Choice of Product under Government Regulation:
The Case of Chile’s Privatized Pension System

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Abstract

Many countries are considering adopting individual retirement investment accounts to replace traditional pay-as-you-go social security systems that are projected to become insolvent. Chile was one of the first countries to transit to a private accounts system 28 years ago. Since then it has served as a role model for pension reforms in several other Latin American countries. Under the Chilean pension system, workers are mandated to contribute 10% of earnings to a retirement investment account at a pension fund firm of their choice. Workers who contribute at least 20 years are guaranteed a minimum pension benefit, so government obligations depend on fund performance. The pension fund industry is privatized but subject to regulations aimed at reducing the riskiness of industry products. This paper uses data from Chile to study the operation of the privatized pension fund industry under alternative regulatory schemes related to pension fund returns and fees. It estimates an equilibrium demand and supply model using individual-level data on fund choices and firm-level data on costs, pricing, and market performance. The estimated model is used to investigate how alternative regulatory schemes affect firm pricing behavior, consumer choices, consumer balances and government obligations. We find that in the absence of any pension fund return regulation, the industry offers very risky products, which induces highly volatile consumer balances and government obligations. Another finding is that the government’s regulation that requires firms to deliver returns close to the industry average has the intended consequence of limiting product diversity but it is not very effective in reducing the riskiness of the industry’s products. This discourages individuals to participate in the pension system. An alternative form of regulation that is found to be more effective places explicit restrictions on pension fund firms’ portfolio riskiness.
1 Introduction

The United States and many European countries are currently considering how best to reform their pay-as-you-go social security systems. Demographic trends indicate rising numbers of pensioners per worker and pending insolvency of many social security systems. The kinds of reforms being considered include increasing the required social security contribution per worker, raising the standard retirement age, or overhauling the system by transiting to a private retirement accounts system. Chile has been at the forefront of pension reforms, having switched to a private retirement accounts system twenty eight years ago.¹ Numerous other Latin American countries followed suit, building on the Chilean model. These include (with years of adoption in parentheses): Peru (1993), Colombia (1994), Argentina (1994), Uruguay (1996), Bolivia (1997), Mexico (1997), El Salvador (1998), Costa Rica (2001), the Dominican Republic (2003), Nicaragua (2004), and Ecuador (2004).²

Previous research on Chile mainly examined the impact of pension reforms on the macro-economy, capital markets and aggregate savings.³ It found substantial benefits of moving to a private retirement accounts system in promoting the development of well-functioning capital markets and stimulating economic growth. However, critics of Chile’s pension system point to low coverage rates and commissions and fees that are thought to be excessive.⁴ Low coverage rates are mainly due to the presence of a significant informal sector of the economy, where workers do not contribute to the pension system, and to low labor force participation among some groups in the population, such as women.⁵

¹University of Chicago economists played a role in the early adoption of the privatized account system.
²Cogan and Mitchell (2003) discuss prospects for funded individual defined contributions account pensions in the United States.
⁴A recent critique citing the problem of low coverage rates is Holzmann, Hinz et. al. (2005).
⁵See Arenas de Mesa, Bravo, Behrman, Mitchell, and Todd (2006).
The proposed plans for pension reform in the US and in Europe have many features in common with Chile’s current pension system. They outline a system under which all workers are mandated to contribute a pre-specified part of their income to their pension account, which is managed by money manager(s) (either a government owned company or a competitive industry of money managers). The government serves as a last resort guarantor, supplementing pension income if pension accumulations are insufficient upon retirement (below pre-specified minimal level) either because of low income or unfavorable returns on investment. All these features are present in the Chilean pension fund system, called the Administradoras de Fondos de Pensiones (AFPs). Workers are mandated to contribute 10% of their earnings to a retirement account. Workers who contribute to the pension account for at least 20 years receive a minimum pension benefit guarantee from the government.

Two important concerns have been raised about an individual retirement accounts pension system. The first is that government obligations can be large, particularly in years with unfavorable market returns on investments. Second, the government guarantee of minimal support may induce moral hazard problems by providing incentives for consumers with low income to choose risky investment options. If the system is run by a competitive industry, then money managers may offer products to meet this riskier demand, which, in turn, can raise government obligations. This is an undesirable feature of a competitive pension fund industry, although competition can also provide incentives for cost efficiency, quality improvement, and efficient pricing.

Chile has a competitive pension fund industry overseeing pension investment that is subject to government regulations designed to limit fees, to facilitate switching among funds by promoting transparency of fees and pension fund returns and to limit the riskiness of the investment products offered. A particularly important regulation is a return requirement under which money managers are responsible with their capital for delivering a rate of return which must not fall below the industry average return
by two percentage points.\textsuperscript{6} This regulation essentially shifts some of the risk of investment from consumers to the pension fund firm. Another important regulation restricts firms to charge fees only on new contributions and not on existing balances.

This paper investigates how the government regulation of the pension fund industry affects its operation. To this end, we estimate an equilibrium demand and supply model of the pension investment market and use the model to study the effects of actual regulatory rules and of hypothetical alternative regulatory rules. The question of whether and to what extent governmental regulations imposed on a privatized account system can protect investors from risk and high fees without too greatly compromising investment returns is pertinent not only for Chile but also for any other country considering a move to a privatized account system.

Our empirical analysis combines data from multiple sources: longitudinal household survey data gathered in 2002 and 2004, administrative data on contributions and fund choices from 1981-2004 that were obtained from the pension fund regulatory agency, market data on the performance of the various funds, and a data series on the fees charged by funds as well as accounting cost data. The household survey data come from the 2002 Historia Laboral y Seguridad Social (HLLS) survey and the 2004 Enquesta Proteccion Sociale (EPS) follow up survey. The data contain demographic and labor market information on 17,246 individuals of age 15 or older, including information on household demographics, work history, pension plan participation, and savings, as well as more limited information on health, assets, disability status and utilization of medical services.

We develop a demand model of consumer choices among AFP funds that as-

\textsuperscript{6}There is also a regulation that requires AFPs that earn returns in excess of two percentage points above the market average to keep the excess in a reserve fund to be used in the event of reaching the lower return boundary. In practice, the upper limit was only reached once by two firms (Fomenta and Valora). In that case, the excess return was paid out to the investors when the firms merged. In this paper, we ignore the upper bound on AFP returns, which essentially assumes that consumers get any excess returns. The upper bound was eliminated in 2008.
sumes that the consumer chooses an AFP to manage his/her pension savings at the beginning of each period (annually). Under the Chilean pension system, consumers are required to invest all their pension accumulations with one money manager; however, they can freely move their savings from one money manager to another. The funds are not allowed to charge a fee to set up an account or to withdraw money. Our model assumes that the consumer’s choice of investment fund at a given point in time depends on the product’s characteristics (mean return and risk) and on the fees charged by that fund, and that consumers make an annual choice about where to invest their funds. Chilean pension funds charge fixed and variable fees that depend on contribution levels. Hence, a fund with a high fixed fee but a low variable fee might be the one that comes at the lowest cost for a consumer with a high contribution level, whereas a fund with zero fixed fee but a high variable fee might be the one that comes at the lowest cost for someone with a low contribution level. In addition to making a choice about where to invest pension accumulations, consumers also decide each period whether to contribute to their pension plan by working in the formal sector where pension contributions are mandated. Thus, each period each consumer chooses jointly where to invest his/her pension accumulations and whether to contribute or not.

The demand side model is specified as a random coefficients random utility model, which leads to a multinomial discrete choice framework. Our model allows for both observable and unobservable sources of heterogeneity across consumers as well as unobservable attributes of pension fund firms that may, in addition to fund performance, affect the consumers’ perceptions of fund quality. Repeated pension fund choices over time determine the consumers’ balance accumulation. Aggregation over consumers generates the market demand for an AFP’s product.

The supply side of the market is modeled as an oligopolistic environment in which AFPs sequentially choose product location (the mean return and risk of their
fund) and choose their two-part (fixed and variable) fee structure, taking into account the distribution of consumers’ preferences and consumer types. At the time of our data collection, each AFP firm offered a distinct investment product. After 2002, however, there was a regulatory reform that introduced regulations requiring firms to offer 5 different investment products that vary in risk characteristics. This paper focuses on the regulatory environment that was in place prior to 2002, because allowing each firm to offer multiple product-types and for consumers to make portfolio choices within firms substantially increases both the modeling and computational complexity.

We view our study of the effects of regulation in the simpler pre-2002 environment of each firm offering one product as a first step towards a more general understanding of how regulations of a privatized pension industry affect its operation. Our study also provides insight as to why reform was needed to increase the diversity of products offered. As previously noted, another regulation on firms that was instituted in 1988 and remains in place today is that firms are allowed to charge fees only on new contributions and not on balances. Our analysis of the pension fund industry shows why fees charged to consumers are lower with this type of regulatory restriction.

There are a few features of the pension fund industry that make it inherently different from a standard product market. One is that once an individual accumulates some pension funds, he/she is essentially forced to consume one of the industry’s products by choosing an investment fund. On the firm side, firms service consumers who in some periods are not contributing and therefore not paying fees. Another nonstandard feature of the industry relative to a standard product market is that the relative costs of different products is consumer-specific.

Our demand model is estimated using the simulated method of moments technique of McFadden(1990). The estimates are based on micro-moments that capture

\footnote{Consumers cannot easily opt out of participating, because there are stringent rules in place in Chile that prohibit early withdrawal of pension funds, except in cases where individuals can demonstrate very high accumulations to assure that they are not at any risk of relying on minimum pension or welfare pension support.}
the contribution of different consumer characteristics to the consumer’s propensity to make a specific choices. We examine the goodness of fit of the model both to the moments used in estimation and with respect to aggregate statistics on market balance shares that were not used in the estimation and find that the model has a reasonably good fit. The demand side estimation recovers estimated coefficients of the risk aversion, which are in line with estimates reported in other studies. We find that the risk aversion is inverse U-shaped with the youngest and oldest being more willing to take on risk.

The supply side of the model is estimated using standard panel data techniques. The pension fund cost function is specified as a flexible translog function, which allows for scale economics with respect to the number of consumers and total balance managed by the fund. The estimates point to the existence of scale economics and also to an optimal scale both with respect to the number of consumers and total balance managed. Notably, we do not find evidence of increasing returns to scale throughout, supporting the role for more than one firm in supplying the pension fund market.

After estimating the parameters of the demand and supply model, i.e. the distribution of consumers’ tastes and companies’ cost functions, we use the model to conduct counterfactual experiments that investigate firm and consumer behavior under four different regulatory environments. As a benchmark, we first study behavior in the absence of any regulation on firms’ pension fund returns. Second, we consider the set of products that would be offered by a social planner, under the constraint that the level of government obligations is fixed at the level implied by the data. We consider this to be a constrained optimal set of products. Third, we analyze the existing regulation that requires AFP firms to deliver returns that are close to the industry average. Fourth, we evaluate the effect of an alternative regulation that explicitly regulates the choice of investment instruments, by placing an upper limit
on the riskiness of the firm’s portfolio (the CAPM beta). Our analysis yields the following key findings:

(i) In the absence of regulation on pension fund returns, the industry offers a very risky set of products which, in turn, implies a large variance in consumer balances and in the level of government obligations.

(ii) The actual governmental regulation does not achieve its goal of reducing the riskiness of industry products. Rather, it results in relatively risky products, low product diversity and low pension plan participation rates.

(iii) A simple regulation in the form of an explicit restriction on the portfolio riskiness is more effective than the actual regulation in that it more closely resembles the constrained optimum.

The paper is organized as follows. Section two provides some background information on the Chilean private accounts system and reviews related literature. Section three describes the consumer’s choice problem and outlines the oligopolistic model of the firms’ price and location decisions. Section four describes the estimation strategy. Section five presents the empirical results and section six concludes.

2 Industry Description and Related Literature

2.1 Industry Description

As previously described, investors in the Chilean pension system are permitted to hold their money in only one AFP at a time. The rules governing switching between money managers changed several times over the years, but beginning in 1984 investors could switch funds without incurring any monetary costs. Pension funds charge fees for their services. Initially, the fee was a three part non-linear tariff consisting of a fixed fee, a variable fee proportional to the participant’s contribution, and a fee proportional to the participant’s balance. Some companies also charged fees for withdrawal of funds,
but in 1984 the government passed a regulation to disallow fees on the balance or on withdrawal. Currently, most AFPs charge a two-part tariff consisting of a fixed fee and a fee that is proportional to the participant’s contribution.

From the inception of the private accounts system, the government exerted control over the investment choices. Initially, pension investments could only be held in government bonds, but over time the options expanded to include riskier assets, namely stocks and a higher degree of foreign investments. As an additional measure to reduce the riskiness of the system, the government required that AFPs deliver a real return within a 2 percentage point band around the industry average, making the AFP firms responsible for covering low realizations of returns with their own capital. During the period after 1987, a number of AFPs had financial difficulties because of these restrictions and had to exit the market.⁸

Up until 2000, each AFP firm essentially offered a single investment product. Starting in 2000, however, they were allowed to offer four instruments which differ according to the riskiness of the investment.⁹ In the analysis here, we use data from the time period prior to offering multiple investment instruments to simplify the modeling of the firm’s choice of their product characteristics.

### 2.2 Related literature

There exists a substantial literature based on US data that studies to what extent performance of mutual fund managers, stock analysts etc. can be predicted from publicly available data on their characteristics and past performance. However, we are aware of only one study by Hortacsu and Syverson (2003) that focused on consumer choice among money managing companies. It explores consumer choices of S&P

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³⁸In each case, the exit was organized as a merger with one of the existing AFPs. The clients of an exiting AFP were transferred to its merging partner, though they could easily switch funds afterwards.

³⁹Each of these instruments has a targeted age group. An investor’s contributions are allocated by default into an age-appropriate fund unless he/she chooses otherwise.
500 index funds, which exhibit return homogeneity and sizable dispersion in fund fees. They find that consumer choices are largely driven by search costs, i.e. the cost of acquiring information about a fund which would be indicative of the fund’s future performance. The authors conclude that this property combined with consumer heterogeneity in search costs and large proportion of consumers with high costs leads to a large dispersion in funds fees.

The literature on consumer choice of services/products in the presence of switching costs emphasizes entry barriers that arise as a result of switching costs, low incentives to invest in quality and adverse selection which arises if the switching cost is private information of the consumer. Some of the empirical and theoretical papers in this area include Beggs and Klemperer (1992), Calem, Gordy and Mester (2005), Gravelle and Masiero (2000), Kiser (2002), Klemperer (1987), Knittel (1997), Rhoades (2000), Stango (2002).

3 Model

In this section we lay out how we incorporate the peculiarities of the Chilean private account system into a formal model of the consumers’ AFP choice and the AFPs’ pricing and portfolio choice decisions. We set out with the formulation of the basic features of the environment. Then we formalize the demand side and subsequently we develop the supply side model of the AFPs pricing and location decisions.

3.1 Framework

There are $J$ AFPs, $J \in \mathbb{N}$. A consumer considers fund $j$ as being characterized in period $t$ by its return, $\bar{R}_{jt}$, its fees, and other non-portfolio characteristics, e.g., features related to the convenience of obtaining service from this fund. Each AFP $j$ offers a single product denoted by $\phi_j$. All products lie in a $p + np$-dimensional space
of portfolio and non-portfolio characteristics, \(\Phi^{p+np} \subset \mathbb{R}^{p+np}\), where \(p\) denotes the “portfolio dimension” and \(np\) denotes the “non-portfolio dimension”. We concentrate on the AFPs’ portfolio choice and subsume the non-portfolio characteristics of AFP \(j\)’s product by fund fixed effects \(\xi_{jt}\).

Each AFP \(j\) constructs a portfolio out of the assets in the investment opportunity set. The random (gross) return of AFP \(j\)’s portfolio is denoted by \(\tilde{R}_{jt}\). We assume that \(\tilde{R}_{jt}\) is distributed according to a continuous probability density function denoted by \(g(\cdot)\) for all \(j \in \{1, ..., J\}\). Moreover, \(g(\cdot) > 0\) on its domain and the associated c.d.f is denoted by \(G(\cdot)\). We assume that the distribution of \(\tilde{R}_{jt}\) is identical for each AFP \(j\) up to the moments of the return distribution. Put differently, the moments of the return distribution constitute the portfolio characteristics of AFP \(j\)’s product.

Consumers regard \(\tilde{R}_{jt}\) as a random variable. We assume that consumers know the portfolio compositions of all AFPs and, therefore, the joint and marginal distributions of the AFPs returns. AFPs are subject to a minimum return (or profitability) requirement for the pension fund under their management. The return requirement is set in relation to the average performance of all pension funds over any twelve-month period. More specifically, if the real investment return is lower than the average by 2 percentage points, the AFP is required to make up the difference from its investment reserves.

Consumers incorporate this minimum return requirement into their decision process. This implies that a consumer anticipates that if she deposits her retirement savings at AFP \(j\), then her actual return given the minimum return requirement in period \(t\) is

\[
\tilde{R}_{jt}^r = \max\left\{\tilde{R}_{jt}, \tilde{R}_{jt}^\prime\right\},
\]

with \(\tilde{R}_t := \frac{\sum_{n=1}^J \tilde{R}_{nt}}{J} - 0.02\).

Whereas AFPs can decide freely about the fee’s amount, the types of fees
that they are allowed to charge are regulated. Authorized fees include a fixed fee per contribution and a pro rata fee on wages on which contributions are based. Thus, the fees associated with participating at AFP $j$ differ by consumers. More specifically, we denote the fee that AFP $j$ charges a customer $i$ in period $t$ by

$$
\bar{p}_{jt} + \hat{p}_{jt}y_{it},
$$

where $\bar{p}_{jt}$ denotes the fixed fee, $\hat{p}_{jt}$ denotes AFP $j$’s pro-rata fee, and $y_{it}$ denotes the current contribution of customer $i$. Another peculiarity of the Chilean system is that the maximum fee that a fund may charge a consumer is according to the amount equal to the mandatory contribution. Thus, the fee that an AFP may charge customer $i$ is given by

$$
p_{ijt} = \min\{y_{it}, \bar{p}_{jt} + \hat{p}_{jt}y_{it}\}.
$$

Thus, both the mandatory contribution and the fee are deducted from an individual’s current income.

### 3.2 Demand side

If an individual already participates in the pension program, then each period he/she chooses to which AFP he/she should allocate his/her current pension balance and whether he/she should contribute to his/her account. If an individual never participated in the pension program, which implies that he/she has never worked in the formal sector, then he/she decides whether to remain in the informal sector or to work in the formal sector and which pension fund to choose. We assume that decisions are made on an annual basis.

In the following we formalize the multinomial choice problem that an individual faces each period.

**Choice Set** An individual is, in a given period, either not affiliated, $\omega = 0$, or affiliated to the system. If an individual is affiliated he/she has to choose an AFP
\( j \in \{1, \ldots, J\} \) that manages his/her retirement savings. In addition, he/she has to
decide whether to contribute actively to her account \((j, 1)\) or to skip contemporaneous
contribution \((j, 0)\). Thus, each individual’s choice set comprises \(\{0, (j, 1), (j, 0)\}\), for
all \(j \in \{1, \ldots, J\}\).

More concisely, the set of a customer’s possible choices in period \(t\) is denoted
by \(\Omega\), \(\Omega := \{0\} \cup \mathcal{D}\), where

\[
\mathcal{D} := \bigcup_{j=1}^{J} \{(j, 1), (j, 0)\}.
\]

We denote a general element of the set \(\Omega\) by \(\omega\) and a general element of the set \(\mathcal{D}\)
by \(d\). In the remainder, we refer to the choice \((j, 1)\) as “active affiliation” to AFP \(j\),
to the choice \((j, 0)\) as “ passive affiliation” to AFP \(j\), and to the choice \(0\) as “non-
participation”.

Now, we characterize the contemporaneous utility that an individual \(i\) derives
from each possible choice in period \(t\).

**Consumer (Expected) Utility**  We assume that an affiliated individual \(i\) at time
\(t\) receives utility from two different components: contemporaneous consumption and
expected retirement wealth. Our modeling choice is to assign each component a
different utility function: The natural logarithm for contemporaneous consumption
and the quadratic utility function for expected retirement consumption. In addition,
we augment the formulation by the parameter \(\tau_{it}\) which measures how an individual
trades-off contemporaneous consumption and future expected retirement consump-
tion at a given point in time.

The reason for this formulation is the following. Ultimately, we are interested
in the AFPs’ product choice. In discrete choice theory it is common to exogenously
impose that an individual’s utility function is composed of a part that measures
his/her utility from consuming a numeraire commodity and a transport cost function
that measures an individual’s disutility from not consuming his/her most preferred product. In this respect our formulation is more general. Instead of directly assuming a transport cost function we employ an utility function that is commonly used in portfolio choice theory in order to endogenously derive an individual’s disutility from not choosing his/her most preferred portfolio.

(j,1): Active Affiliation

The utility individual $i$ derives from the choice $(j, 1)$ at time $t$ is given by

$$U_{i(j,1)t} = \tau_{it} \ln(Y_{i1t} - y_{i1t} - p_{ijt}) + w_{i(j,1)t}\tilde{R}_{jt} - \gamma_{it}w_{i(j,1)t}^2 \left(\tilde{R}_{jt}\right)^2 + \xi_{jt} + \epsilon_{i(j,1)t},$$

(3)

where $Y_{i1t}$ denotes the labor income individual $i$ earns in period $t$ in the formal sector and $y_{i1t} = 0.1Y_{i1t}$ is the mandatory contribution. The beginning-of-period retirement wealth in period $t$ is denoted by $w_{i(j,1)t} = b_{it} + y_{i1t}$, where $b_{it}$ represents the retirement balance of individual $i$, i.e. the sum of past mandatory contributions including interest. The variable $\gamma_{it} > 0$ denotes a parameter that affects an individual $i$’s coefficient of risk aversion and $\tau_{it} > 0$ denotes a parameter that affects individual $i$’s elasticity of substitution between current and retirement consumption. In the remainder we will refer to $\tau$ as the elasticity of substitution and to $\gamma$ as the risk aversion parameter in order to ease exposition. The unobservable component of consumer $i$’s preferences is captured by $\epsilon_{i(j,1)t}$ and $\xi_{jt}$ captures the unobserved product-specific fixed effect.\footnote{Note that $\epsilon_{i\omega t}$ is observable to AFPs and that $\xi_{jt}$ is observable to consumers. However, both are unobservable to the modeler.}

(j,0): Passive Affiliation

Each affiliated individual has the option to skip contemporaneous contribution, e.g., in the eventuality of becoming unemployed. We assume that if an individual ceases to contribute actively to an AFP, he receives his type’s annual income in the informal sector. In addition, the Chilean regulation stipulates that AFPs are not permitted to levy commissions on inactive accounts. Thus, the utility a customer $i$ derives from
choosing \((j, 0)\) is

\[ U_{i(j, 0)t} = \tau_{it} \ln(Y_{i0t}) + w_{i(j, 0)t} \tilde{R}_{jt}^r - \gamma_{it} w_{i(j, 0)t}^2 \left( \tilde{R}_{jt}^r \right)^2 + \xi_{jt} + \epsilon_{i(j, 0)t}, \quad (4) \]

where \(Y_{i0t}\) denotes the annual income consumer \(i\) earns in the informal sector and \(w_{i(j, 0)t} = b_{it}\). Since the individual does not contribute in the current period its beginning-of-period retirement wealth in period \(t\) corresponds to the sum of past mandatory contributions including interest, i.e., its retirement balance.

Finally, we state an unaffiliated individual's utility at time \(t\).

**0: Non-participation**

If an individual has never been affiliated to the system before \(t\) and does not choose to become affiliated in period \(t\) either, her utility is given by

\[ U_{i0t} = \tau_{it} \ln(Y_{i0t}) + \epsilon_{i0t}. \]

Thus, corresponding to the set of possible choices, the expected utility of consumer \(i\) at time \(t\) is: \(^{11}\)

\[
\begin{align*}
E[U_{i(j, 1)}] &= \tau_1 \ln(Y_{i1} - y_{i1} - p_{ij}) + \alpha_{1,i(j,1)} E[\tilde{R}_j^r] + \alpha_{2,i(j,1)} E[(\tilde{R}_j^r)^2] + \xi_j + \epsilon_{i(j,1)} \quad (5) \\
E[U_{i(j, 0)}] &= \tau_1 \ln(Y_{i0}) + \alpha_{1,i(j,0)} E[\tilde{R}_j] + \alpha_{2,i(j,0)} E[(\tilde{R}_j)^2] + \xi_j + \epsilon_{i(j,0)} \quad (6) \\
E[U_{i0}] &= \tau_1 \ln(Y_{i0}) + \epsilon_{i0} \quad (7)
\end{align*}
\]

where

\[
\begin{align*}
\alpha_{1, id} &= w_{id}, \\
\alpha_{2, id} &= -\gamma_i w_{id}^2, \\
\alpha_{3, i\omega} &= \epsilon_{i\omega}, \\
\alpha_{1, i0} &= \alpha_{2, i0} = 0.
\end{align*}
\]

\(^{11}\)In the remainder of this subsection we suppress the time index \(z\) for the ease of exposition.
From introspection of (5) to (7) it is evident that expected consumer utility is concave in the numeraire commodity “contemporaneous consumption” and linear in the preference parameters $\alpha_{l,i\omega}$, $l \in \{1, 2, 3\}$.

Note that the identity of an AFP does not affect an individual’s beginning-of-period retirement wealth $w_i$, i.e. $w_{ij} = w_i \forall j$. What matters is whether an individual chooses to be actively or passively affiliated to a fund. We state the identity of the AFP to which an individual is affiliated in the retirement wealth in addition to the individual’s contribution decision in order to unambiguously identify an individual’s choice.

From the previous analysis it follows that a consumer $i$ in period $t$ is characterized by the $(9 + (J + 1))$-tuple $(\alpha, \tau, Y_0, Y_1) \in \mathbb{B}$ where $\mathbb{B}$ is a convex subset of $\mathbb{R}^{9+(J+1)}$.

We complete the description of the demand side by an assumption concerning the joint density of consumer preference parameters that will turn out to be crucial for the equilibrium existence in the pricing stage.

**Assumption A1** The logarithm of the joint density of consumers’ utility parameters, elasticity of substitution and income, $\ln[f(\alpha, \tau, Y_0, Y_1)]$, is a concave function in $(\alpha, \tau, Y_0, Y_1)$ over its support, $\mathbb{B}$, which is a convex subset of $\mathbb{R}^{9+(J+1)}$.

### 3.3 Supply Side

The demand model described above assumes that consumers have quadratic utility with respect to expected retirement wealth. In this context, the mutual fund separation theorem implies that money managers – the AFPs – can satisfy consumers by choosing a convex combination of the risk-free asset and the market portfolio. Therefore, the moments of AFP $j$’s return distribution are uniquely determined by $x_{jt}$, the share of assets that AFP $j$ invests in the market portfolio in period $t$. We
refer to this share as AFP \( j \)'s product choice or “location”.

In order to proceed to the profit function of AFP \( j \) we have to characterize the demand for its product. It follows from the discussion of the demand side that consumers can, in principle, choose between two modes of investing their retirement wealth at AFP \( j \): active or passive affiliation. In the remainder we will refer to individual (aggregate) demand for active affiliation as individual (aggregate) active demand and to individual (aggregate) demand for passive affiliation as individual (aggregate) passive demand. In addition, we refer to the sum of individual (aggregate) passive and active demand as total individual (aggregate) demand.

We begin the characterization of an AFP’s demand components by establishing that aggregate active, aggregate passive, and therewith total aggregate demand are continuous in the AFPs’ prices.

**Proposition 1** Under Assumption A1 and preferences given by (5) - (7), aggregate active, aggregate passive, and total aggregate demand are continuous in the price vector \((p_j)_{j=1}^J\) whenever the \( J \) products are distinct.

**Proof** See Appendix.

We continue by establishing that individual (active) demand features the reservation price property. There are two reasons for proceeding in this way: First, this property implies that aggregate (active) demand for a given firm’s product is non-increasing in the respective firm’s price vector. Second, the reservation price functions allow in conjunction with the continuity of demand a concise representation of the demands for an AFP’s product.

**Reservation price property of individual demand** In the course of the derivation we focus on a given type \( i \) in a given period \( t \). Thus, we suppress the subscripts \( i \) and \( t \) in this subsection for the ease of exposition. Moreover, we consider the set of products in the market, \( \bar{x} = \{x_1, ..., x_J\} \), to be constant. In addition, we denote by \( k \)
the AFP that an individual chooses from the set \( \{1, \ldots, J\} \) and by \( j \) a general element from this set.

First, we turn to individual active demand. That individual active demand features the reservation price property results directly from unit demand and the strict monotonicity of preferences in contemporaneous consumption, that is in \( z_j = 0.9Y_1 - p_j \), where \( p_j = \min\{y_1, \bar{p}_j + \hat{p}_j y_1\} \). Put differently, preferences are non-increasing in \( p_j \). Thus, each individual has a reservation price for contributing actively to AFP \( k \). Formally, type \((\omega, \tau, Y_0, Y_1)\) maximizes his/her contemporaneous utility by choosing \((k, 1)\) if and only if

\[
p_k \leq R^a_k(\omega, \tau, \bar{x}, Y_0, z_{j \neq k} | \omega \in \Omega \setminus \{(k, 1)\}).
\]  

(8)

In order to characterize the reservation price further, we denote the best alternative utility level of type \((\alpha(j,1), \tau, Y_0, Y_1)\) as \( A^a_k(\omega, \tau, \bar{x}, Y_0, z_{j \neq k} | \omega \in \Omega \setminus \{(k, 1)\}) \), which is defined as

\[
\max \left\{ \max_{j \neq k} E[U(\alpha(j,1), \tau, x_j, z_j)], \max_j E[U(\alpha(j,0), \tau, x_j, Y_0)], E[U(\alpha_0, \tau, Y_0)] \right\}
\]

Thus, individual \( i \)'s reservation price for active affiliation is

\[
R^a_k(\omega, \tau, \bar{x}, Y_0, z_j | \omega \in \Omega \setminus \{(k, 1)\}) = \begin{cases} 
y_1 & \text{if } E[U(\alpha(k,1), \tau, x_k, 0.8Y_1)] \geq A^a_k; 
p_k & \text{if } p_k \text{ solves} 
E[U(\alpha(k,1), \tau, x_k, z_k)] = A^a_k; 
-\infty & \text{if } E[U(\alpha(k,1), \tau, x_k, z)] < A^a_k, 
\end{cases}
\]

\forall z \in [0.8Y_1, 0.9Y_1].

The amount 0.8\( Y_1 \) in the reservation price formulation comes about as follows: Only individuals which are employed in the formal sector may contribute. These individuals earn an annual income of \( Y_1 \). The mandatory contribution amounts to 10% of their annual income. In addition, AFPs charge consumers a fee for their services. The maximum fee that a fund may charge a consumer is according to the amount equal to the mandatory contribution. Thus, if an AFP charges an actively
contributing individual the maximum fee, the individual’s disposable income amounts to 0.8Y1.

The reservation price property implies that aggregate active demand is non-increasing in (\(\bar{p}_j, \hat{p}_j\)). In a similar way, we can establish the reservation price property of individual demand for AFP \(k\). In order to formalize this, we introduce the indicator function \(\mathbb{I}_{(\alpha(1), \alpha(0), \tau, Y_0, Y_1)}\), where

\[
\mathbb{I}_{(\alpha(1), \alpha(0), \tau, Y_0, Y_1)} = \begin{cases} 
1 & \text{if } \max\{E[U(k,1)], E[U(k,0)]\} = E[U(k,1)] \\
0 & \text{if } \max\{E[U(k,1)], E[U(k,0)]\} = E[U(k,0)]
\end{cases}
\]

Now, type \((\alpha, \tau, Y_0, Y_1)\) maximizes her contemporaneous utility by choosing AFP \(k\)'s product if and only if

\[
\psi_k := (1 - \mathbb{I}_{(\alpha(1), \alpha(0), \tau, Y_0, Y_1)})p_k 
\]

\[
\leq R_k(\alpha, \tau, \bar{x}, Y_0, z_j \neq k | \omega \in \Omega \setminus \{(k, 1), (k, 0)\}).
\]

Correspondingly, the best alternative utility level of type \((\alpha(1), \alpha(0), \tau, Y_0, Y_1)\), which is denoted by \(A_k(\alpha, \tau, \bar{x}, Y_0, z_j \neq k | \omega \in \Omega \setminus \{(k, 1), (k, 0)\})\), is defined as

\[
\max \left\{ \max_{j \neq k} E[U(\alpha(1), \tau, x_j, z_j)], \max_{j \neq k} E[U(\alpha(0), x_j, \tau, Y_0)], E[U(\alpha_0, \tau, Y_0)] \right\}.
\]

Hence, individual \(i\)'s reservation price for being affiliated to AFK \(k\) is

\[
R_k(\alpha, \tau, \bar{x}, Y_0, z_j \neq k | \omega \in \Omega \setminus \{(k, 1), (k, 0)\}) = \begin{cases} 
y_1 & \text{if } \max\{E[U(k,1)], E[U(k,0)]\} \\
& = E[U(\alpha(1), \tau, x, 0.8Y_1)] \geq A_k; \\
\psi_k & \text{if } \psi_k \text{ solves} \\
& \max\{E[U(k,1)], E[U(k,0)]\} = A_k; \\
-\infty & \text{if } \max\{E[U(k,1)], E[U(k,0)]\} < A_k \\
& \forall z \in [0.8Y_1, 0.9Y_1].
\end{cases}
\]

This implies that aggregate demand – i.e., the sum of aggregate active and passive demand – is non-increasing in \(\bar{p}_j\) and \(\hat{p}_j\). The reasoning behind this is as follows: Suppose that AFP \(j\) raises either \(\bar{p}_j\) or \(\hat{p}_j\). Now, it might happen that those
consumers who no longer maximize utility with \((j, 1)\) due to this price increase switch to \((j, 0)\). This does not alter aggregate demand.

With continuous demand functions, the reservation price functions provide a concise representation of aggregate demand for a given firm’s product. Once the other firms’ prices are fixed, demands for AFP \(k\) may be viewed from the perspective of a monopolist facing consumers with the respective reservation price functions; \(R^a_k(\cdot)\) and \(R_k(\cdot)\). Thus, aggregate active demand and total aggregate demand of AFP \(k\) are given by

\[
D^a_k(p_k, p_{-k}, x_k, x_{-k}) = \int_{(\tau, Y_0, Y_1) \in B} f(\tau, Y_0, Y_1) D^a_k(p_k, p_{-k}, x_k, x_{-k}|\bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\tau dY_0 dY_1,
\]

and

\[
D_k(p_k, p_{-k}, x_k, x_{-k}) = \int_{(\tau, Y_0, Y_1) \in B} f(\tau, Y_0, Y_1) D_k(p_k, p_{-k}, x_k, x_{-k}|\bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\tau dY_0 dY_1.
\]

**Proposition 2** Consider preferences represented by (5) to (7) and \(f(\alpha, \tau, Y_0, Y_1)\) a log-concave probability density function on \(\mathbb{R}^{(9+(J+1))}\) with convex support \(B\). Active aggregate demand (9) and total aggregate demand (10) are log-concave over the price interval in which demand is strictly positive.

**Proof** See Appendix.

After having characterized the demand for an AFP’s product we complete the description of an AFP’s objective function by formulating the production process that an AFP uses to provide its service. We think of the output of an AFP as being two-dimensional, consisting of the number of customers that receive its service as well as the total retirement wealth that the AFP manages on their behalf. We assume that all AFPs have access to the same technology, which may have economies of scale related to both inputs as well as the relative level of inputs. Additionally, we assume that AFPs may have firm-specific cost factors that reflect managerial talent or any
other firm-specific productivity factor.

We denote by

\[ D_{TRW}^{jt}(p_k, p_{-k}, x_k, x_{-k}) := D_j^{a,y}(p_k, p_{-k}, x_k, x_{-k}) + D_j^b(p_k, p_{-k}, x_k, x_{-k}), \]

the total retirement wealth that AFP \( j \) manages on behalf of its customers, where \( D_j^{a,y}(\cdot) \) captures the aggregate contemporaneous contribution to AFP \( k \) at price \( p_k \) and \( D_j^b(\cdot) \) represents the aggregate contemporaneous retirement balance that AFP \( k \) manages if it charges \( p_k \) for its service. Then

\[
D_j^{a,y}(p_k, p_{-k}, x_k, x_{-k}) = \int_{(\tau,Y_0,Y_1) \in B} y_1 f(\tau,Y_0,Y_1) D_k^a(p_k, p_{-k}, x_k, x_{-k}|\tilde{\tau}, \tilde{Y}_0, \tilde{Y}_1) d\tau dY_0 dY_1,
\]

\[
D_j^b(p_k, p_{-k}, x_k, x_{-k}) = \int_{(b,\tau,Y_0,Y_1) \in B} b f(b,\tau,Y_0,Y_1) D_k^b(p_k, p_{-k}, x_k, x_{-k}|\tilde{b}, \tilde{\tau}, \tilde{Y}_0, \tilde{Y}_1) db d\tau dY_0 dY_1.
\]

Concerning the characteristics of \( D_j^{a,y}(\cdot) \) and \( D_j^b(\cdot) \) it follows immediately from A1 and the arguments developed in the course of the proof of Proposition 2 presented in the Appendix that both are log-concave in \( p_k \).

We specify an AFP \( j \)'s cost of producing its product as a translog function

\[ C(D_{jt}, D_j^{TRW}) \]

such that

\[
\ln(C(D_{jt}, D_j^{TRW})) = \beta_1 \ln(D_{jt}) + \beta_2 \ln(D_j^{TRW}) + \beta_3 (\ln(D_{jt}))^2 + \beta_4 (\ln(D_j^{TRW}))^2 + \beta_5 \ln(D_{jt}) \ln(D_j^{TRW}) + \nu_j + \eta_{jt}, \tag{11}
\]

where \( \nu_j \) is a firm fixed effect, and \( \eta_{jt} \) is an idiosyncratic shock to the cost of AFP \( j \) in period \( t \). In order to guarantee the existence of a price equilibrium we have to make a technical assumption on the cost function.

**Assumption A2** The translog cost function, \( \ln(C(D_{jt}, D_j^{TRW})) \) is concave in an AFP's price vector over its support \( \mathbb{B} \) which is a convex subset of \( \mathbb{R}^n \).

Chile’s regulation requires that firms have to provide their customers a minimum return which is no less than the average industry return minus 2 percentage points. If an AFP does not achieve this return than it is required to make up the
difference between the realized and the minimum return from its investment reserve. Thus, an AFP faces an additional regulatory cost which is, in expectation, equal to

\[ C_{\text{reg}}^{jt} = E[(\hat{R}_{jt} - \bar{R}_d) | \hat{R}_{jt} < \bar{R}_d] (D_j^a + D_j^b), \] (12)

in period \( t \).

Thus, in period \( t \) the expected profit of an AFP \( j \) that chooses the location \( x_j \) and charges fees \( p_j = (\bar{p}_j, \hat{p}_j) \) when its competitors locate at \( x_{-j} \) and charge \( p_{-j} \) is

\[ E[\Pi_j(p_j, p_{-j}, x_j, x_{-j})] = \bar{p}_j D_j + \hat{p}_j D_j^{a-y_1} - \ln(C(D_j, D_{\text{TRW}})) - C_{\text{reg}}^{jt}, \] (13)

where we assumed that \( p_j \) and \( y_i \) are so that \( p_{ij} = \bar{p}_j + \hat{p}_j y_{i1} \leq y_{i1} \), for all types \( i \) for the ease of exposition. The last term in (13) is zero in the absence of the minimum return requirement.

**Proposition 3** Under Assumptions A1 and A2 and preferences given by (5) - (7) each AFP \( j \)'s expected profit function is quasi-concave in \((\bar{p}_j, \hat{p}_j)\).

**Proof** See Appendix.

### 3.4 Equilibrium Conditions

At a given point in time, competition between AFPS takes the form of a two-stage game. In the first stage, AFPS simultaneously and irrevocably choose their locations. Thus, portfolio characteristics are fixed at the second stage when AFPS engage in price competition. Given the set of portfolios, \( \bar{x} = \{x_1, \ldots, x_J\} \), funds simultaneously choose prices given locations. Thereafter, the rate of return on the market portfolio is realized, interest is paid on consumers’ retirement wealth, and AFPS’ profits accrue. We solve for the subgame perfect equilibrium by backward induction.

**Price Game** Having established that each AFPS profit function is quasi-concave we are in a position to derive an important result, which is contained in the following Proposition.
**Proposition 4** Under Assumptions A1 and A2, preferences given by (5) to (7), for any J firms and arbitrary portfolios $\bar{x}$, there exists a pure strategy Bertrand-Nash equilibrium.

**Proof** Given the continuity of firms’ profit functions with respect to own prices, rival prices, and the quasi-concavity of a firm’s profit with respect to fixed and variable fee, an equilibrium necessarily exists as the set of feasible price combinations is compact and convex (by Kakutani’s fixed point theorem). ■

However, the pricing equilibrium is not necessarily unique. We take the potential non-uniqueness of second stage equilibria into account in our choice of the estimation strategy and when we perform the counterfactual experiments. More specifically, we assume in the counterfactual simulations that firms select the payoff-dominant equilibrium if multiple equilibria exist. The solution of the above problem yields optimal prices as a function of AFPs locations, i.e. $p_{jt}^*(x_{jt}, x_{-jt})$.

**The Location Stage** Given the other AFP’s locations and taking into account the impact on prices, firm $j \in \{1, \ldots, J\}$ maximizes its expected profit by choosing the location $x_{jt} = x_{jt}^*$, where $x_{jt}^*$ solves

$$x_{jt}^* = \text{argmax} \{E[\Pi_{jt}(p_{jt}^*(x_{jt}, x_{-jt}), p_{-jt}^*(x_{jt}, x_{-jt}))]\}. \tag{14}$$

Thus, the necessary conditions for the equilibrium of the two-stage location-then-price game are

$$x_{jt}^* = \text{argmax} \{E[\Pi_{jt}(p_{jt}^*(x_{jt}, x_{jt}^*), p_{-jt}^*(x_{jt}, x_{-jt}^*))]\}, \forall j \in \{1, \ldots, J\}. \tag{15}$$

We provide a proper existence and uniqueness proof of the two-stage location-then-pricing game in a stylized version of the model in the Appendix. More specifically, in the stylized version we assume that consumers can only be actively affiliated to an AFP, AFPs charge only the fixed fee, and consumers are heterogenous with respect to their retirement balance and their risk attitude.
4 Estimation Approach

This section describes how we estimate the fund’s locations, the parameters of consumer preferences and of the industry cost structure from the data. In this environment, the funds’ location choices are not directly observed and need to be inferred from the data. We estimate them from historical data on returns using a variant of the CAPM model. Our estimation procedure consists of two steps. In the first step we estimate funds’ locations and then, in the second step, we use estimated locations to recover consumer preferences and firms’ cost structure. We take two-step nature of our estimator into account in estimating the variance of the estimated coefficients.

4.1 Recovering Fund’s Locations

The pension funds’ choice of location or riskiness of fund’s portfolio is an important component of both the demand and supply side models. As noted, we use the fund’s estimated CAPM beta to represent its location choice. More specifically, we work with a model of time varying beta and GARCH errors to recover funds’ betas, which approximates consumers’ forecast of the funds’ expected returns and return volatilities. Our approach to estimating the CAPM betas is based on Bollerslev at al.’s (1989) CAPM model with time varying covariances. Denote by $Y_{jt}$ an excess return of fund $j$ at time $t$ and by $Y_{m,t}$ an excess market return at time $t$. We assume that the vector $(Y_{jt}, Y_{m,t})$ changes over time according to

$$
Y_{jt} = b_j + \delta h_{jm,t} + \epsilon_{jt} \tag{16}
$$

$$
Y_{m,t} = b_m + \epsilon_{mt}
$$
where $\epsilon_t = (\epsilon_{jt}, \epsilon_{mt})$ is distributed according to $N(0, H_t)$ and the elements of the variance-covariance matrix are:

$$
\begin{align*}
  h_{jj,t} &= \gamma_{jj} + \alpha_{jj}\epsilon_{jt}^2 - 1 + \beta_{jj}h_{jj,t-1} \\
  h_{mm,t} &= \gamma_{mm} + \alpha_{mm}\epsilon_{mt}^2 - 1 + \beta_{mm}h_{mm,t-1} \\
  h_{jm,t} &= \gamma_{jm} + \alpha_{jm}\epsilon_{jt} - 1 \epsilon_{mt} - 1 + \beta_{jm}h_{jm,t-1}
\end{align*}
$$

(17)

Model parameters are estimated via maximum likelihood. The beta values and forecasts are obtained using rolling 18 months window.

4.2 Estimating Consumer Preference Parameters

The demand side of the model represents the consumer’s choice among multiple discrete pension investment fund alternatives. As noted in the previous section, his/her preference for alternative options is described by a random utility model, where utility depends on balances, income, fees, and the location of the firm. Consumers differ in risk aversion and in the intertemporal rate of substitution in a way that depends on observable demographics as well as on unobservables, which yields a random coefficients model. The demand model also allows includes alternative-specific fixed effects to accommodate unobserved differences in the perception of fund quality.

The demand model is estimated using McFadden’s (1989) simulated method of moments (SMM) approach. The parameter vector $\theta$ is recovered as

$$
\theta = \arg\min_{\theta} (d - P(\theta))'W'(d - P(\theta)),
$$

(18)

where $d$ denotes $Jn \times 1$ vector of consumer choices with $d_{ij} = 1$ if individual $i$ chose alternative $j$ and $P(\theta)$ represents the predicted choice given a vector of coefficients $\theta$. Therefore, $d - P(\theta)$ is a vector of residuals stacked by alternatives for a given individual and by individuals. The weighting matrix $W$ is $K \times Jn$ array of instruments of rank $K \geq k$ where $k$ is the length of parameter vector.
The choice probability is estimated using a frequency simulator. McFadden(1989) shows that with a suitable choice of a simulator and matrix of instruments proportional to $\partial \ln(P(\theta^*)/\partial \theta$, the method is asymptotically efficient. We implement the method using an iterative process. First, we find an ‘initial consistent’ estimator of $\theta^*$ using a matrix of non-optimal instruments ($X_{kij}, X_{kij}^2$). Then we use the first-stage estimator to construct optimal instruments which are used to obtain final estimate of $\theta^*$. Mcfadden’s SMM method requires for consistency using two independent sets of random draws in estimation: the first set to construct the instruments and the second set to simulate choice probabilities.

We compute the variance - covariance matrix of estimated coefficients according to

$$\Sigma = (R'R)^{-1}R'G(R'R)^{-1}$$  \hspace{1cm} (19)

where

$$R = \lim_{n \to \infty} n^{-1} WP_\theta(\theta^*)$$  \hspace{1cm} (20)

and

$$G = \lim_{n \to \infty} n^{-1} (1 + r^{-1}) \sum_{t=n}^{t} \sum_{j=1}^{J} (P(j|\theta^*, X)W_{jt}W'_{jt} - W_{j}W'_t).$$  \hspace{1cm} (21)

Here, $r$ is the number of draws used in the frequency simulator.

### 4.3 The Cost Function

The parameters of the cost function are estimated using annual data on various components of firms’ operational costs. This estimation is informative on potential scale effects both with respect to the number of customers served and the total balance managed by a particular fund, while allowing for the interdependence of these two factors in determining costs. We estimate the cost function using a flexible ‘translog’ functional form, where cost depend on the number of customers ($D_{jt}$) and the total
retirement wealth of AFP $j$'s customers ($D_{jt}^{TRW}$). Specifically, we assume that

$$\ln(C(D_{jt}, D_{jt}^{TRW})) = \beta_1 \ln(D_{jt}) + \beta_2 \ln(D_{jt}^{TRW}) + \beta_3 (\ln(D_{jt}))^2 +$$

$$+ \beta_4 (\ln(D_{jt}^{TRW}))^2 + \beta_5 \ln(D_{jt}) \ln(D_{jt}^{TRW}) + \nu_j + \eta_{jt},$$

where $D_{y}$ are year effects. The $\beta$ parameters are estimated using standard panel data methods.

5 Empirical Results

5.1 Descriptive statistics

Table 1 presents some descriptive statistics derived from the administrative pension data, which includes men and women. The pension plan participant sample is fairly young, with a median age of 34 and an interquartile range of 27-42. The median years of contribution is 3.83 years with an interquartile range of 1.41-7.58. This indicates that consumers do not contribute in every year that they are affiliated with pension system. The median pension fund balance is close to the median of one year’s annual income. There is rising dispersion with age, particularly over the age 35-45 range. At subsequent ages, the dispersion remains roughly constant. There is also increasing dispersion in income up through age 40, as exhibited by the interquartile range, after which it declines. The dispersion in income is not as large as the dispersion in balances, which might be expected given that balances represent a stock measure and income a flow measure.

We next turn to descriptive characteristics of the AFP firms. Table 2 shows the fixed and variable fees charged by the AFP firms in year 1999 and reveals substantial variation in the fees charged across firms. A number of funds do not charge any fixed fee. The AFP firm Habitat has the lowest variable fee at 2.84% of monthly contributions and no fixed fee. The firm Concordia has the highest fixed fee at 3.48%
Table 1
Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>25% centile</th>
<th>median</th>
<th>75% centile</th>
</tr>
</thead>
<tbody>
<tr>
<td>age</td>
<td>27</td>
<td>34</td>
<td>42</td>
</tr>
<tr>
<td>years contributing</td>
<td>1.41</td>
<td>3.83</td>
<td>7.58</td>
</tr>
<tr>
<td>annual income (all ages)</td>
<td>$1230</td>
<td>$1342</td>
<td>$3196</td>
</tr>
<tr>
<td>annual income (age=30)</td>
<td>$1127</td>
<td>$1250</td>
<td>$2916</td>
</tr>
<tr>
<td>annual income (age=40)</td>
<td>$1342</td>
<td>$1512</td>
<td>$3705</td>
</tr>
<tr>
<td>annual income (age=55)</td>
<td>$760</td>
<td>$834</td>
<td>$3235</td>
</tr>
<tr>
<td>balance (all ages)</td>
<td>$783</td>
<td>$1090</td>
<td>$3381</td>
</tr>
<tr>
<td>balance (age=30)</td>
<td>$590</td>
<td>$930</td>
<td>$2231</td>
</tr>
<tr>
<td>balance (age=40)</td>
<td>$1500</td>
<td>$2500</td>
<td>$6210</td>
</tr>
<tr>
<td>balance (age=55)</td>
<td>$1653</td>
<td>$2670</td>
<td>$7161</td>
</tr>
</tbody>
</table>

and also a relatively high fixed fee at 230 pesos per month. The variation in fees indicates that products are differentiated.
<table>
<thead>
<tr>
<th>AFP</th>
<th>Percent Fee</th>
<th>Fixed Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concordia</td>
<td>3.48</td>
<td>230</td>
</tr>
<tr>
<td>Cuprum</td>
<td>2.99</td>
<td>0</td>
</tr>
<tr>
<td>Habitat</td>
<td>2.84</td>
<td>0</td>
</tr>
<tr>
<td>Planvital</td>
<td>3.45</td>
<td>280</td>
</tr>
<tr>
<td>Provida</td>
<td>2.85</td>
<td>195</td>
</tr>
<tr>
<td>Santa Maria</td>
<td>3.15</td>
<td>100</td>
</tr>
<tr>
<td>Summa</td>
<td>3.15</td>
<td>230</td>
</tr>
<tr>
<td>Magister</td>
<td>3.4</td>
<td>220</td>
</tr>
<tr>
<td>Union</td>
<td>3.7</td>
<td>290</td>
</tr>
<tr>
<td>Proteccion</td>
<td>2.94</td>
<td>0</td>
</tr>
<tr>
<td>Futuro</td>
<td>3.25</td>
<td>0</td>
</tr>
<tr>
<td>Formenta</td>
<td>3.25</td>
<td>0</td>
</tr>
</tbody>
</table>

In Table 3, we compare the market shares in 1999 of the different funds in terms of the share of clients and the share of the total market balance under each firm’s management. The table also shows the estimated CAPM-beta, with lower betas indicating lower risk. The fund with the largest market share both in terms of customers and balances is Provida, which manages pensions for about one-third of all pension plan participants. Provida is also one of the least risky funds.

The AFP firm Santa Maria has the second largest market share in terms of clients but ranks lower in terms of balance shares. Its portfolio allocation is in the median risk range. The firm with the lowest fees, Habitat, is relatively low ranking in terms of numbers of share of customers but is in the top three in terms of share of total balances. There are also a number of funds in the market with very low shares of customers and of balances. For example, Fomenta has the riskiest portfolio, measured in terms of the beta and also attracts few clients. In summary, there is substantial heterogeneity across firms in fee structures, in shares of clients and in shares of balances.
Table 3
Market Structure in 1999

<table>
<thead>
<tr>
<th>Company</th>
<th>N_Share</th>
<th>B_Share</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concordia</td>
<td>0.191</td>
<td>0.223</td>
<td>0.356</td>
</tr>
<tr>
<td>Cuprum</td>
<td>0.018</td>
<td>0.005</td>
<td>0.540</td>
</tr>
<tr>
<td>Habitat</td>
<td>0.058</td>
<td>0.133</td>
<td>0.400</td>
</tr>
<tr>
<td>Planvital</td>
<td>0.027</td>
<td>0.011</td>
<td>0.525</td>
</tr>
<tr>
<td>Provida</td>
<td>0.345</td>
<td>0.450</td>
<td>0.376</td>
</tr>
<tr>
<td>Santa Maria</td>
<td>0.207</td>
<td>0.074</td>
<td>0.330</td>
</tr>
<tr>
<td>Summa</td>
<td>0.076</td>
<td>0.053</td>
<td>0.500</td>
</tr>
<tr>
<td>Magister</td>
<td>0.014</td>
<td>0.008</td>
<td>0.530</td>
</tr>
<tr>
<td>Union</td>
<td>0.051</td>
<td>0.024</td>
<td>0.520</td>
</tr>
<tr>
<td>Proteccion</td>
<td>0.013</td>
<td>0.013</td>
<td>0.550</td>
</tr>
<tr>
<td>Futuro</td>
<td>0.000</td>
<td>0.001</td>
<td>0.380</td>
</tr>
<tr>
<td>Fomenta</td>
<td>0.001</td>
<td>0.005</td>
<td>0.640</td>
</tr>
</tbody>
</table>

5.2 Model Estimates

Tables 4, 5 and 6 present estimated model parameters and evidence on goodness of fit. As described above, we allow risk aversion (the $\gamma$ parameter) to depend on demographics and also allow for an unobservable component intended to capture unobservable sources of heterogeneity in people’s attitudes towards risk.

Table 4: Demand Estimation: Risk Aversion

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td></td>
</tr>
<tr>
<td>age $\geq$ 35</td>
<td>-3.36</td>
</tr>
<tr>
<td>35 $\leq$ age $\leq$ 50</td>
<td>-7.34</td>
</tr>
<tr>
<td>age $\geq$ 50</td>
<td>-5.36</td>
</tr>
<tr>
<td>sigma</td>
<td>2.05</td>
</tr>
</tbody>
</table>

Theoretical models of dynamic savings accumulation decisions would suggest that age is an determinant of risk aversion, although its net effect on risk aversion
is ambiguous. Older individuals are typically less willing to take on investment risk, because of a shorter time horizon until retirement, but may also be more willing to take on risk, because they have higher balances.

Table 5 presents estimates of absolute and of relative risk aversion at different ages. The Arrow (1965)-Pratt (1964) measure of absolute risk aversion (as a function of consumption) is given by

$$\frac{-u''(c)}{u'(c)}$$

and the measure of relative risk aversion by

$$\frac{-cu''(c)}{u'(c)}.$$  

These are standard measures of risk aversion that stay constant up to affine transformations of the utility function.\(^\text{12}\) As seen in the table, people are estimated to be more risk averse at age 40 than at age 30 or 50.

<table>
<thead>
<tr>
<th>Age</th>
<th>Absolute Risk Aversion</th>
<th>Relative Risk Aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>-0.030</td>
<td>-2.24</td>
</tr>
<tr>
<td>40</td>
<td>-0.048</td>
<td>-2.98</td>
</tr>
<tr>
<td>50</td>
<td>-0.031</td>
<td>-2.11</td>
</tr>
</tbody>
</table>

Table 5: Implied Risk Aversion

In Table 6, we examine the importance of unobservables to the fit of the model. Specifically, we evaluate the fit of the moments under the original model and under two restricted version of the model, one that sets the alternative-specific fixed effects to zero (i.e. shuts down permanent unobservable firm heterogeneity) and one that, in addition, suppresses the utility shock. We find that the fit of the moments is

\(^{12}\)The advantage of the relative measure vis-a-vis the absolute measure is that it accommodates the situation varying degrees of risk aversion at different levels of \(c\) (for example, switching from being risk averse to risk loving and then back to risk averse).
not greatly compromised by shutting down unobservable sources of heterogeneity, although the fit is certainly improved by including these components.

Table 6: Role of Unobservables

<table>
<thead>
<tr>
<th>Proportion explained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observable part of utility function</td>
</tr>
<tr>
<td>Observable part plus fixed effects</td>
</tr>
<tr>
<td>Observable part plus fixed effects plus Weibull errors</td>
</tr>
</tbody>
</table>

Table 7 compares the model’s aggregate predicted shares of annual contributions to pension funds to the empirical shares. Recall that in estimation we only used moments pertaining to shares of customers and share of balances. The moments related to shares of contributions were not used in estimating the model parameters, so this comparison could be viewed as a form of model validation. Generally, the model is able to identify the top five AFP firms in terms of shares of contributions and is fairly accurate in terms of predicting the actual contribution share for four of the five funds. The AFP firm Provida had the largest contribution share in the data, which is also predicted by the model. For the third (Concordia), though, the model overpredicts the contribution share.

Table 7: Aggregate Fit to Contribution Shares

<table>
<thead>
<tr>
<th></th>
<th>Actual</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concordia</td>
<td>18.5%</td>
<td>30.4%</td>
</tr>
<tr>
<td>Cuprum</td>
<td>1.3%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Habitat</td>
<td>12.1%</td>
<td>8.4%</td>
</tr>
<tr>
<td>Planvital</td>
<td>2.0%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Provida</td>
<td>29.0%</td>
<td>30.7%</td>
</tr>
<tr>
<td>Santa Maria</td>
<td>18.6%</td>
<td>22.3%</td>
</tr>
<tr>
<td>Summa</td>
<td>9.3%</td>
<td>8.5%</td>
</tr>
<tr>
<td>Magister</td>
<td>1.7%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Union</td>
<td>4.4%</td>
<td>2.8%</td>
</tr>
<tr>
<td>Proteccion</td>
<td>2.5%</td>
<td>1.0%</td>
</tr>
<tr>
<td>Futuro</td>
<td>0.1%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>
Table 8 provides AFP cost function estimates that are derived from panel data on firms, costs and cost components. We assume that the cost depends on the number of clients served, and the total balance under management. We specify the cost function flexibly as a function of linear and interaction effects in these variables. According to the estimates, once the pension fund reaches a certain size there are decreasing returns to scale. This implies that the market is efficiently served by more than one pension fund firm.

Table 8: Parameter Estimates of the Translog Cost Function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-4.5</td>
<td>0.95</td>
</tr>
<tr>
<td>log(affiliates)</td>
<td>0.98</td>
<td>0.28</td>
</tr>
<tr>
<td>log(assets)</td>
<td>0.45</td>
<td>0.14</td>
</tr>
<tr>
<td>((\log(affiliates))^2)</td>
<td>0.015</td>
<td>0.021</td>
</tr>
<tr>
<td>((\log(assets))^2)</td>
<td>0.038</td>
<td>0.012</td>
</tr>
<tr>
<td>log(affiliates)*log(assets)</td>
<td>-0.086</td>
<td>0.027</td>
</tr>
</tbody>
</table>

6 Policy Experiments

Next, we use our estimated demand and supply-side model to evaluate the effects of several regulatory regimes on AFP products choices, consumer participation, balance accumulation, and the level of government obligations. For reasons of computational difficulty, our current results are for a market with three firms only (the largest firms in the market).

We study four regulatory regimes: (a) unrestricted competition or no regulation, (b) constrained optimization, (c) current regulation, (d) an alternative regulation that places an upper limit on the riskiness of the firm’s portfolio (the CAPM beta). Constrained optimization describes a set of products which would be chosen by a planner while holding the government guarantees fixed at the current level. In the in-
dustry game, the funds’ locations (CAPM betas) and fees are strategic complements. The constrained optimum eliminates this complementarity.

Table 9 shows the averages and standard deviations of CAPM betas for the portfolios offered under the four regulatory regimes. We also separately investigate the industry equilibrium during the periods of high and low market volatility. In a market with low volatility, we find that under no regulation the industry chooses to offer a very risky but well diversified set of portfolios. The selection of portfolios under constraint optimum is less risky but still reasonably well diversified. However, the current regulation that requires rates of return within 2% of the industry average induces a choice of products that are very similar to each other. This effect arises, because the regulation penalty is related to the factors that affects firm’s profitability, i.e. the amount of contributions received by firm. In a market with high volatility and no regulation, there is a move into even riskier products than under low volatility, catering to the demand from low income consumers seeking a high risk gamble. The products offered under the constrained optimization regime reflects the planner’s intention to control overall risk in the system by reducing the exposure to aggregate risk when it is high. Interestingly, the current regulation lead to similar outcomes as in the no regulation regime with the industry choosing higher risk products relative to constrained optimum. The products also exhibit low diversification. We also consider an alternative regulation that places explicit restrictions on portfolio riskiness of firms (the CAPM betas), which lead to product characteristics that are similar to those chosen under the constrained optimum.

Table 9: Product Characteristics Under Different Regulatory Regimes
Table 10 compares the proportion of consumers participating in the pension plan under the four different cases considered previously. As noted in the introduction, a major concern of the Chilean government is low contribution rates, with a substantial fraction of workers opting to work outside the formal sector of the economy and therefore to not participate in the pension program. The estimates in Table 10 show that the current regulation results in the lowest participation rate both in the high and low volatility markets. It is significantly lower than under a constrained optimum or under the alternative regulation that restricts portfolio risk directly. The lower contribution rate arises because of low product diversity that results under current regulation.
Table 10: Pension Plan Participation Rates under Alternative Regulatory Schemes

<table>
<thead>
<tr>
<th></th>
<th>Participation Rate</th>
<th>Balance (Mean)</th>
<th>Balance (Std. Dev.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low market volatility, ( \sigma_m = 0.10 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Regulation</td>
<td>65%</td>
<td>1.25</td>
<td>1.35</td>
</tr>
<tr>
<td>Current Regulation</td>
<td>58%</td>
<td>0.8</td>
<td>0.75</td>
</tr>
<tr>
<td>Portfolio Restrictions</td>
<td>75%</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>Constrained Optimum</td>
<td>75%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>High market volatility, ( \sigma_m = 0.25 )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Regulation</td>
<td>55%</td>
<td>1.35</td>
<td>1.60</td>
</tr>
<tr>
<td>Current Regulation</td>
<td>56%</td>
<td>1.21</td>
<td>1.18</td>
</tr>
<tr>
<td>Portfolio Restrictions</td>
<td>70%</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Constrained Optimum</td>
<td>70%</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 11 analyses the size of government obligations under the four regulatory regimes. We find that the no regulation regime results in extremely high volatility of government obligations. The current regulation induces high expected government liabilities as well as substantial variation in government liabilities. The first effect arises because of the low participation rate whereas the second effect arises because of the high riskiness of industry portfolios offered under current regulation.
### Table 11 Government Obligations

<table>
<thead>
<tr>
<th></th>
<th>Retirees on support</th>
<th>Retirees on support</th>
<th>Total amount</th>
<th>Total amount</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(mean)</td>
<td>(std. dev.)</td>
<td>(mean)</td>
<td>(mean)</td>
</tr>
<tr>
<td><strong>Low market volatility</strong>, $\sigma_m = 0.10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Regulation</td>
<td>28%</td>
<td>10%</td>
<td>0.476</td>
<td>0.17</td>
</tr>
<tr>
<td>Current Regulation</td>
<td>25%</td>
<td>3%</td>
<td>0.425</td>
<td>0.051</td>
</tr>
<tr>
<td>Portfolio Restrictions</td>
<td>20%</td>
<td>5%</td>
<td>0.34</td>
<td>0.085</td>
</tr>
<tr>
<td>Constrained Optimum</td>
<td>20%</td>
<td>5%</td>
<td>0.34</td>
<td>0.085</td>
</tr>
<tr>
<td><strong>High market volatility</strong>, $\sigma_m = 0.25$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Regulation</td>
<td>34%</td>
<td>25%</td>
<td>0.731</td>
<td>0.425</td>
</tr>
<tr>
<td>Current Regulation</td>
<td>32%</td>
<td>15%</td>
<td>0.544</td>
<td>0.255</td>
</tr>
<tr>
<td>Portfolio Restrictions</td>
<td>25%</td>
<td>9%</td>
<td>0.425</td>
<td>0.136</td>
</tr>
<tr>
<td>Constrained Optimum</td>
<td>25%</td>
<td>9%</td>
<td>0.425</td>
<td>0.136</td>
</tr>
</tbody>
</table>

The amount of government obligations is given in billions of US dollars.

### 7 Conclusion

Chile has one of the oldest individual-account pension systems and therefore provides a unique opportunity to study firm and consumer behavior under a well established private accounts system. The design of the Chilean pension system includes insurance features, in the form of a minimum return guarantee and a minimum pension guarantees, that are intended to protect investors against low levels of pension accumulations. These guarantees create the potential for moral hazard in consumers’ investment decisions.

In this paper, we developed a demand and supply model of the Chilean pension fund market. In the demand model, a consumer chooses an AFP to manage his/her investments, taking into account pension fund fees and historical pension fund performance. Consumers are heterogeneous in terms of their demographics and in risk aversion. The supply side is modeled as an oligopolistic environment in which AFPs sequentially choose product location (mean and variance of the return), and the fixed and variable fees they charge for service while taking into account consumers’ preferences for risk.
After estimating the parameters of the model, we use the model to assess the impact of government regulations on pension funds’ choices of locations. We also study implications of regulations for the consumers’ accumulated balances, and for government obligations. We find that Chilean regulatory rule that mandates firms to guarantee returns within 2% of the industry average creates incentives for the AFP firms to invest in the riskier portfolios than they would choose under an alternative regulation that instead restricts the riskiness of their portfolio limited. Surely, this is an unanticipated effect of the regulation. Because the portfolio location choices that firms make are riskier, fewer people participate in the pension program, which is a particularly worrisome finding considering that the government places a high priority on increasing coverage rates. Also, the choice of the portfolios under the current regulation is riskier than would be the selection of portfolios that a social planner would choose. Not surprisingly, it leads to a higher than desirable (by social planner) volatility in accumulated balances. We find that from the point of view of social welfare, an alternative regulation that restricted directly the investment instruments of the pension fund rather than requiring them to achieve a performance near the mean would be more effective.
References


A Price Equilibrium Existence Proof

The proof proceeds along the lines of Caplin and Nalebuff (1991b). Though the involved steps and techniques are similar to their analysis it is indispensable to provide the present price equilibrium existence proof due to the following reasons: First, the regulations that are in place in the Chilean system make the pension fund market inherently different from a standard product market. More specifically, the most prominent features that draw the distinction are that consumers have the option to be passively affiliated to an AFP and the fact that AFPs charge two-part tariffs.

Second, our chosen modeling of consumer utility implies that consumer preferences are only partially covered by the regularity conditions in Caplin and Nalebuff (1991b). In particular, in our model consumers are heterogeneous with respect to income and preferences are non-linear in contemporaneous consumption. By contrast, in their framework income heterogeneity comes with preferences that are linear in income. In addition, we scale the utility of contemporaneous consumption and therewith prices with the consumer characteristic \( \tau \). This constitutes yet another point of departure from their framework.

As will become clear below, the existence of the price equilibrium depends solely on consumer preferences and the joint distribution of consumer preference parameters. Thus, the price equilibrium existence proof is valid for all regulatory settings, i.e. with or without the minimum return requirement, and conducted counterfactual experiments since these merely affect the portfolio characteristics of AFP \( j \).

The structure is as follows: First, we proof that each aggregate demand for a firm’s product is continuous (Proposition 1) and log-concave in an AFP’s price vector (Proposition 2). Second, given the log-concavity of each demand component we provide the proof of Proposition 3 that shows that an AFP’s expected profit function is quasi-concave.
A.1 Proof of Proposition 1

It follows from (5) that individuals who are indifferent between contributing actively to AFP $k$ and AFP $j$, $j \neq k$ are defined by

$$\alpha \eta_{k,j} = \phi_{k,j},$$

where

$$\eta_{k,j} = \left( E[\hat{R}_k] - E[\hat{R}_j]; E[\hat{R}_k^2] - E[\hat{R}_j^2] \right)^T,$$

and

$$\phi_{k,j} = \epsilon_{i(k,1)} - \epsilon_{i(j,1)} + \xi_k - \xi_j + \tau_i \left( \ln(z_{ik}) - \ln(z_{ij}) \right).$$

By assumption $\eta_{k,j} \neq 0$, so that (23) defines a hyperplane in $\mathbb{R}^n$.

In addition, the hyperplane that describes the consumers indifferent between contributing actively to AFP $k$ and the outside options is defined by

$$E[U(\alpha_{i,(k,1)}, \tau_i, x_k, z_{ik})] = \max \left\{ \max_j E[U(\alpha_{i,(j,0)}, \tau_i, x_j, Y_{i0})], E[U(\alpha_{i0}, \tau_i, Y_{i0})] \right\}. \quad (24)$$

It follows from Assumption A1 that the distribution of consumer types is hyperdiffuse. Thus, the sets of indifferent consumers – (23) and (24) – have zero measure. The continuity of demand for active contribution to AFP $k$ then follows from the continuity of the $\ln(\cdot)$ function.

Similar arguments imply the continuity of aggregate passive demand and therewith, the continuity of aggregate demand. ■

A.2 Proof of Proposition 2

The proof proceeds as follows: First, we show that demand functions are log-concave in a market segment in which consumers have identical elasticities of substitution and income, that is for fixed $(\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$. Second, we establish that aggregate demands – which are generated by aggregating the demand functions of all market segments – are log-concave under Assumption A1.
Active Aggregate Demand  In the first part of the proof, the strategy is to show the following: Suppose that an individual of type \((\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) prefers choice \((k, 1)\) when the price of AFP \(k\)'s product is \(p_k\) and that the same choice is preferred for type \((\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) when the price of \(k\)'s product is \(p'_k\), with \(p'_k > p_k\). Then it follows that \((k, 1)\) is among the most preferred products for type \((\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) at price \(p^\lambda_k = [\lambda \bar{p}_k + (1 - \lambda) \bar{p}'_k, \lambda \hat{p}_k + (1 - \lambda) \hat{p}'_k]\).

Thus, in principle we have to check this condition for active demand in three different cases, i.e.

1. customers prefer \((k, 1)\) to \((h, 1)\), \(h \neq k\); prices of other products are fixed at \(p_h\),
2. customers prefer \((k, 1)\) to \((j, 0)\), for all \(j \in \{1, ..., J\}\),
3. customers prefer \((k, 1)\) to \((0)\).

For the sake of exposition we focus on case (1). Arguments similar to the one the developed in case (1) imply that the required condition holds in the cases (2) and (3) as well.

Case (1)  Consider the comparison between actively contributing to AFP \(k\) and another AFP \(h\), \(h \neq k\). Substitution in the expected utility function (5) yields the following inequalities:

\[
\sum_{l=1}^{3} \alpha_l t_l(x_k) + \bar{\tau} \ln(z_{ik}) + t_4(x_k) \geq \sum_{l=1}^{3} \alpha_l t_l(x_h) + \bar{\tau} \ln(z_{ih}) + t_4(x_h),
\]

\[
\sum_{l=1}^{3} \alpha'_l t_l(x_k) + \bar{\tau} \ln(z'_{ik}) + t_4(x_k) \geq \sum_{l=1}^{3} \alpha'_l t_l(x_h) + \bar{\tau} \ln(z_{ih}) + t_4(x_h),
\]

where \(t_1(x_j) = E[\bar{R}_j], t_2(x_j) = E[(\bar{R}_j)^2], t_3(x_j) = 1,\) and \(t_4(x_j) = \xi_j.\)

Combining these inequalities yields

\[
\sum_{l=1}^{3} \alpha^\lambda_l t_l(x_k) + \lambda \bar{\tau} \ln(z_{ik}) + (1 - \lambda) \bar{\tau} \ln(z'_{ik}) + t_4(x_k) \geq \sum_{l=1}^{3} \alpha^\lambda_l t_l(x_h) + \bar{\tau} \ln(z_{ih}) + t_4(x_h).
\]

(25)
It follows from concavity that
\[
\bar{\tau} \ln(z_{ik}^\lambda) \geq \lambda \bar{\tau} \ln(z_{ik}) + (1 - \lambda) \bar{\tau} \ln(z_{ik}^{'})
\]

Thus the portfolio of AFP \( k \) at price \( p_{k}^\lambda \) is among the most preferred products for consumers of type \((\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\).

When products \( k \) and \( h, h \neq k \) are distinct, the mass of consumers for which (25) is an equality is zero. Hence, the Prékopa-Borell Theorem applies.\(^{13}\)

If observed and unobserved characteristics of the products \( k \) and \( h \) are identical, product \( k \) must still be the most preferred product at price \( p_{k}^\lambda \) for type \((\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\). This is due to the fact that \( p_{k}^\lambda < p_k' \) by definition and that \( p_h \geq p_k' \) as otherwise type \((\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) would have strictly preferred AFP \( h' \)'s portfolio. Hence the set of individuals investing at AFP \( k \) contains the Minkowski average of consumers choosing \((k, 1)\) at prices \( p_k \) and \( p_k' \). Thus, the Prékopa-Borell Theorem applies directly.

Similar arguments imply that cases (2) and (3) are covered as well.

**Aggregate demand** Aggregate demand also comprises those customers who prefer to be passively affiliated to an AFP. It follows directly from the preceding analysis that AFP \( k' \)'s product is the most preferred choice for every type that can be represented as a linear combination of types whose most preferred choice is passive affiliation to AFP \( k \) and whose most preferred alternative is either non-affiliation or affiliation to another AFP.

In addition, we have to account for types whose most preferred alternative is active affiliation to AFP \( k \). In order to see that the required condition also holds in this case, suppose that type \((\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) prefers AFP \( k' \)'s product if it is priced at \( p_k \) and that type \((\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) also prefers \( k' \)'s product if it is priced at \( p_k' \), with \( p_k' > p_k \). Moreover, the former type prefers \((k, 1)\) to \((k, 0)\) and the latter type prefers \((k, 0)\) to \((k, 1)\) at the corresponding prices.

\(^{13}\)See Caplin and Nalebuff (1991a) for an in-depth treatment of the Prékopa-Borell Theorem.
By construction, type \((\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) also prefers \((k, 1)\) at price \(p'_k\) to the other alternatives. Thus, affiliation to AFP \(k\) is among the most preferred products for type \((\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) at price \(p'_k^\lambda = [\lambda \bar{p}_k + (1 - \lambda) \bar{p}'_k, \lambda \hat{p}_k + (1 - \lambda) \hat{p}'_k]\).

Hence, the set of individuals who are affiliated to AFP \(k\) contains the Minkowski average of the consumers who prefer affiliation to AFP \(k\) at prices \(p_k\) and \(p'_k\). Thus, the Prékopa-Borell Theorem applies.

The arguments developed above imply that aggregate active demand and aggregate demand in a sub-market characterized by identical \((\bar{\tau}, \bar{Y}_0, \bar{Y}_1)\), which are given by

\[
D_{a_k}(\bar{p}_k, \hat{p}_k | \bar{\tau}, \bar{Y}_0, \bar{Y}_1) = \int_{\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1 : p_k \geq p_k^\alpha} f(\alpha | \bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\alpha, \tag{26}
\]

and

\[
D_{k}(\bar{p}_k, \hat{p}_k | \bar{\tau}, \bar{Y}_0, \bar{Y}_1) = \int_{\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1 : p_k \geq p_k^\alpha} f(\alpha | \bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\alpha, \tag{27}
\]

are log-concave. This completes the first part of the proof.

In the second part we show that aggregate active and total aggregate demand are log-concave under Assumption A1. Aggregating both \(D_{a_k}^a(\bar{p}_k, \hat{p}_k | \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) and \(D_k(\bar{p}_k, \hat{p}_k | \bar{\tau}, \bar{Y}_0, \bar{Y}_1)\) over all sub-markets characterized by identical incomes and elasticities yields

\[
D_{a_k}^a(p_k) = \int_{(\tau, Y_0, Y_1) \in B} f(\tau, Y_0, Y_1) D_{a_k}^a(p_k | \tau, Y_0, Y_1) d\tau dY_0 dY_1, \tag{28}
\]

and

\[
D_k(p_k) = \int_{(\tau, Y_0, Y_1) \in B} f(\tau, Y_0, Y_1) D_k(p_k | \tau, Y_0, Y_1) d\tau dY_0 dY_1. \tag{29}
\]

Due to A1 it follows from Theorem 6 in Prékopa (1973) that \(f(\tau, Y_0, Y_1)\), i.e. the marginal density of consumer incomes and elasticity of substitution, is also log-concave. Thus, each term of the products \(f(\tau, Y_0, Y_1) D_{a_k}^a(p_k)\) and \(f(\tau, Y_0, Y_1) D_k(p_k)\) is log-concave. From this it follows that (28) and (29) are log-concave as well. ■
A.3 Proof of Proposition 3

We set out with the case in which \( p_j \) and \( y_{i1} \) are so that \( p_{ij} = \tilde{p}_j + \hat{p}_j y_{i1} \leq y_{i1} \), for all types \( i \). Then the expected profit function of AFP \( j \) is given by

\[
E[\Pi_j(\tilde{p}_j, \hat{p}_j)] = \tilde{p}_j D_j + \hat{p}_j D_{a,y}^{a,y_1} - \ln(C(D_j, D_{jTRW}^j)) - C_j^{reg},
\]

The profit function is monotonically increasing in \( p_j \) in the region in which \( E[\Pi_j(\tilde{p}_j, \hat{p}_j)] < 0 \); an increase in \( \tilde{p}_j \) and \( \hat{p}_j \) reduces sales and per-unit losses. Hence we can restrict attention to \( p_j \) so that \( E[\Pi_j(\tilde{p}_j, \hat{p}_j)] \geq 0 \). Because we consider exclusively the profit function of AFP \( j \), we will suppress the index \( j \) in the remainder of the proof.

A failure of quasi-concavity requires that there exist \( \bar{p} < \bar{p}', \bar{p} < \hat{p}', \) and \( 0 < \lambda < 1 \) so that

\[
\bar{p} D(p) + \bar{p} D_{a,y}^{a,y_1}(p) - C(D(p), D_{jTRW}^j) - C_j^{reg}(p) > \]

\[
\bar{p}' D(p') + \bar{p}' D_{a,y}^{a,y_1}(p') - C(D(p'), D_{jTRW}^j) - C_j^{reg}(p') > \]

\[
\bar{p}' D(p') + \bar{p}' D_{a,y}^{a,y_1}(p') - C(D(p'), D_{jTRW}^j) - C_j^{reg}(p') > \]

\[
\bar{p}' D(p') + \bar{p}' D_{a,y}^{a,y_1}(p') - C(D(p'), D_{jTRW}^j) - C_j^{reg}(p') > \]

Now, suppose that \( \bar{p}, \bar{p}', \hat{p} \) and \( \hat{p}' \) are such that

\[
\hat{p} D_{a,y}^{a,y_1}(p) - C(D(p), D_{jTRW}^j) - C_j^{reg}(p) < \]

\[
\hat{p}' D_{a,y}^{a,y_1}(p') - C(D(p'), D_{jTRW}^j) - C_j^{reg}(p') < \]

Then, it follows from (30) and (31) that a failure of quasi-concavity requires that

\[
\bar{p} D(p) > \bar{p}' D(p'),
\]

\[
\bar{p}' D(p') > \bar{p}' D(p').
\]
Taking logs, weighting the logged inequalities (32) and (33) with $\lambda$ and $1 - \lambda$, respectively, and adding the left- and right-hand sides yields

$$\lambda \ln \left( \tilde{p} D(p) \right) + (1 - \lambda) \ln \left( \tilde{p}' D(p') \right) > \ln \left( \tilde{p}^\lambda D(p^\lambda) \right),$$

which is contradicted by the log-concavity of $D(p)$.

Next, suppose that $\bar{p}$, $\bar{p}'$, $\hat{p}$ and $\hat{p}'$ are such that

$$\hat{p} D_{a,y}^{1}(p) - C(D(p), D_{\text{TRW}}(p)) - C_{j}^{\text{reg}}(p) >$$

$$\hat{p}^\lambda D_{a,y}^{1}(p^\lambda) - C(D(p^\lambda), D_{\text{TRW}}(p^\lambda)) - C_{j}^{\text{reg}}(p^\lambda),$$

$$\hat{p}' D_{a,y}^{1}(p') - C(D(p'), D_{\text{TRW}}(p')) - C_{j}^{\text{reg}}(p') <$$

$$\hat{p}^\lambda D_{a,y}^{1}(p^\lambda) - C(D(p^\lambda), D_{\text{TRW}}(p^\lambda)) - C_{j}^{\text{reg}}(p^\lambda).$$

Then, it follows from (30), (31), and the preceding analysis, i.e. (32) to (34), that a failure of quasi-concavity requires that

$$\bar{p} D(p) < \bar{p}^\lambda D(p^\lambda),$$

$$\bar{p}' D_{a,y}^{1}(p') > \bar{p}^\lambda D_{a,y}^{1}(p^\lambda).$$

In conjunction, (35) and (36) imply that there exists a $\lambda \in (0, 1)$, denoted by $\hat{\lambda}$ so that

$$\hat{\lambda} \ln \left( \bar{p} D(p) \right) + (1 - \hat{\lambda}) \ln \left( \bar{p}' D(p') \right) = \ln \left( \bar{p}^\hat{\lambda} D(p^\hat{\lambda}) \right),$$

which is contradicted by the log-concavity of $D(p)$.

Finally, suppose that $\bar{p}$, $\bar{p}_f$, $\hat{p}$ and $\hat{p}'$ are such that

$$\hat{p} D_{a,y}^{1}(p) - C(D(p), D_{\text{TRW}}(p)) - C_{j}^{\text{reg}}(p) >$$

$$\hat{p}^\lambda D_{a,y}^{1}(p^\lambda) - C(D(p^\lambda), D_{\text{TRW}}(p^\lambda)) - C_{j}^{\text{reg}}(p^\lambda),$$

$$\hat{p}' D_{a,y}^{1}(p') - C(D(p'), D_{\text{TRW}}(p')) - C_{j}^{\text{reg}}(p') >$$

$$\hat{p}^\lambda D_{a,y}^{1}(p^\lambda) - C(D(p^\lambda), D_{\text{TRW}}(p^\lambda)) - C_{j}^{\text{reg}}(p^\lambda).$$

By repeatedly applying the steps developed above, we can show that (??) and (40) are in conjunction contradicted by the log-concavity of $D_{a,y}^{1}(p)$, $D(p)$, $D_{b}(p)$, and $C(D(p), D_{\text{TRW}}(p))$. We omit this for the sake of exposition.
Conducting the same analysis for each demand component of an AFP’s profit function yields the desired result. Obviously, the result also holds in the absence of the minimum return requirement, i.e. $C_j^{reg}(p) = 0$ and $\tilde{R}_{jt} = \tilde{R}_{jt}$. Similar arguments imply that an AFP’s expected profit function is also quasi-concave if $p_{ij} = y_{i1} \leq \bar{p}_j + \hat{p}_j y_{i1}$, for some types $i$. ■