
Robert Moffitt
Johns Hopkins University

Peter Gottschalk
Boston College

September, 2008
Revised, December, 2008

The authors would like to thank the participants at many seminars and conferences for comments, Joe Tracy and John Abowd for discussant comments, and Tom DeLeire and Gary Solon for helpful conversations. Outstanding research assistance was provided by Yonatan Ben-Shalom, Kai Liu, Shannon Phillips, and Sisi Zhang.
Abstract


We estimate the trend in the transitory variance of male earnings in the U.S. using the Michigan Panel Study of Income Dynamics from 1970 to 2004. Using both an error components model as well as simpler but more approximate methods, we find that the transitory variance increased substantially in the 1980’s and then remained at this new higher level through 2004. We also find a strong cyclical component to the transitory variance. Its increase accounts for between 30 and 65 percent of the total rise in cross-sectional variance, depending on the time period. The cross-sectional variance has recently increased but this reflects a rise in the variance of the permanent component, not the transitory component. Increases in transitory variance occurred for less educated in the early 1980s and for more educated workers in the later 1980s and early 1990s.
A substantial literature has accumulated on trends in various measures of instability in individual earnings and family income in the US over the last thirty or so years (Gottschalk and Moffitt, 1994; Moffitt and Gottschalk, 1995; Dynarski and Gruber, 1997; Cameron and Tracy, 1998; Haider, 2001; Hyslop, 2001; Moffitt and Gottschalk, 2002; Dahl et al., 2008; Dynan et al., 2008; Jensen and Shore, 2008; Shin and Solon, 2008) as well as in Canada (Baker and Solon, 2003; Beach et al., 2003, 2006). Interest in this question has been motivated from several different directions. One is that greater instability of earnings or income decreases welfare because families cannot completely smooth income fluctuations, at least not without significant cost, though other work implies that transitory shocks are mostly insurable (Dynarski and Gruber, 1997; Blundell et al., forthcoming). Another is that increased instability contributes, statistically, to a rising cross-sectional variance of income and can therefore partially explain rising cross-sectional inequality. For example, Gottschalk and Moffitt (1994) found that one-half of the increase in cross-sectional earnings inequality of white men from the 1970s to the 1980s arose from an increase in the transitory variance of earnings.

In this paper we report estimates of the transitory variance of earnings of U.S. males through 2004. The literature to date has generally found increases in instability of individual earnings from the 1970s to the 1980s, but whether that instability declined over the rest of the 1980s and 1990s (Dynarski and Gruber, 1997; Cameron and Tracy, 1998; Haider, 2001) or stabilized at the new higher level (Dahl et al., 2008; Shin and Solon, 2008) is not widely agreed upon. One reason for the difference in findings, as we will show, is that instability is highly cyclical and can be mistaken for a trend if the time period ends near a cyclical peak or trough.
Another reason for the difference in findings is that different measures of instability are being used. Our study uses the most recent PSID data available and uses the classical definition of the variance of a transitory component of earnings.

We use several methods to estimate trends in transitory variances. Our preferred method is a formal error components model which is an extension of that originally developed by Moffitt and Gottschalk (1995), but here adding several features that have since gained prominence in the literature. We also use two simpler methods, one an extension of the method originally suggested by Gottschalk and Moffitt (1994) and the other a new nonparametric method which provides consistent estimates of the transitory variance under a particular maintained assumption. All three methods show rising transitory variances during the 1980’s and a leveling off starting in the early 1990s and continuing through 2004. These trends are net of the large cyclical changes that we also document.

The first section of the paper briefly gives the intuition for how trends in transitory variances are identified with a panel data set. The next section describes the data set we construct and the third section lays out our methods and results. We present results disaggregated by education and then provide a final section that compares our results to others in the literature and provides explanations for the differences. A brief summary concludes.
I. Identification of Trends in Transitory Variances

The intuition for identification of trends in a transitory variance can be seen from the canonical error components model with permanent and transitory components:

\[ y_{it} = \mu_i + v_{it} \]  

(1)

where \( y_{it} \) is log earnings or residual log earnings for individual \( i \) at age \( t \), \( \mu_i \) is a time-invariant, permanent individual component and \( v_{it} \) is a transitory component. The typical assumptions are that \( E(\mu_i) = E(v_{it}) = 0 \), \( Var(\mu_i) = \sigma^2_\mu \) and \( Var(v_{it}) = \sigma^2_v \). Identification and estimation of this basic random effects model has been known since the 1960s. However, typically these models assume \( E(v_{it}v_{i\tau}) = 0 \) for \( t \neq \tau \) but this has been shown not to hold in most earnings applications. When it does not hold, identification is less obvious. Carroll (1992) was, to our knowledge, the first to point out explicitly that identification can be obtained from “long” autocovariances. The covariance of \( y_{it} \) between periods \( \tau \) apart is

\[ \text{Cov}(y_{it}, y_{i,t-\tau}) = \sigma^2_\mu + \text{Cov}(v_{it}, v_{i,t-\tau}) \]  

(2)

and hence \( \sigma^2_\mu \) is identified from \( \text{Cov}(y_{it}, y_{i,t-\tau}) \), which is observed in the data, provided \( \text{Cov}(v_{it}, v_{i,t-\tau}) = 0 \). But \( \text{Cov}(v_{it}, v_{i,t-\tau}) = 0 \) is essentially the definition of a transitory component in the first place, because this covariance represents its persistence (that is, whether a transitory shock at time \( t-\tau \) is still present, in reduced magnitude, by time \( t \)). Since the definition of a

---

\(^1\) The textbook random-effects ANOVA expression for the permanent variance, which involves the variance of the mean of \( y \) for each individual over time periods, should, under the assumptions of the model, equal this long autocovariance if that mean is taken over periods far apart.
transitory component is something that eventually goes away, the permanent variance must be identified at sufficiently high values of \( \tau \).

Once the permanent variance is identified, the transitory variance is identified as the residual:

\[
\sigma_u^2 = \text{Var}(y_{it}) - \sigma_\mu^2
\]

(3)
because \( \text{Var}(y_{it}) \) is observed in the data. This exercise can be conducted in different calendar periods, thereby revealing whether transitory variances are changing.

This method of identification of permanent and transitory variances from the long autocovariances of \( y_{it} \) is employed in the richer error components model as well as the nonparametric method described below. The data requirements are therefore for a sufficiently long panel which allows not only calculation of variances but also long autocovariances, and for different periods of calendar time.

II. Data

The Michigan Panel Study on Income Dynamics (PSID) satisfies these requirements, for it covers a long calendar time period (1968-2005) and, because it is a panel, we can compute autocovariances of earnings for the same individuals between periods quite far apart. We use the data from interview year 1971 through interview year 2005.\(^2\) Earnings are collected for the previous year, so our data cover the calendar years 1970 to 2004. The PSID skipped interviews

\(^2\) We do not use earnings reported in 1969 or 1970 since these are reported only in bracketed form.
every other year starting in interview year 1998, so our last five observations are for earnings years 1996, 1998, 2000, 2002, and 2004. We follow many previous studies by focusing only on males in order to reduce the effects of trends in labor force participation. We take any year in which these males were between the ages of 30 and 59, a head of household (wage and salary information is not available for males who are not heads), not a student, and had positive annual wage and salary income and positive annual weeks of work. We include men in every year in which they appear in the data and satisfy these requirements. We therefore work with an unbalanced sample because of aging into and out of the sample in different years, attrition, and movements in and out of employment. Fitzgerald et al. (1998) have found that attrition in the PSID has had little effect on its cross-sectional representativeness, although less is known about the effect of attrition on autocovariances. Measurement error in earnings reports is another potential problem when using survey data to estimate covariance matrices. However, Pischke (1995) has shown that measurement error in the PSID has little effect on earnings covariances and Gottschalk and Huynh (forthcoming) show that this is a result of the non-classical structure of measurement error in earnings found in many surveys. We exclude men in all PSID oversamples (SEO, Latino). All earnings are put into 1996 CPI-U-RS dollars. The resulting data set has 2,883 men and 31,054 person-year observations, for an average of 10.8 year-observations per person. Means of the key variables are shown in Appendix Table A-1.

Rather than form a variance-autocovariance matrix directly from these earnings observations, we work with residuals from regressions of log earnings on education, race, a polynomial in age, and interactions among these variables, all estimated separately by calendar year. Our analysis, therefore, within-group transitory variances of log earnings. We use these
residuals to form a variance-autocovariance matrix indexed by year, age, and lag length. Thus a
typical element consists of the covariance between earnings of men at ages a and a’ between
years t and t’. Because of sample size limitations, however, we cannot construct such
covariances by single years of age. Instead, we group the observations into three age groups--30-
39, 40-49, and 50-59--and then construct the variances for each age group in each year, as well as
the autocovariances for each group at all possible lags (back to 1970 or age 20, whichever comes
first). We then compute the covariance between the earnings of the group in the given year and
each lagged year, using the individuals who are in common in the two years.³ The resulting
autocovariance matrix represents every individual variance and covariance between every pair of
years only once, and stratifies by age so that life cycle changes in the variances of permanent and
transitory earnings can be estimated.⁴ The covariance matrix has 1,197 elements over all years,
ages, and lag lengths. A few specimen elements are illustrated in Appendix Table A-2.

III. Models and Results

We present results on trends in transitory variances from three models: a parametric error
components model, which is our preferred model; an approximate nonparametric implementation
of that model; and an even simpler method used originally by Gottschalk and Moffitt (1994)
which also only approximates the variance of interest.

³ We trim the top and bottom one percent of the residuals within age-education-year cells
to eliminate outliers and top-coded observations.

⁴ The first stage regression controls for changes in the mean of earnings. Standard human
capital or search models imply that variances as well as means change with age.
Error Components Model. We first formulate an error components model of life cycle earnings dynamics process in the absence of calendar time shifts. There is a large literature on the formulation of such models (Hause, 1977, 1980; Lillard and Willis, 1978; Lillard and Weiss, 1979; MaCurdy, 1982; Abowd and Card, 1989; Baker, 1997; Geweke and Keane, 2000; Meghir and Pistaferri, 2004; Guvenen, 2007; see MaCurdy, 2007, for a review). These models have suggested that the permanent component is not fixed over the life cycle but evolves, typically with variances and covariances rising with age. This pattern can be captured by a random walk or random growth process in the permanent effect. The literature has also shown that the transitory error is serially correlated, usually by a low-order ARMA process. Our model contains all these features:

\[ y_{ia} = \mu_{ia} + \nu_{ia} \]  \hspace{1cm} (4)

\[ \mu_{ia} = \mu_{i,a-1} + \delta_i + \omega_{ia} \]  \hspace{1cm} (5)

\[ \nu_{ia} = \rho \nu_{i,a-1} + \xi_{ia} + \theta \xi_{i,a-1} \]  \hspace{1cm} (6)

with \( \text{E}(\mu_{ia})=\text{E}(\delta_i)=\text{E}(\omega_{ia})=\text{E}(\xi_{ia})=0 \), orthogonality between all four of these variables, and initial conditions \( \mu_{i0} \neq 0, \nu_{i0} = 0 \) (the life cycle begins at \( a=1 \)). Eqn(4) again posits a permanent-transitory model but with an age-varying permanent effect \( (\mu_{ia}) \). The latter evolves over the life cycle from a random growth factor \( (\delta_i) \) which allows each individual to have a permanently higher or lower growth rate than that of other individuals, and from a random walk factor \( (\omega_{ia}) \) that arrives randomly but is a permanent shock. The transitory error evolves accordingly to a ARMA(1,1) process typically found in the literature, with the underlying transitory shock \( (\xi_{ia}) \)
fading out at rate $\rho$ but deviating from that smooth fade-out rate by $\theta$ in the next period ($\theta$) (the MA(1) parameter $\theta$ improves the fit of the lag process significantly). Our tests also show, consistent with past work, that higher order ARMA parameters are not statistically significant. We assume all forcing errors to be i.i.d. except $\xi_{ita}$ whose variance we assume to vary with age because transitory shocks are likely to be greater at younger ages. We allow $\mu_{i0}$ and $\delta_i$ to be correlated in light of the Mincerian theory that they should be negatively related (those who have higher initial investments in human capital will start off low but rise at a faster rate). Hence $E(\delta_i^2) = \sigma_\delta^2$, $E(\omega_{ia}^2) = \sigma_\omega^2$, $E(\xi_{ita}) = \pi_0 + \pi_1 a$, $E(\mu_{i0}^2) = \sigma_{\mu}^2$ and $E(\mu_{i0} \delta_i) = \sigma_{\mu \delta}$.

In this more realistic model, compared to the simple canonical model outlined previously, the permanent variance, and hence the transitory variance, is identified only by extrapolation. Because of the AR(1) assumption on the transitory error, $\text{Cov}(u_{it}, u_{i,t-\tau})$ never hits zero in a finite lifetime. However, provided $\rho$ is not too high, that covariance will be small within the finite lifetime of an individual and the permanent variance can be identified implicitly by extrapolation, but again primarily by the long autocovariances.$^5$

With this identification condition satisfied, the parameters of the model can be identified for a single cohort. Determining whether there are calendar time shifts can therefore be identified from changes in parameters across multiple cohorts. Although all the parameters of the model could potentially shift with calendar time, for reasons of convenience we allow calendar time shifts in only two places in the model, in the permanent component and the forcing

---

$^5$ In an AR(1) model, another way to state the identification condition for the permanent variance is that we require $\rho < 1$. If $\rho = 1$, transitory shocks are equivalent to permanent shocks and hence the two cannot be separately identified. In practice, we have found this sometimes to be an important issue because estimates of $\rho$ can be close to 1.
transitory component:

\[ y_{iat} = \alpha_t \mu_{ia} + \nu_{iat} \]  
\[ \mu_{ia} = \mu_{i,a-1} + \delta_i + \omega_{ia} \]  
\[ \nu_{iat} = \rho \nu_{i,a-1,t-1} + \beta_t \xi_{ia} + \theta (\beta_{t-1} \xi_{i,a-1}) \]  

where \( t \) is calendar time. The parameter \( \alpha_t \) alters the variance of the permanent effect, which is now \( \alpha_t^2 \text{Var}(\mu_{ia}) \). This formulation coincides with an interpretation of \( \mu_{ia} \) as a flow of human capital services and \( \alpha_t \) as its time-varying price, consistent with the literature on changes in the returns to skill. We force it to be the same for all ages although this could be relaxed. The parameter \( \beta_t \) likewise allows calendar time shifts in the variance of the transitory component, which is now \( \beta_t^2 \text{Var}(\xi_{ia}) \).

The introduction of time-varying parameters introduces a problem of left-censoring because those parameters cannot be identified prior to 1970 yet their evolution prior to that year affects variances and covariances after 1970. To address this issue, we introduce a new parameter \( \gamma \) which allows the transitory variances in 1970 to deviate from what they would be if \( \beta_t = 1 \) for \( t < 1970 \), with \( \gamma = 0 \) implying no deviation. The details are given in Appendix B.

For any values of the parameters, the model in (7)-(9) generates a predicted set of variances and covariances in each year and for each age and lag length, and therefore a predicted value for each of the 1,197 elements of our covariance matrix. We estimate the parameters by
minimizing the sum of squared deviations between the observed elements and elements predicted
by the model, using an identity weighting matrix and computing robust standard errors. The
formal statement of the model and estimating procedure appears in Appendix B.6

The estimates of the model parameters are given in Appendix Table A-3. The transitory
component is significantly serially correlated both through the AR(1) and MA(1) component
implying, as discussed before, that long autocovariances are needed to identify the model.
However, our main interest is in the estimates of \( \alpha_t \) and \( \beta_t \), which are shown graphically in
Figures 1 and 2 along with smoothed trendlines (both are normalized to 1 in 1970). Figure 1
shows that the permanent variance rose in the 1980s, leveled off through the mid-1990s, and then
started rising again in the mid-1990s. This pattern is roughly consistent with rises in the return to
education and other indicators of skill differentials. This pattern reflects, as has already been
emphasized and as will be shown further below, trends in the long autocovariances in the data.

Of more direct interest to our focus on instability of earnings is Figure 2, which shows the
estimated values of \( \beta_t \) along with a trendline. The transitory variance rose sharply starting in the
mid-1970s and continued to rise, albeit at a slowing rate, until 1990, after which time it has
remained flat, although with fluctuations (related to the business cycle; see below). Thus our
answer to the question we posed in the introduction is: the transitory variance of male earnings
was higher in the 1980s than in the 1970s, and also higher in the 1990s than in the 1980s on

6 We estimate the model in levels rather than differences. The individual effect \( \mu_{i0} \)
does not cancel out in differences in our model because of the \( \alpha_t \). In addition, the covariance
matrix of the differences of \( y_{i,t} \) is a function of the same covariance matrix we are fitting with
our levels model. Fitting in levels is more convenient for our purposes because we wish to
decompose the trend in the cross-sectional variance of \( y_{i,t} \) into permanent and transitory
variances. We will show how our model fits the variances of differences of \( y_{i,t} \) below.
average, although its rate of rise declined in the later 1980s and stopped altogether by the early 1990s.

Drawing the implications of these estimated parameters for the variances of $\alpha_{ia}$ and $u_{iat}$--the permanent and transitory variances in the model--requires applying the formulas in Appendix B. Figure 3 shows the pattern for those age 30-39 (the variances differ by age, but a similar time pattern obtains for other ages). Both the permanent and transitory variances follow roughly the same pattern as $\alpha_t$ and $\beta_t$, as should be expected since these are the only calendar-time varying parameters in their formulas. From 1970 to 1984, the rise in the transitory variance accounted for 49 percent of the total rise in the cross-sectional variance, consistent with prior work on the 1980s by Gottschalk and Moffitt (1994) and Moffitt and Gottschalk (1995). From 1970 to 1993, the transitory variance accounted for 65 percent of the total cross-sectional variance rise. However, from 1970 to 2004, it has accounted for only 31 percent of the total rise because of the increase in the permanent variance in the last several years.

Much of the fluctuation in the transitory variance is business-cycle related. Figure 4 shows the same transitory variance but plotted along with the national unemployment rate for men 20 and over. The variance is clearly positively correlated with the unemployment rate, albeit with something of a lag in the first half of the period. This explains much of the variability of the transitory variance. Nevertheless, the variance has drifted upward, never coming back to its original level after a recession, and ending in 2004 at a higher level than at the beginning even though the unemployment rate has returned to its levels of the 1970s.

An important question for understanding these results is whether they can be demonstrated with simpler econometric methods, and whether they hold under more flexible
specifications of the permanent and transitory components than assumed in our parametric model. We consider two simpler methods to address this question.

Approximate Nonparametric Method. One approach is to follow the simple model described in Section I by using the long autocovariances to estimate the variance of the permanent effect and by then subtracting that value from the total variance to obtain an estimate of the transitory variance. Figure 5 shows the actual variance of those 30-39 in each year, for illustration, along with the autocovariances at lags 6 and 10, which might be considered to be sufficiently long so that the transitory shocks are no longer correlated. The figure shows this not to be the case, for the lag-10 autocovariance is below that at lag 6; that is, the autocovariance is still falling. Taking the difference between the upper line and the lag-10 autocovariance results in the transitory variance estimate shown in Figure 6, which necessarily cannot be computed until year 1980. This estimate shows an increase in the transitory variance in the early 1980s but a fall in the later 1980s, a rise in the early 1990s, and another fall in the later 1990s. This is a significantly different pattern than that in the error components model.

There are two drawbacks of this method. One is that the more realistic model of the age-evolution of the permanent effect shown in (4)-(6) with its random walk and random growth specifications no longer makes the autocovariance of the permanent component equal to its variance (see Appendix eqns (B15) and (B16)). The second is that the method does not work well when \( \alpha_t \) is evolving over the period covered by the long autocovariance, for in that case the autocovariance of \( y_{iat} \) is

\[
\text{Cov}(y_{iat}, y_{ia-t-\tau}) = \alpha_t \alpha_{t-\tau} \text{Cov}(\mu_{ia}, \mu_{i,t-\tau})
\]  

(10)
and therefore the long autocovariance will not equal $\alpha_t \text{Cov}(\mu_{ia}^{t}, \mu_{i,a-t})$ because of the presence of the lagged effect $\alpha_{t-\tau}$. Thus, for example, $\text{Cov}(y_{iat}, y_{i,a-\tau,t-\tau})$ will continue to rise even if $\alpha_t$ is falling, so long as $\alpha_{t-\tau}$ was rising; lagged trends in the permanent variance affect the current variance. Errors in the specification of $\alpha_t$ lead, in turn, to errors in the estimates of $\beta_t$ with this method.\footnote{This problem has been noted by Gottschalk and Moffitt (2006) and Shin and Solon (2008).}

Nevertheless, (10) forms a more appropriate basis for approximate nonparametric estimation because

$$\log[\text{Cov}(y_{iat}, y_{i,a-\tau,t-\tau})] = \log \alpha_t + \log \alpha_{t-\tau} + \log[\text{Cov}(\mu_{ia}, \mu_{i,a-\tau})]$$

$$= \log \alpha_t + \log \alpha_{t-\tau} + f(a, \tau) \quad (11)$$

where $f(a, \tau)$ is a flexible parametric approximation to $\log[\text{Cov}(\mu_{ia}, \mu_{i,a-\tau})]$. Eqn(11) can be estimated by OLS using year dummies to capture the $\alpha_t$ and (say) a polynomial approximation for $f$. This method is nonparametric because it makes no assumption on the way in which the permanent effect evolves--random walk, random growth, or something else--with that evolution approximated by an arbitrary function of age and lag length. But it is only approximate because the effects of past transitory shocks are never exactly zero because of the presence of the AR(1) process. Still, the transitory variance can be approximating by subtracting from the total variance the product of the square of the estimates of $\alpha_t$ and the estimated $f(a, \tau)$ evaluated at $\tau=0$ because the latter is an estimate of the permanent variance at age $a$. 

\footnote{This problem has been noted by Gottschalk and Moffitt (2006) and Shin and Solon (2008).}
Figure 7 shows the estimates of the transitory variance obtained in this way using a second-order polynomial for  and  for the function  and using all lags of order 10 and over in the regression eqn (11). The pattern is much closer to that obtained from the error components model, differing only in the period of the 1990s and after, when the variance gradually falls instead of flattening out. This latter difference is a result of the failure of the key assumption of negligible transitory autocovariance. At lag 10 that autocovariance is small but it is still trending upward (a plot demonstrating this is not shown for brevity). The transitory autocovariance is trending upward because the transitory variances themselves are; and, because the transitory components are serially correlated, an increase in the variance of the underlying transitory shocks necessarily increases all transitory autocovariances as well (see eqn (B19)). This spuriously pulls up the permanent variance when (11) is estimated, pushing the transitory variance down.

Nevertheless, this simple nonparametric method is close enough to the error components model to provide some support for it using this simpler and less parametric method.

Gottschalk-Moffitt (“BPEA”) Method. An even simpler method used by Gottschalk and Moffitt (1994) and applied in some subsequent studies (e.g., Beach et al., 2003,2005) is to estimate the permanent and transitory variances with standard random-effects formulas within moving calendar time windows of fixed length. To estimate the transitory variance in year  , average the  residuals in the calendar time window [t-w, t+w], for each individual i, to obtain an estimate of the individual’s permanent component; then compute the variance of these means to obtain an estimate of the permanent variance, and then compute the variance of the
transitory component from the $2w+1$ deviations of the residuals from each individual’s mean.\footnote{The exact formulas used for the permanent and transitory variance are given in Gottschalk and Moffitt (1994). The above description is only approximate because a term involving the deviations around individual means must be subtracted from the variance of the means to obtain consistent estimates of the permanent variance.} Repeat this process for each successive year $t$ in the data, providing a trend in the estimated transitory variance. This method, like some of the other simple methods, produces consistent estimates under the canonical model described in Section I. However, it is also again problematic in more general models, particularly those with serial correlation in the transitory component. It can be shown that the variance of the deviations of the residuals from individual means in this case includes the short-term transitory autocovariances with a negative sign. Thus, if those autocovariances are rising, the trend in the estimated transitory variance will be too small.

Figure 8 shows the results of this exercise using a nine-year window (thus the earliest date is 1974 and the latest is 2000 because of the requirement of four years of data on either side of the year in question). The results are quite similar to those from the error components model, rising in the 1980s and leveling off around 1990. While the transitory variance again takes a dip in the mid-1990s, arising from the bias just mentioned, it is small. The same substantive conclusion of an essentially flat profile of the variance in the 1990s would also be reached with these results.
IV. Differences by Educational Level

A question of interest is whether the increases in the transitory variance from the mid-1970s to 1990 were concentrated primarily in the less-skilled workforce, the more-skilled, or both. Gottschalk and Moffitt (1994), applying a variant of the BPEA method described above through 1987, found the variances to have risen at all education levels but at a much greater rate for the less educated than the more educated. Cameron and Tracy (1998) also find greater increases in earnings instability in the early 1980s for less-educated workers. The fact that the 1980s were also a period of marked deterioration in the low-skilled labor market in terms of levels of wages as well, suggests a possible general deterioration in the quality and stability of jobs for low-skilled workers.

We address this issue by estimating our error components model separately for those with years of education less than or equal to 12, and those more than 12. Our sample sizes do not permit further disaggregation. The results are shown in Figures 9 and 10, which show the implied estimates of permanent and transitory components at age 30-39. While the transitory variance for the more educated grew slightly in the late 1970s and early 1980s, the increase in the transitory variance across all age groups in the first half of the 1980s was entirely the result of increases for the less-educated workers, consistent with Gottschalk and Moffitt (1994). However, the transitory variance for the more educated workers rose in the late 1980s and through the early 1990s, although cyclical factors began to affect the trends at that time. In fact, the transitory variances for the less educated fell a bit in the later 1990s and, by 2004, the transitory variance was higher for the more educated than for the less educated. It is worth noting that this higher transitory variance for the more educated in the 1990s coincides with a period in which cross-
sectional inequality was also notably increasing in the upper percentiles of the distribution, again showing a correlation between cross-sectional measures of inequality and our estimates of transitory variances.⁹

Although it is not the focus of our analysis, we note that these results yield new findings on the trends in permanent variances within education groups, which we interpret as a measure of changes in the within group price of skill. While the permanent variance rose for the less educated from the 1970s to the early 1980s, it has not increased much since then, on average, although an upward trend beginning in 1995 may continue into the future. The permanent variance for the more educated, while also increasing in the 1980s and leveling off thereafter, shows a more notable upward trend since the mid-1990s. This finding is consistent with those of Lemieux (2006) on heterogeneous returns to college. Increases in returns to college not only increase mean earnings of college educated workers but also increase the within group inequality of college graduates if there is heterogeneity in returns among college graduates. However, our evidence on the recent growth in the transitory variance implies that the increase in the total cross sectional variance of more educated workers reflects increases in the transitory component as well as the increase in the permanent component stressed by Lemieux.

⁹ Application of the BPEA method by education yields the same findings; results available from the authors.
V. Comparison with Other Work

Our results are mostly comparable with the few prior studies that have estimated error components models with allowance for calendar-time shifts.\textsuperscript{10} Gottschalk and Moffitt (1994) and Moffitt and Gottschalk (1995) found increases in the transitory variance through 1987 using the same PSID data we use, Hyslop (2001) found increases in that variance from 1979 to 1985 also using PSID data, Cameron and Tracy (1998) found increases in that variance through 1982 using matched CPS data, Baker and Solon (2003) found increases in transitory variance in the early 1980s in Canada--a labor market closely linked to the US labor market--and Jensen and Shore found increases in the transitory variance from 1970s and 1980s (although also finding increases in the later 1990s and early 2000s, unlike our results).\textsuperscript{11} However, Haider (2001), Moffitt and Gottschalk (2002) and Baker and Solon (2003) found that the transitory variance continued to rise in the 1990s rather than flattening out, but the explanation for this difference is that those studies ended in 1992 or 1993, when the variance was rising for what can now be seen to be only cyclical reasons (see Figure 2 or 3).

Our results also differ from those of Dynarski and Gruber (1997), Cameron and Tracy (1998), Dahl et al.(2008) and Shin and Solon (2008), who find no increase in male earnings instability over the course of the 1980s.\textsuperscript{12} The explanation for this difference is that these studies

\textsuperscript{10} We compare our results only to prior studies that have estimated the transitory variance of male earnings (many estimate the transitory variance of family income).

\textsuperscript{11} Beach et al. (2006) found no increases in the transitory variance in Canada after 1982, however.

\textsuperscript{12} Dahl et al. only have data after 1984 and find that male earnings instability declined slightly after that point. Shin-Solon find declines in male earnings instability starting about 1982. Dynan et al. (2008), on the other hand, find increases in male earnings volatility in the
use a different measure of earning instability, namely, the variance of the change in earnings from a year $t$ to $t+1$ (one-year changes for Dahl et al. and two-year changes for Shin-Solon) or some measure related to it.\footnote{13} The relationship between the variance of the change in $y_{it}$ and our measure can be seen by examining the former, which is

\[
\text{Var}(y_{it} - y_{i,t-1}) = \text{Var}(y_{it}) + \text{Var}(y_{i,t-1}) - 2 \text{Cov}(y_{it},y_{i,t-1})
\]  

(12)

In the canonical permanent-transitory model described in Section I, this expression equals our measure of interest, the transitory variance, $\sigma^2_u$, exactly. However, that is not the case when the transitory errors are serially correlated, for then the covariance on the right-hand-side of (12) that must be used to obtain our measure should be a long autocovariance. This matters for the measurement of trends because, as we have shown, the short autocovariances of the transitory component have been increasing over time. Including these rising short autocovariances in the covariance in (12) will lead the resulting calculated trends in $\text{Var}(y_{it} - y_{i,t-1})$ to be “biased” downward (biased if it is the classical transitory variance that is aimed to be estimated). Using a long autocovariance in (12) should provide estimates closer to our series on trends in the variance.

\footnote{13} These authors are very clear in their discussion that they are using a different measure than the classical transitory variance, and are considering short-term earnings instability for interest in its own right.

1980s, consistent with our results, but another, smaller increase in the late 1990s and early 2000s (Shin-Solon also find an uptick in that later period). We have not explored the reasons for the difference in the latter finding or the reasons for the difference in findings with Dahl et al. or Shin-Solon. We should also note that Haider et al. (2001) found a decline in transitory variance after 1982, using an error components model.
of the transitory component.\textsuperscript{14}

Figures 11 and 12 demonstrate this principle with our data and our fitted error components model. Figure 11 shows the variance of two-year changes in our earnings measure when going from age 33 to 35 (our model requires that age be specified because the levels, though not the patterns, differ by age). As with Dynan et al. and Shin-Solon, we use two-year changes so that the later years, when the PSID conducted biannual interviewing, can be included. The figure necessarily starts only in 1972 since a two-year lag is required. As the figure shows, the actual data show no trend in the variance after the early 1980s, although the data are quite noisy, typical of differenced data (there is an actual decline from the early 1980s to the late 1980s, consistent with prior differenced data). Thus this finding coincides with past work. However, the figure also shows that our fitted error components model replicates this pattern, despite the fact that that model also contains a rising transitory variance.

Figure 12 reconciles these results by showing the actual and fitted trends in the variance of changes in earnings 10 years apart. This figure can necessarily begin only in 1980. The actual data now show an upward trend as does our fitted model its estimate of an upward trend in the transitory variance. Thus the use of long autocovariances more accurately estimates the variance of the transitory component when taking variances of differences.

Our measure and method provide insight into the reasons for the lack of trend in short-term instability. We find that that lack of trend is the result of two offsetting forces. One is the rising underlying variance of transitory shocks, which leads to increases in short term volatility.\textsuperscript{14}

\textsuperscript{14} Cameron and Tracy (1998) have a discussion of this issue and attempt to adjust for it, but they only use one-year matched CPS data and hence have no direct information on long autocovariances.
The other is the rising persistence of those autocorrelated shocks. The latter pulls down short-term instability, which partially reflects the changing variance of past shocks. Our estimates suggest that these two offsetting factors roughly cancel each other out.

VI. Summary

We have provided new estimates of the trend in the transitory variance of within-group male earnings in the U.S. using the Michigan Panel Study of Income Dynamics through 2004. Our study uses the classical definition of a transitory component that fades out over time and eventually disappears altogether, as distinct from a permanent component which never goes away. Using both a preferred error components model as well as simpler but more approximate methods, we find that the transitory variance increased substantially in the 1980s and then remained at this new higher level through 2004. We also find a strong cyclical component to the transitory variance. The trend increase in the transitory variance accounted for about half of the increase in cross-sectional inequality through the late 1980s but only for about a third through 2004 because the permanent variance has been rising markedly since the mid-1990s. We find that the transitory variance rose in the early 1980s for less-educated workers and in the later 1980s and early 1990s for more-educated workers but, by 2004, the transitory variance was higher for the latter group. We also have reconciled our results with past work suggesting no upward trend in instability since 1980 by showing that different measures of earnings instability are being used.
Figure 1: Error Components Model Estimates of Alpha

In this and subsequent figures, the four PSID non-interview years are interpolated from the two adjacent points. Trend line is fit from a fifth-order polynomial.
Figure 2: Error Components Model Estimates of Beta
Figure 3: Fitted Permanent, Transitory, and Total Variances of Male Log Annual Earnings, Age 30-39
Figure 4: Transitory Variance for Age 30-39 with Unemployment Rate

Year

Transitory

Unemployment Rate
Figure 5: Variances and Autocovariances, Age 30-39
Figure 6: Implied Transitory Variance Using 10-Lag Autocovariance, Age 30-39
Figure 7: Approximate Nonparametric Estimate of Transitory Variance

Averaged over all ages.
Figure 8: Transitory Variance Estimated by BPEA Method, 9-Year Window
Figure 9: Estimated Permanent, Transitory, and Total Variances for Those 30-39 with Education Less Than or Equal to 12
Figure 10: Permanent, Transitory, and Total Variances for those 30-39 with Education Greater than 12
Figure 11: Actual and Fitted Variance of Two-Year Change in Log Residual Male Annual Earnings from Age 33 to 35
Figure 12: Actual and Fitted Variance of Ten-Year Change in Log Residual Male Annual Earnings from Age 35 to 45
### Appendix Table A-1

#### Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Stnd Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log annual real earnings</td>
<td>10.461</td>
<td>0.746</td>
<td>-0.104</td>
<td>14.964</td>
</tr>
<tr>
<td>Log annual real earnings residual</td>
<td>0.697&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.683</td>
<td>-11.040</td>
<td>4.471</td>
</tr>
<tr>
<td>Age</td>
<td>42.8</td>
<td>8.4</td>
<td>30.0</td>
<td>59.0</td>
</tr>
<tr>
<td>Year</td>
<td>1985.7</td>
<td>9.6</td>
<td>1970.0</td>
<td>2004.0</td>
</tr>
</tbody>
</table>

#### Notes:

Means taken over all person-year observations (NT=31,054).
Means taken prior to 1-percent trimming within age-education-year cells for paired observations.

<sup>a</sup> Multiplied by 10,000; residual mean is close to zero.
Appendix Table A-2
Specimen Elements of the Covariance Matrix: Year 1974

<table>
<thead>
<tr>
<th>Year</th>
<th>Age</th>
<th>Lag Year</th>
<th>Lag Age</th>
<th>Covariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>35</td>
<td>1970</td>
<td>31</td>
<td>.0783</td>
</tr>
<tr>
<td>1974</td>
<td>35</td>
<td>1971</td>
<td>32</td>
<td>.0941</td>
</tr>
<tr>
<td>1974</td>
<td>35</td>
<td>1972</td>
<td>33</td>
<td>.1150</td>
</tr>
<tr>
<td>1974</td>
<td>35</td>
<td>1973</td>
<td>34</td>
<td>.1211</td>
</tr>
<tr>
<td>1974</td>
<td>35</td>
<td>1974</td>
<td>35</td>
<td>.1861</td>
</tr>
<tr>
<td>1974</td>
<td>45</td>
<td>1970</td>
<td>41</td>
<td>.1117</td>
</tr>
<tr>
<td>1974</td>
<td>45</td>
<td>1971</td>
<td>42</td>
<td>.1225</td>
</tr>
<tr>
<td>1974</td>
<td>45</td>
<td>1972</td>
<td>43</td>
<td>.1283</td>
</tr>
<tr>
<td>1974</td>
<td>45</td>
<td>1973</td>
<td>44</td>
<td>.1420</td>
</tr>
<tr>
<td>1974</td>
<td>45</td>
<td>1974</td>
<td>45</td>
<td>.1945</td>
</tr>
<tr>
<td>1974</td>
<td>55</td>
<td>1970</td>
<td>51</td>
<td>.0993</td>
</tr>
<tr>
<td>1974</td>
<td>55</td>
<td>1971</td>
<td>52</td>
<td>.1075</td>
</tr>
<tr>
<td>1974</td>
<td>55</td>
<td>1972</td>
<td>53</td>
<td>.1094</td>
</tr>
<tr>
<td>1974</td>
<td>55</td>
<td>1973</td>
<td>54</td>
<td>.1196</td>
</tr>
<tr>
<td>1974</td>
<td>55</td>
<td>1974</td>
<td>55</td>
<td>.1762</td>
</tr>
</tbody>
</table>

Notes:
Ages in the table denote midpoints of a ten-year group (35=30-39, 34=29-38, etc).
Covariance element values are after 1-percent trimming.
A set of covariance elements of this type exist for each year, 1970-2004, for all three age groups
in each, and for lags back to 1970 or age 25 (=20-29), whichever comes first.
## Appendix Table A-3

Estimates of the Error Components Model

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971</td>
<td>0.9320</td>
<td>0.0471</td>
</tr>
<tr>
<td>1972</td>
<td>0.9942</td>
<td>0.05158</td>
</tr>
<tr>
<td>1973</td>
<td>1.0376</td>
<td>0.0590</td>
</tr>
<tr>
<td>1974</td>
<td>1.0182</td>
<td>0.0658</td>
</tr>
<tr>
<td>1975</td>
<td>1.0565</td>
<td>0.0748</td>
</tr>
<tr>
<td>1976</td>
<td>1.2024</td>
<td>0.0896</td>
</tr>
<tr>
<td>1977</td>
<td>1.1055</td>
<td>0.0819</td>
</tr>
<tr>
<td>1978</td>
<td>1.0285</td>
<td>0.0752</td>
</tr>
<tr>
<td>1979</td>
<td>1.0900</td>
<td>0.0905</td>
</tr>
<tr>
<td>1980</td>
<td>1.1055</td>
<td>0.0892</td>
</tr>
<tr>
<td>1981</td>
<td>1.1730</td>
<td>0.0928</td>
</tr>
<tr>
<td>1982</td>
<td>1.2930</td>
<td>0.1030</td>
</tr>
<tr>
<td>1983</td>
<td>1.2764</td>
<td>0.1040</td>
</tr>
<tr>
<td>1984</td>
<td>1.2518</td>
<td>0.1006</td>
</tr>
<tr>
<td>1985</td>
<td>1.3603</td>
<td>0.1097</td>
</tr>
<tr>
<td>1986</td>
<td>1.3314</td>
<td>0.1077</td>
</tr>
<tr>
<td>1987</td>
<td>1.2546</td>
<td>0.1024</td>
</tr>
<tr>
<td>1988</td>
<td>1.3781</td>
<td>0.1141</td>
</tr>
<tr>
<td>1989</td>
<td>1.2614</td>
<td>0.0998</td>
</tr>
<tr>
<td>1990</td>
<td>1.2339</td>
<td>0.1010</td>
</tr>
</tbody>
</table>
Table A-3 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>1.1910</td>
<td>0.1023</td>
</tr>
<tr>
<td>1992</td>
<td>1.1843</td>
<td>0.1019</td>
</tr>
<tr>
<td>1993</td>
<td>1.2284</td>
<td>0.1036</td>
</tr>
<tr>
<td>1994</td>
<td>1.2594</td>
<td>0.1082</td>
</tr>
<tr>
<td>1995</td>
<td>1.3126</td>
<td>0.1127</td>
</tr>
<tr>
<td>1996</td>
<td>1.2041</td>
<td>0.1094</td>
</tr>
<tr>
<td>1998</td>
<td>1.2750</td>
<td>0.1205</td>
</tr>
<tr>
<td>2000</td>
<td>1.3377</td>
<td>0.1327</td>
</tr>
<tr>
<td>2002</td>
<td>1.3812</td>
<td>0.1465</td>
</tr>
<tr>
<td>2004</td>
<td>1.6133</td>
<td>0.1715</td>
</tr>
<tr>
<td>Beta</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>1.0461</td>
<td>0.1328</td>
</tr>
<tr>
<td>1972</td>
<td>0.7143</td>
<td>0.0880</td>
</tr>
<tr>
<td>1973</td>
<td>0.6804</td>
<td>0.0974</td>
</tr>
<tr>
<td>1974</td>
<td>0.8274</td>
<td>0.1165</td>
</tr>
<tr>
<td>1975</td>
<td>0.8659</td>
<td>0.1234</td>
</tr>
<tr>
<td>1976</td>
<td>0.9554</td>
<td>0.1424</td>
</tr>
<tr>
<td>1977</td>
<td>0.9737</td>
<td>0.1340</td>
</tr>
<tr>
<td>1978</td>
<td>1.0116</td>
<td>0.1414</td>
</tr>
<tr>
<td>1979</td>
<td>0.9817</td>
<td>0.1394</td>
</tr>
<tr>
<td>1980</td>
<td>0.7937</td>
<td>0.1131</td>
</tr>
<tr>
<td>1981</td>
<td>1.0708</td>
<td>0.1578</td>
</tr>
<tr>
<td>1982</td>
<td>1.1747</td>
<td>0.1709</td>
</tr>
<tr>
<td>Year</td>
<td>Estimate</td>
<td>Standard Error</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>----------------</td>
</tr>
<tr>
<td>1983</td>
<td>1.2086</td>
<td>0.1736</td>
</tr>
<tr>
<td>1984</td>
<td>1.1462</td>
<td>0.1594</td>
</tr>
<tr>
<td>1985</td>
<td>1.3068</td>
<td>0.1789</td>
</tr>
<tr>
<td>1986</td>
<td>1.2053</td>
<td>0.1675</td>
</tr>
<tr>
<td>1987</td>
<td>1.0044</td>
<td>0.1370</td>
</tr>
<tr>
<td>1988</td>
<td>1.0982</td>
<td>0.1560</td>
</tr>
<tr>
<td>1989</td>
<td>1.2699</td>
<td>0.1692</td>
</tr>
<tr>
<td>1990</td>
<td>1.0718</td>
<td>0.1439</td>
</tr>
<tr>
<td>1991</td>
<td>1.4059</td>
<td>0.1963</td>
</tr>
<tr>
<td>1992</td>
<td>1.3934</td>
<td>0.1860</td>
</tr>
<tr>
<td>1993</td>
<td>1.2409</td>
<td>0.1676</td>
</tr>
<tr>
<td>1994</td>
<td>1.3267</td>
<td>0.1778</td>
</tr>
<tr>
<td>1995</td>
<td>1.2486</td>
<td>0.1770</td>
</tr>
<tr>
<td>1996</td>
<td>1.1748</td>
<td>0.1592</td>
</tr>
<tr>
<td>1998</td>
<td>1.1053</td>
<td>0.1552</td>
</tr>
<tr>
<td>2000</td>
<td>1.2802</td>
<td>0.1847</td>
</tr>
<tr>
<td>2002</td>
<td>1.3354</td>
<td>0.1942</td>
</tr>
<tr>
<td>2004</td>
<td>1.1726</td>
<td>0.1807</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\sigma^2_{\mu 0} & = 0.0901 & \sigma^2_{\omega \cdot 100} & = 0.2669 \\
\sigma^2_{\delta \cdot 10000} & = 0.3830 & \sigma_{\mu \delta \cdot 1000} & = -1.9033
\end{align*}
\]
Table A-3 (continued)

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.8468</td>
<td>0.0510</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.5740</td>
<td>0.0495</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>0.0625</td>
<td>0.0216</td>
</tr>
<tr>
<td>$\pi_1 \times 100$</td>
<td>0.1985</td>
<td>0.0859</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-0.0330</td>
<td>0.0348</td>
</tr>
</tbody>
</table>

Notes:

R-squared = .105  
Chi-squared = 5.27
Appendix B

Model and Estimating Procedure

The model, restated, is

\[ y_{iat} = \alpha_{t} \mu_{ia} + v_{iat} \]  \hspace{1cm} (B1)

\[ \mu_{ia} = \mu_{i,a-1} + \delta_{i} + \omega_{ia} \]  \hspace{1cm} (B2)

\[ v_{iat} = \rho v_{i,a-1,t-1} + \beta_{t} \xi_{ia} + \theta (\beta_{t-1} \xi_{i,a-1}) \]  \hspace{1cm} (B3)

with the following normalizations, variance assumptions, and initial conditions:

\[ \alpha_{70} = 1, \quad \beta_{70} = 1 \]  \hspace{1cm} (B4)

\[ \text{Var}(\mu_{i0}) = \sigma_{\mu 0}^{2} \]  \hspace{1cm} (B5)

\[ \text{Var}(\delta_{i}) = \sigma_{\delta}^{2} \]  \hspace{1cm} (B6)

\[ \text{Var}(\omega_{ia}) = \sigma_{\omega}^{2} \]  \hspace{1cm} (B7)

\[ \text{Cov}(\mu_{i0}, \delta_{i}) = \sigma_{\mu \delta} \]  \hspace{1cm} (B8)

\[ \text{Var}(\xi_{ia}) = \sigma_{\xi}^{2} = \pi_{0} + \pi_{1} a \]  \hspace{1cm} (B9)

and with \( a = 1 \) defined as age 20. For the left-censored (1970) observations, define \( a_{70} \) as the individual’s age in 1970. Those with \( a_{70} > 1 \) are left-censored. We define the variance of the transitory component in 1970 for these left-censored observations in the following way (the variance of the permanent component does not require knowledge of \( \alpha_{t} \) prior to 1970):
\[ v_i,_{a,70}^{70} = \rho v_i,_{a,69,69}^{69} + \beta_{70}^{70} \xi_i,_{a,70}^{70} + \theta \beta_{69}^{69} \xi_i,_{a,69}^{69} \]  \hspace{1cm} (B10)  

\[ \text{Var}(\rho v_i,_{a,69,69}^{69} + \theta \beta_{69}^{69} \xi_i,_{a,69}^{69}) = [1 + \gamma(a_{70}^{-1})]\text{Var}(v_i,_{a,70}^{70}) = g(a_{70}) \]  \hspace{1cm} (B11)  

\[ \text{Var}(v_i,_{a,70}^{70}) = \sum_{s=0}^{a_{70}^{-2}} \rho^2(\rho+\theta)^2\text{Var}(\xi_i,_{a,70-s-1}^{70-s-1}) \]  \hspace{1cm} (B12)  

Eqn(B10) is the ARMA(1,1) expression for the 1970 transitory component. Only the first and third terms are prior to 1970 and hence only they must be approximated. Eqn(B12) gives the formula for the variance of the transitory component for someone who is age a_{70} in 1970 and whose transitory component has followed its age evolution from age a=1 to that age with \beta_{70}^{70} = 1 in all those years. Eqn(B11) allows that age profile of transitory variances to be modified by the parameter \gamma, and it is assume that the deviation is a function of age—the lower the age, the fewer years prior to 1970 have occurred, and hence the smaller the expected deviation. We denote the expression in (B11) as g(a_{70}) for use in the formulas below.

An alternative treatment of the left-censored observations would simply allow the 1970 age-profile of transitory variances to be some unknown function of g(a_{70}) whose parameters would be estimated. However, this approach would result in a misspecification in the present case because it would make all succeeding transitory variances a function of calendar time (we do not demonstrate this for brevity). As a result, even in a model with \alpha_t = \beta_t = 1, the model would predict calendar time evolution of the variances and covariances. Thus true calendar time shifts after 1970 would be confounded with distance from the left-censoring point, generating incorrect estimates of \alpha_t and \beta_t (see MaCurdy, 2007, pp.4094-4098 for a related discussion).

The unknown parameters in the model are \alpha_t, \beta_t, \rho, \theta, \pi_0, \pi_1, \gamma, \sigma^2_{\mu_0}, \sigma^2_\delta, \sigma^2_\omega, and
They generate variances and covariances for all years, ages, and lag lengths.

\[ \text{Var}(y_{iat}) = \alpha_t^2 \text{Var}(\mu_{ia}) + \text{Var}(u_{iat}) \]  

(B13)

\[ \text{Cov}(y_{iat}, y_{i,a-\tau,t-\tau}) = \alpha_t \alpha_{t-\tau} \text{Cov}(\mu_{ia}, \mu_{i,a-\tau}) + \text{Cov}(u_{iat}, u_{i,a-\tau,t-\tau}) \]  

(B14)

**Permanent Variances and Covariances**

\[ \text{Var}(\mu_{ia}) = \sigma_{\mu_0}^2 + a \sigma_\delta^2 + a \sigma_\omega^2 + 2a \sigma_{\mu\delta} \]  

(B15)

\[ \text{Cov}(\mu_{ia}, \mu_{i,a-\tau}) = \sigma_{\mu_0}^2 + a(a-\tau) \sigma_\delta^2 + (a-\tau) \sigma_\omega^2 + [a+(a-\tau)] \sigma_{\mu\delta} \]  

(B16)

**Transitory Variances and Covariances**

If \( a_{\tau0} \leq 1 \) (non-left-censored):

\[ \text{Var}(u_{iat}) = \beta_t^2 \sigma_{\xi 1}^2 \]  

(B17)

\[ \text{Var}(u_{iat}) = \sum_{s=0}^{a-2} \rho^{2s} (\rho+\theta)^2 \beta_{t-s-1}^2 \sigma_{\xi,a-s-1}^2 + \beta_t^2 \sigma_{\xi a}^2 \]  

(B18)

\[ \text{Cov}(u_{iat}, u_{i,a-\tau,t-\tau}) = \rho^\tau \text{Var}(u_{i,a-\tau,t-\tau}) + \rho^{\tau-1} \theta \beta_{t-\tau}^2 \sigma_{\xi,a-\tau}^2 \]  

(B19)

If \( a_{\tau0} > 1 \) (left-censored):

\[ \text{Var}(u_{i70,70}) = g(a_{70}) + \beta_{70}^2 \sigma_{\xi 70}^2 \]  

(B20)

\[ \text{Var}(u_{iat}) = \rho^{2(t-70)} g(a_{70}) + \sum_{s=0}^{t-71} \rho^{2s} (\rho+\theta)^2 \beta_{t-s-1}^2 \sigma_{\xi,a-s-1}^2 + \beta_t^2 \sigma_{\xi a}^2 \]  

(B21)

\[ \text{Cov}(u_{iat}, u_{i,a-\tau,t-\tau}) = \rho^\tau \text{Var}(u_{i,a-\tau,t-\tau}) + \rho^{\tau-1} \theta \beta_{t-\tau}^2 \sigma_{\xi,a-\tau}^2 \]  

(B22)
We estimate the model with minimum distance. Let \( s_{im} = y_{ij} y_{ik} \), where \( y_{ij} \) and \( y_{ik} \) are the log earnings residuals for individual \( i \) for age-year "locations" \( j \) and \( k \), and where \( m=1,\ldots,M \) indexes the moments generated by the product of residuals at all locations \( j \) and \( k \). In our case, \( M=1,197 \). Write the model in generalized form as

\[
s_{im} = f(\theta_{j,k}) + \epsilon_{im} \quad i=1,\ldots,N, \ m=1,\ldots,M
\]

(B23)

where \( \theta \) is a \( L \times 1 \) vector of parameters. Then the set of \( M \) equations in (B23) constitutes an SUR system whose efficient estimation requires an initial consistent estimate of the covariance matrix of the \( \epsilon_{im} \). However, following the findings and recommendations of Altonji and Segal (1996) on bias in estimating covariance structures of this type, we employ the identity matrix for the estimation. Hence we choose \( \theta \) to minimize the sum of squared residuals:

\[
\min_{\theta} \sum_{i=1}^{N} \sum_{m=1}^{M} [s_{im} - f(\theta_{j,k})]^2
\]

(B24)

or, equivalently, since \( f \) is not a function of \( i \),

\[
\min_{\theta} \sum_{m=1}^{M} [s_{im} - f(\theta_{j,k})]^2
\]

(B25)

where \( \bar{s}_m \) is the mean (over \( i \)) of \( s_{im} \) (i.e., a covariance).

To obtain standard errors, we apply the extension of Eicker-White methods in the manner suggested by Chamberlain (1980), using the residuals from (B24), each of which we denote \( e_{im} \).
Let \( \Omega \) be the MxM covariance matrix of the \( e_{im} \), each element of which is estimated by: \(^{15}\)

\[
\hat{\sigma}_{mm'} = \frac{1}{N} \sum_{i=1}^{N} e_{im} e_{im}'
\]

(B26)

Define \( \Delta \) as the NMxNM covariance matrix of individual residuals which is a block diagonal matrix with the matrix \( \Omega \) on the diagonals. Then

\[
\hat{\text{Cov}}(\hat{\theta}) = (G'G)^{-1}G'\Delta G(G'G)^{-1}
\]

(B27)

where \( G \) is the NMxL matrix of gradients \( \partial f(\theta,j,k)/\partial \theta \).

---

\(^{15}\) Each individual in our data set contributes to only a subset of the moments in \( \Omega \); we do not adjust the notation in (B26) for this.
References


