) stereogram with equal amounts of red and green.

For comparison we also measured thresholds for monochrome and isoluminant stereograms that were not blurred. Except for the lack of blurring, these stereograms were identical to those in the main experiment.

**Elimination of Chromatic Aberration Artifacts**

As mentioned above, we found it possible to remove the luminance artifacts from the stimuli by blurring. Blurring was achieved by filtering the patterns with an exponential function: \( \exp[-2\pi\beta u^2 + v^2] \), where \( u \) and \( v \) are the horizontal and the vertical spatial frequencies and \( \beta \) is the exponential decay constant. The appropriate decay constant was determined by computing the size of the luminance artifacts (produced by both longitudinal and transverse chromatic aberrations) and then adjusting \( \beta \) until the magnitude of the computed artifacts became negligible. To make sure that the blurring was sufficient, we tried to err in the direction of overestimating the size of the artifacts.

In this section we first develop a formal description of the chromatic aberration artifacts occurring at isoluminance. This is followed by a description of how the sizes of the artifacts were estimated for our isoluminant stereograms. Finally, the sizes of the artifacts remaining after the stereograms were blurred with the exponential function are described.

In the Fourier domain the luminance distributions of the red and the green patterns are given by

\[
I_r(u, v) = I_r[C_0(u, v) + \delta(u, v)], \\
I_g(u, v) = I_g[-C_0(u, v) + \delta(u, v)],
\]

where \( u \) and \( v \) are the horizontal and the vertical spatial frequencies (in cycles per degree) and \( \delta(u, v) \) is an impulse (or delta) function at the origin of the spatial frequency plane [cf. Eqs. (1) and (2)]. After passing through the optics of the eye, the Fourier transforms of the retinal illumination distributions of the red and the green patterns (in trolands) are

\[
I_r(u, v) = s[I_t(u, v)A(u, v) + \delta(u, v)], \\
I_g(u, v) = s[I_t(u, v)A(u, v) + \delta(u, v)],
\]

where \( T_t \) and \( T_g \) are the optical transfer functions (OTFs's) for the red and the green primaries and \( s \) is the area of the pupil in square millimeters. At isoluminance \( I_r \) and \( I_g \) are equal. Under this condition the sum of \( I_r(u, v) \) and \( I_g(u, v) \) is the Fourier transform, \( I(u, v) \), of the retinal
illuminance distribution, \(i(x, y)\), resulting from chromatic aberration:

\[
I(u,v) = \text{sl}(CA(u,v)\left[T_r(u,v) - T_s(u,v)\right]/2 + \delta(u,v)),
\]

(7)

where \(l\) is the mean luminance (i.e., \(l = l_1 + l_2\)).

For the purpose of evaluating the detectability of the chromatic aberration artifacts, it is useful to rewrite Eq. (7) in the following form:

\[
I(u,v) = \text{sl}(T_g(u,v)CA(u,v)\times [T_r(u,v)/T_g(u,v) - 1]/2 + \delta(u,v)).
\]

(8)

Now consider a luminance distribution, \(e(x, y)\), drawn in the green primary alone with a mean luminance of \(l\), whose Fourier transform is given by

\[
E(u,v) = [CA(u,v)\left[T_r(u,v)/T_g(u,v) - 1\right]/2 + \delta(u,v)].
\]

(9)

A comparison of Eqs. (8) and (9) shows that, when \(e(x, y)\) is viewed (i.e., when it is passed through the optics of the eye), the resulting retinal illuminance distribution is exactly \((x, y)\). In other words, \(e(x, y)\) is the isochromatic illuminance distribution on the CRT, which when viewed mimics the chromatic aberration artifacts produced by the isoluminant red–green pattern.

If the ratio of the OTF’s for the red and the green primaries \([T_r(u,v)/T_g(u,v)]\) is known, then \(e(x, y)\) can be computed from Eq. (9) by inverse transformation of \(E(u,v)\). If subjects can reliably discriminate crossed and uncrossed green (isochromatic) stereograms, computed with Eq. (9), then the chromatic aberration artifacts are potentially large enough to contribute to the stereo thresholds at isoluminance; if they cannot, then the artifacts are likely to be unimportant.

To compute the luminance artifact distribution, \(e(x, y)\), it is necessary to estimate \(T_r(u,v)/T_g(u,v)\), which we term the chromatic aberration transfer function, \(T_r(u,v)\). In general, the farther \(T_r(u,v)\) is from 1.0 the greater the size of the luminance artifacts. The values of \(T_r(u,v)\) are dependent on a number of factors: the variation in the index of refraction of the ocular media with wavelength, the state of accommodation, the size of the pupil, the position of the pupil with respect to the nodal points, the eccentricity of the stimulus from the optic axis, and the sizes of the monochromatic aberrations of the eye. A precise model containing all these factors is not available. However, our only need was to get an upper bound on the size of the luminance artifacts. Therefore, in estimating \(T_r(u,v)\), we attempted to err in the direction of overemphasizing the effects of the above factors.

In all the calculations, it was assumed that the variation in the index of refraction of the ocular media with wavelength is adequately described by the mean results of Wald and Griffin and Bedford and Wyszecki. This is a reasonable assumption because there is good agreement between the different studies.

Longitudinal chromatic aberrations increase with pupil size. Crude measurements of pupil size in our apparatus, at the mean luminance of the CRT screen (10 cd/m²), showed that subject WSG had the largest pupil, which was between 4 and 4.5 mm. Therefore the diameter of the entrance pupil was assumed to be 4.5 mm.

If subjects accommodate to the yellow background (which is the most reasonable assumption), the longitudinal chromatic aberration artifacts would be minimized because both the red and the green patterns would be blurred approximately equally by the optics of the eye. However, focusing errors have a big effect on the size of the longitudinal artifacts. Thus, to be conservative, we assumed accommodation to the dominant wavelength of the green primary, which was 550 nm. (We also evaluated the assumption of accommodation to the red primary, 610 nm, but smaller artifacts were obtained.)

The size of the lateral chromatic aberration artifacts depends on the location of the pupil with respect to the nodal points and the optic axis. We assumed that the pupil was located at the front surface of the lens. In the LeGrand schematic eye, this places the pupil 7.5 mm in front of the image nodal point. Simonet and Campbell point out that the pupil may be shifted off the optic axis toward the visual axis, thereby reducing the lateral chromatic aberrations at the fovea. However, to be conservative we assumed that the pupil was centered on the optic axis.

Lateral chromatic aberration artifacts increase with eccentricity; the artifacts are nil at the optic axis and grow with distance from the optic axis. The shifted squares in the random-dot stereograms were 3.2° in width. Under the assumption that the visual axis is 5° eccentric to the optic axis, the farthest edge of the shifted square fell 6.6° eccentric to the optic axis. A stimulus eccentricity of 6.6° was assumed in the calculations. (Note that pixels beyond the edge of the square were irrelevant because the disparity in those regions did not change between the first and the second stimulus intervals.)

The higher-order monochromatic aberrations have a modest effect on the chromatic aberration transfer function. To include realistic levels of the monochromatic aberrations, we used the average values reported by Walsh et al. (see their Table 2). As a check on the reasonableness of the coefficients of Walsh et al., we computed, using their coefficients, the OTF for the stimulus conditions of the Campbell–Gubisch experiment (white light, pupil diameter 4.9 mm). Specifically, we added the effects of diffraction and chromatic aberration to the monochromatic aberrations of Walsh et al. The resulting OTF was in good agreement with the Campbell–Gubisch OTF. There are, however, substantial individual differences in the sizes of the monochromatic aberrations. To evaluate the effects of these individual differences, we computed \(T_r\) when the aberration coefficients of Walsh et al. were scaled by ±50% and found that \(T_r\) varied only slightly.

All the above assumptions were incorporated into a computer program that calculated the OTF’s for the red and the green primaries by standard methods. The spectral power distributions of the red and the green primaries were assumed to be the same as those reported by Cowan for the same Tektronics monitor. The program was checked (a) by comparing the OTF’s generated by the program with special cases for which there are known analytical solutions, e.g., square apertures and simple defocus, and (b) by comparing the OTF the program computed (at 555 nm) by using the aberration coefficients of
Walsh et al.\textsuperscript{17} with the one that Walsh et al. computed using the same coefficients.

Figure 1A shows the computed transfer functions for the red and the green primaries assuming accommodation to the dominant wavelength of the green primary. As can be seen, when the eye is accommodated to green, the red OTF falls off at a faster rate than the green OTF. The solid curve in Fig. 1B shows the ratio of the transfer functions for the red and the green primaries ($T_r$). As mentioned above, $T_r$ can be used in Eq. (9) to compute a pure luminance pattern (drawn in the green primary) that produces the same retinal luminance distribution as an isoluminant red–green pattern. Figure 2A shows such an artifact luminance distribution for the unblurred isoluminant random-dot pattern in one eye. Stereograms of these luminance artifacts (when presented as green images on the CRT) were easily fused by our subjects. Although the sizes of the computed luminance artifacts are undoubtedly somewhat overestimated, the easy fusion of the patterns suggests that the chromatic aberration artifacts might be large enough to drive fusion. Thus it is possible that some previous reports of stereo fusion at isoluminance were due to chromatic aberration artifacts.

The dashed curve in Fig. 1B shows the exponential decay transfer function that was selected to filter both the red and the green patterns before presentation on the CRT [i.e., $A(u, v)$ was multiplied by $\exp[-2\pi\beta(u^2 + v^2)^{1/2}]$]. The value of the exponential decay constant, $\beta$, was 0.21. This value of $\beta$ removes essentially all the frequencies from the random-dot patterns above 3 cycles/deg. As can be seen, the chromatic aberration transfer function has a value close to 1.0 below 3 cycles/deg, and hence the artifacts after blurring should be small. Indeed, Fig. 2B shows the chromatic aberration artifacts after blurring. The rms contrast of the luminance artifact distribution was 0.0026 (the rms contrast is defined below). Stereograms of these luminance artifacts could not be fused by any of our subjects. Thus we concluded that all the significant artifacts were removed.

**Control for Blue-Cone Response**

The red–green patterns were obtained by mixing light from the red and the green guns in the CRT. Thus there was some modulation produced in the short-wavelength (B) cones at isoluminance. At a mean luminance of 10 cd/m$^2$, photon absorptions in the B cones must be low. Nonetheless, the B cones may have contributed to stereopsis at isoluminance.\textsuperscript{12,13} To determine whether this happened, we included control conditions in which B-cone absorptions were held constant. This was done by adding an appropriate amount of blue (computed by using the

![Fig. 1. Computed OTF's. A, OTF's for the red and the green primaries assuming (a) a 4.5-mm pupil located on the optic axis, 7.5 mm in front of the image nodal point in the Legrand schematic eye, (b) a stimulus eccentricity of 6.6°, (c) accommodation to the green primary (dominant wavelength 550 nm), and (d) monochromatic aberrations equal to the average reported by Walsh et al.\textsuperscript{17} B, The solid curve is the ratio of the OTF's for the red and the green primaries in panel A. The dashed curve is the transfer function used to blur the random-dot patterns before presentation on the CRT.](image)

![Fig. 2. Simulation of luminance artifacts that are due to longitudinal and transverse chromatic aberrations. A, Aberration artifacts for one side of a red–green, random-dot stereogram pair at isoluminance, with square elements 16 arcmin in width. The longitudinal artifacts were computed by using the chromatic aberration transfer function in Fig. 1B [see Eq. (9)]. Stereograms with these artifacts (when presented as gray-scale images on a CRT) were easily fused by our subjects, suggesting that luminance artifacts resulting from chromatic aberrations might be large enough to drive fusion. B, Aberration artifacts that remained after the red–green stereogram was blurred by using an exponential filter function (see method). The artifact stereogram, after blurring, could not be fused by any of our subjects, indicating that all the significant artifacts were removed.](image)
Fig. 3. Rms contrast thresholds for front–back discriminations in blurred random-dot stereograms plotted as a function of the ratio, \( p \), of the red luminance to the red + green luminance. Photometrically defined isoluminance occurs at a ratio of 0.5. Each data point is the average of four staircases; error bars indicate \( \pm 1 \) standard error of the mean. The open symbols are the threshold for a monochrome yellow (pure luminance) stereogram. The curve is the locus of constant equivalent contrast calculated by using an ideal observer located at the level of the photopigments. The good fit between the measured thresholds and the equivalent-contrast curve indicates that LVS's visual system is using chromatic and luminance information with equal efficiencies in this task.

Fig. 4. Rms contrast thresholds for front–back discriminations in blurred random-dot stereograms plotted as a function of the ratio, \( p \), of the red luminance to the red + green luminance. Although ACS is able to fuse random-dot stereograms at isoluminance, the difference between his contrast thresholds and the equivalent-contrast curve indicates that near isoluminance his visual system uses chromatic information with less efficiency than it uses luminance information.

Fig. 5. Rms contrast thresholds for front–back discriminations in blurred random-dot stereograms plotted as a function of the ratio, \( p \), of the red luminance to the red + green luminance. The good fit between the measured thresholds and the equivalent-contrast curve indicates that WSG's visual system is using chromatic and luminance information with equal efficiencies in this task.

RESULTS

The contrast threshold data (as a function of the ratio of the red to the red + green luminance, \( \rho \)) are plotted in Figs. 3, 4, and 5 for subjects LVS, ACS, and WSG, respectively. The symbols are the average thresholds obtained from four staircase runs; the error bars indicate \( \pm 1 \) standard error. The thresholds are reported as rms contrasts (of the pattern in each primary), as defined in the following equation:

\[
C_{\text{rms}} = \left\{ \frac{1}{A} \sum \sum [l(x, y) - \bar{l}^2] / \bar{l} \right\}^{1/2},
\]

where \( A \) is the area of the stereogram, \( l(x, y) \) is the luminance at point \( (x, y) \) and \( \bar{l} \) is the mean luminance. Substi-
Table 1. Contrast Thresholds at Isoluminance for Constant S-Cone Absorption

<table>
<thead>
<tr>
<th>Subject</th>
<th>Constant S Cone</th>
<th>Varying S Cone</th>
<th>Monochrome</th>
</tr>
</thead>
<tbody>
<tr>
<td>LVS</td>
<td>0.023 (0.0064)</td>
<td>0.023 (0.0065)</td>
<td>0.0054 (0.0014)</td>
</tr>
<tr>
<td>ACS</td>
<td>0.083 (0.0452)</td>
<td>0.074 (0.088)</td>
<td>0.0085 (0.0017)</td>
</tr>
<tr>
<td>WSG</td>
<td>0.043 (0.0051)</td>
<td>0.043 (0.009)</td>
<td>0.013 (0.0066)</td>
</tr>
</tbody>
</table>

The open circles in the figures indicate the thresholds for the monochrome yellow stimulus; they were placed at both ends of the horizontal axis for reference. The thresholds for all values of \( p \) are higher than the threshold for the monochrome yellow. All three subjects show an increase in threshold around isoluminance but not loss of fusion. Thus it appears that stereoscopic fusion can occur at isoluminance even when chromatic aberrations are eliminated.

The thresholds for the monochrome (yellow) stereogram serve as a useful reference for verifying that the aberration artifacts were in fact undetectable. For each color in an isoluminant stereogram (at 100% contrast before blurring) the rms luminance contrast was 0.239 after blurring. As described above, the rms luminance contrast produced by all the chromatic aberrations in our blurred stereograms was, by our calculations, at most 0.0026. The measured rms thresholds at isoluminance were 0.0211, 0.0948, and 0.0502 for LVS, ACS, and WSG, respectively. Thus, at the isoluminance threshold, the contrasts of the aberration artifacts for these three subjects were 0.00005, 0.00025, and 0.00013. The thresholds for the monochrome (yellow) stereogram were 0.0061, 0.0079, and 0.0132 for LVS, ACS, and WSG, respectively. Therefore the luminance contrasts of the aberration artifacts were far (1.5–2 log units) below threshold. In fact, the artifacts could be substantially larger than our maximum estimate and still be well below threshold.

A closer look at the results shows that for subject LVS the highest threshold is not at isoluminance but is centered around a \( p \) of 0.45. Both subjects ACS and WSG, however, show a more symmetric increase in threshold centered around isoluminance. Thus there were some individual differences in the luminance ratio when discrimination was poorest.

At all ratios LVS and WSG have lower contrast thresholds than ACS. However, the differences in sensitivity for the three subjects are not constant across all the ratios. Near isoluminance, the relative threshold for ACS is much higher than for LVS and WSG.

As mentioned above, even at isoluminance there will be some spatial modulation of photon absorptions in the B cones. Table 1 shows the results of the control experiment in which B-cone absorptions were held constant. The right-hand and the middle columns replicate two of the conditions shown in Figs. 3–5. The right-hand column shows stereo contrast thresholds for the monochrome yellow pattern, and the middle column shows them for the isoluminant pattern (\( p = 0.50 \)). The ratios of the thresholds for these two conditions are similar to those in Figs. 3–5 for all three subjects. The left-hand column shows stereo contrast thresholds for the monochrome yellow pattern, and the middle column shows them for Figs. 3–5 for all three subjects. The left-hand column shows stereo contrast thresholds for the isoluminant pattern (\( p = 0.50 \)). The ratios of the thresholds for these two conditions are similar to those in Figs. 3–5 for all three subjects. The left-hand column shows stereo contrast thresholds for the isoluminant pattern (\( p = 0.50 \)).

EFFICIENCY OF LUMINANCE AND CHROMATIC STEREO FUSION

The present experiments showed that isoluminant random-dot stereograms can be fused even when they contain negligible longitudinal and transverse chromatic aberrations. Nonetheless, the stereo contrast thresholds were greatest near isoluminance. Either of two hypotheses might explain this result: (a) loss of effective contrast in the receptors because of the overlap in the long- and the middle-wavelength cone pigments or (b) different postreceptoral processing of color and luminance information. To discriminate between these two hypotheses, we computed the relative efficiency of the postreceptoral mechanisms, using an ideal-observer analysis.\(^8\)

In this ideal-observer analysis, relative efficiency was determined by computing equivalent luminance contrasts.\(^9\)

Two spatially and temporally identical stimuli with the same mean luminance are said to have identical equivalent contrasts if the amounts of information available for a discrimination at the level of the photopigments are identical for both stimuli. In general, because of the overlap in the spectral sensitivities of the cones, the equivalent contrast available at the level of the photopigments decreases when luminance differences in the stimuli are replaced by chromatic differences. Therefore the threshold contrast for chromatic stimuli (when defined as the threshold contrast of R and G stimuli components) is expected to be higher than the threshold contrast for luminance stimuli.

If the postreceptoral mechanisms use both luminance and chromatic cues with equal efficiency, then the chromatic and the luminance stereograms should have equal contrast thresholds for fusion as expressed in terms of equivalent contrast. Equivalent luminance contrasts were computed by assuming the Smith–Pokorny\(^24\) fundamentals and a 2:1 ratio of red to green cones.\(^25,27\) The calculations are briefly described in Appendix A and in more detail in Ref. 9. The curves in Figs. 3–5 are the computed equivalent-contrast curves; if the thresholds fall on these curves, then the chromatic information and the luminance information are being used with equal efficiencies. For LVS (Fig. 3) and WSG (Fig. 5) the obtained contrast thresholds for the various luminance ratios fall on or near the equivalent-contrast curves. Although the fit between the data and the equivalent contrast for LVS is close, there seems to be a slight shift of her data relative to the equivalent-contrast curve. This mismatch might be due to individual differences in the proportions of the R and the G cones, which would shift the curve laterally with little change in shape.

Unlike the thresholds for LVS and WSG, the stereo thresholds for ACS do not fall near the equivalent-contrast curve for the luminance ratios near isoluminance. For the other luminance ratios, the thresholds fall closer to...
the equivalent-contrast curve. Thus, near isoluminance, ACS's visual system does not use color information as efficiently as it uses luminance information. A different proportion of the R, G, B cones could not explain his data. A more likely explanation would be a partial loss of the color input to stereopsis in later neural stages.

Jordan et al. measured luminance and chromatic stereo thresholds for unblurred line stereograms in a paradigm similar to that of the present study; however, in their experiment there was no control for the effects of chromatic aberration. They also found equal efficiencies for luminance and chromatic stereograms (similar to the present results for LVS and WSG) even though the stimuli contained sharp edges. For comparison we measured stereo thresholds on our subjects for unblurred random-dot stereograms in the isoluminant and the monochrome (pure luminance) conditions. Table 2 shows the ratios of the monochrome thresholds to the isoluminant thresholds for the blurred and the unblurred stereograms for each subject. (The thresholds for the blurred conditions were taken from Figs. 3-5.) As can be seen, the ratios of the monochrome to the isoluminant thresholds were approximately the same for both the blurred and the unblurred stereograms. This similarity suggests that the relative efficiencies with which chromatic and luminance cues are used are similar for both blurred and unblurred stereograms. Together, these two studies suggest that chromatic aberration artifacts may not always provide useful stereo cues, even for images with sharp edges.

The fact that the subjects showed approximately equal relative efficiencies for the blurred and the unblurred stereograms seems curious because the present calculations showed that the chromatic aberration artifacts may be quite large (see Fig. 2A). However, there are two factors that might explain this result. First, recall that our calculations of the sizes of the artifacts assumed that the subject was accommodating to one of the two primaries. If the subjects in fact accommodated to the yellow background, then the artifacts would have been substantially smaller and less likely to contribute to stereopsis. Second, the results for the blurred stereogram show that color alone can mediate stereopsis with reasonably high efficiency; thus the chromatic cues may be used sufficiently well by the visual system that the artifact cues make no significant contribution.

**DISCUSSION**

In the present study, reliable stereo (front–back) discrimination was found at isoluminance even when all detectable chromatic aberration artifacts were removed. However, some studies have found stereopsis to be impossible for isoluminant random-dot stereograms. For example, Lu and Fender reported data for one subject who was unable to fuse nonblurred stereograms at isoluminance. An additional subject from the Lu–Fender study, reported by Russell, also could not obtain fusion at isoluminance. However, equivalent-contrast calculations show that fusion at isoluminance should not have been possible even if these subjects were using color and luminance information with equal efficiencies (as were LVS and WSG in the present study). Specifically, the subjects' stereo thresholds for the pure luminance patterns were so high that the loss of effective contrast information at isoluminance necessitated threshold contrasts greater than 100%. One way to appreciate this is to consider what would have happened in the present experiment if the stereo thresholds for the pure luminance case (open circles in Figs. 3–5) were much higher. If these thresholds (and the curves) were shifted upward, there would come a point at which the chromatic thresholds would exceed the maximum obtainable contrast. This is what happened in the study of Lu and Fender.

Grinberg and Williams and Wilson et al. found that fusion is possible for random-dot stereograms that isolate the B cones. Because stereopsis can be mediated by B cones alone, we ran a control experiment in which B-cone absorptions were held constant. The control experiment showed that B cones did not contribute to the fusion of our random-dot stereograms. This result is not consistent with the previous B-cone studies because the mean photon absorption in the B cones for our stimuli was so low. Grinberg and Williams and Wilson et al. note that B cones contribute little to luminance channels. Thus their studies, which also eliminated the effects of chromatic aberration, offer further support for the conclusion that chromatic information alone may support stereopsis.

One of the striking features of stereo perception is the large individual differences. Our study also revealed large individual differences. Of the six subjects pretested, only three were able to fuse the blurred, isoluminant stereograms. Of the remaining three, one could not fuse any stereograms, and the other two could fuse only high-contrast, luminance stereograms. We did not further test these two subjects because, as with the subjects in Lu and Fender's study, it would not have been possible to obtain complete threshold curves. Note that it is quite possible that these subjects used chromatic and luminance information with equal efficiency, but this hypothesis could not be fully tested without a complete threshold curve. Finally, even among the subjects who could fuse isoluminant stereograms, there were some individual differences.

A number of investigators have suggested that chromatic information does not enter the stereo pathway. This suggestion was based on the poor or absent stereo performance observed at isoluminance. The present results, however, show that isoluminant random-dot stereograms can be fused, even when all the detectable longitudinal and transverse aberrations have been removed by blurring. Thus it is clear that luminance artifacts were not the cues permitting fusion. Two of our subjects showed equal neural efficiencies for luminance and chromatic stereograms. Similarly, Jordan et al. found equal efficiencies for luminance and chromatic line stereograms. Although we cannot draw strong inferences concerning the neural pathways for stereopsis from these results, they suggest, if anything, that the same or similar stereo mechanisms are processing color and luminance information.
APPENDIX A: DEFINITION OF EQUIVALENT CONTRAST

We present the definition of equivalent luminance contrast that was used to determine the relative efficiency with which chromatic and luminance information is used in stereopsis. To begin with, consider a random-dot stereogram before blurring. Let \( A \) and \( B \) represent the stereogram elements, which may differ in chromaticity and/or luminance. Next, consider an identical stereogram in which the elements \( A \) and \( B \) have been replaced by elements \( A' \) and \( B' \) that differ in luminance (but not chromaticity). Furthermore, let the mean luminance of the stereogram with elements \( A', B' \) be the same as the mean luminance of the stereogram with elements \( A, B \). The equivalent luminance contrast of the stereogram containing elements \( A, B \) is defined to be the luminance contrast of the stereogram containing elements \( A', B' \); such that the pairs of elements \( A', B' \) and \( A, B \) are equally discriminable by an ideal observer operating at the level of the photopigments.\(^3\) This definition for nonblurred stereograms remains valid after blurring because the blurring reduces ideal-observer discriminability equally for each pair of elements.

The following formula for equivalent luminance contrast, \( C_e \), is derived in Ref. 9:

\[
C_e = \frac{(rp_c d_c + gp_d d_g + bp_c d_b)}{(rp_d d_r + gp_d d_g + bp_d d_b)^2((rp_c + gp_d + bp_c)^2)}.
\]

(A1)

In this equation \( p_r, p_g, \) and \( p_b \) are the proportions of \( R, G, \) and \( B \) cones in the receptor lattice. The quantities \( r_c, g_c, \) and \( b_c \) are the sums of the photon absorptions produced by elements \( A \) and \( B \) in the individual \( R \), \( G \), and \( B \) cones. For example, if \( r_s \) and \( r_b \) are the numbers of photons absorbed in an \( R \) cone for elements \( A \) and \( B \), respectively, then \( r = r_s + r_b \). Similarly, \( r_c, g_c, \) and \( b_c \) are the sums of the photon absorptions produced by elements \( A' \) and \( B' \) in the individual \( R, G, \) and \( B \) cones. The quantities \( c_r, c_g, \) and \( c_b \) are the absorption contrasts in the individual cone types: \( c_r = |\Delta r|/r, c_g = |\Delta g|/g, \) and \( c_b = |\Delta b|/b \), where \( \Delta r, \Delta g, \) and \( \Delta b \) are the differences in the absorptions produced by elements \( A, B \) and \( A', B' \) in the respective cone types. Finally, \( d_r = \ln(1 + c_r)/(1 - c_r) \), \( d_g = \ln(1 + c_g)/(1 - c_g) \), and \( d_b = \ln(1 + c_b)/(1 - c_b) \).

To compute the quantities in this formula, we assumed that the spectral power distributions of the Tektronics 690SR phosphors were the same as those reported by Cowan.\(^21\) That the transmittance function of the color media was that reported by Wyszecki and Stiles,\(^15\) that the receptor absorption spectra were those reported by Smith and Pokorny,\(^24\) and that the proportions of \( R, G, \) and \( B \) cones were as follows: \( p_r = 0.65, p_g = 0.33, \) and \( p_b = 0.02.\)\(^26\)

In calculating the equivalent-contrast curves in Figs. 3–5, we used the elements of the yellow monochrome stereogram as the reference elements, \( A' \) and \( B' \). Thus the equivalent-contrast curves were forced to pass through the yellow monochrome thresholds (the open symbols in the figures).

REFERENCES AND NOTES

28. Another factor might be that the magnification responsible for the transverse chromatic aberration artifacts is centered on the optic axis, which is approximately 5° nasal from the
fovea. Therefore the sizes of the artifacts at corresponding locations in the two eyes will be different; these differences may induce a rivalry that diminishes any potential benefits of the chromatic aberration artifacts. In fact, this rivalry could explain why the ratios of the monochrome to the isoluminant thresholds were slightly smaller for the blurred than the unblurred stereograms.
