

# Comprehensive Exam

August 19, 2013

You have a total of 180 minutes to complete the exam. If a question seems ambiguous, state why, sharpen it up and answer the sharpened-up question. Good luck!

# 1 Menu Costs and Monetary Policy (60min)

**Households:** Consider an economy where time is continuous and the representative household chooses consumption  $C_t$ , labour  $L_t$  and money  $M_t$  to maximise the following utility in time  $t = 0$ :

$$E_0 \max \int_0^\infty e^{-\rho t} \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \alpha L_t + \log \left( \frac{M_t}{P_t} \right) \right] dt$$

$$s.t. \quad \int_0^\infty Q_0^t [P_t C_t + R_t M_t - W_t L_t - \Pi_t] dt \leq M_0$$

where  $\rho$  is the discount factor,  $\gamma$  is the inverse of the intertemporal elasticity of substitution,  $P_t$  is the aggregate price level,  $Q_0^t$  is the value of a period- $t$  dollar in term of period-0 dollars ( $Q_0^0 = 1$ ),  $R_t$  is the interest rate,  $W_t$  are wages the households earn in a competitive labour market,  $\Pi_t$  profits and  $M_0$  the initial money stock. Money supply  $M^s$  follows this exogenous law of motion:

$$d \log M_t^s = \mu dt + \sigma dW_t$$

where  $\mu$  is the expected money growth rate and  $dW_t$  is an increment in a Wiener process.

1. Although the household lives for infinitely many moments, there is only one budget constraint. Explain what implicit assumption is reflected by this modelling. Explain the economic meaning of the term  $R_t M_t$  in the budget constraint.
2. Derive the first-order conditions of the household problem.
3. There exists an equilibrium where  $\forall t : R_t = \rho + \mu$ . Combine this condition with the household FOCs to derive and explain an expression relating wages to the level of money in the economy.

**Firms:** The consumption good is produced competitively from a continuum of varieties as follows

$$C_t = \left[ \int (y_{it})^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where  $\varepsilon$  is the elasticity of substitution between varieties. Each variety  $y_{it}$  is produced with labour by a monopolistic firm using the following technology

$$y_{it} = z_i L_{it}$$

where  $z_i$  is the (constant) productivity level that differs across  $i$ . The firm  $i$  is characterised by its productivity level  $z_i$  and its price level  $p_{it}$ . Whenever it wants to change its price faces menu cost. When changing his price, the monopolist has to pay  $f$  units of labour.

4. Derive demand curve for each intermediate variety, compute an expression of the aggregate price index  $P_t$  and . Also compute the flow profits of a monopolistic intermediate firm  $i$  given price  $p_{it}$ .
5. Write value function for a firm in  $t = 0$  that does not adjust its price now, denoted by  $V_0^n(\cdot)$ , and the value for a firm that does adjust now, denoted by  $V_0^a(\cdot)$ ; be careful to display the state for each value function.
6. Prove that for  $f > 0$  there is a region around the optimal price where the firm will not adjust its price. (Hint: If your proof requires continuity of  $V_t^n(\cdot)$  or  $V_t^a(\cdot)$ , you can rely on the appropriate theorem from SLP.) Explain how that inactivity region changes when  $\sigma$  is very large. What does that mean for aggregate inflation?

**Monetary Policy:** We can think of a central bank as changing parameters  $\mu$  and  $\sigma$ .

7. Suppose  $f = 0$ . Using the household and firm optimality conditions explain what happens to wages, individual prices, aggregate inflation and consumption when the central bank engineers a temporary and unanticipated increase in  $\mu$ .
8. Suppose  $f > 0$ . Using the household and firm optimality conditions explain what happens to wages, individual prices, aggregate inflation and consumption when the central bank engineers a temporary and unanticipated increase in  $\mu$ . Compare your answer to the previous subquestion.

## 2 Term Structure of Interest Rates (30min)

Consider an economy populated by a large number of identical households. The common utility function is

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} \quad \beta \in (0, 1), \theta > 0$$

Each household owns a tree that bears the following stochastic fruit (in units of consumption)

$$c_{t+1} = c^* c_t^\phi \varepsilon_{t+1}$$

where  $c^* > 0$  and  $\phi < 1$  are scalars and  $\varepsilon_t \sim N(1, \sigma^2)$ . The households trade shares in trees and bonds of all maturities.

1. Define a competitive equilibrium.
2. Calculate the term structure of interest rates,  $R_t^{t+j}$ ,  $j = 1, 2, \dots$ , where  $R_t^{t+j}$  denotes the risk-free return in units  $t + j$  good denominated in units of period- $t$  good. In period  $t$ , agents know  $R_t^{t+j}$  and, since it's risk-free,  $1/R_t^{t+j}$  is the price of a unit of period- $(t + j)$  consumption in terms of period- $t$  consumption. Hint: If  $z$  is normal, then  $E[e^z] = e^{E[z] + Var(z)/2}$ .
3. Economist A argues that economic theory predicts that the variance of the log of short-term interest rates (say one-period) is always lower than the variance of long-term interest rates, because short rates are "riskier." Do you agree? Explain why/why not.
4. Economist B argues that short-term interest rates (say one-period) are more responsive to the state of the economy (i.e. consumption), than long-term interest rates ( $j$  large). Do you agree? Explain why/why not.
5. Economist C argues that the Fed should lower interest rates because consumption is high whenever interest rates are low. Do you agree? Explain why/why not.
6. Economist D argues that changes in output (consumption) do not affect interest rates when output (consumption) is very persistent ( $\phi \approx 1$ ). Do you agree? Explain why/why not.

## 2.1 Ramsey Taxation

Consider an infinitely lived consumer with period utility function over consumption and labor supply:  $u(c, h) = \log(c) - \phi \frac{\epsilon}{1+\epsilon} h^{\frac{1+\epsilon}{\epsilon}}$  with  $\epsilon > 0$  and constant discount factor  $\beta$ . Assume that there is a representative firm with production function  $y_t = AK^\alpha H^{1-\alpha}$ . Capital depreciates at rate  $\delta = 100\%$ .

PART 1: Solve for a competitive equilibrium to this economy.

Now assume that the government taxes all sources of income at constant rate  $\tau$  each period in order to fund spending  $g_t$ . The government runs a balanced budget.

2. Define the TDCE for this economy.
3. With  $\phi = 0$ , does the TDCE solve a planner's problem? If so, show which one.
4. With  $\phi = 0$ , solve for output in the TDCE.
5. How does output change as  $\tau$  changes?
6. How would your answers to parts C through E change if  $\phi > 0$ ?

## 2.2 Search With Human Capital

Consider a search model with the following structure: households live forever, discount the future at rate  $\beta$ , have linear preferences over consumption, no access to savings, and are productive in proportion to their "human capital". This is a stock of knowledge or experience, denoted by  $h$ , which grows according to the following law of motion:

$$h_{t+1} = (1 - \delta)h_t + \delta h_t \mathbb{I}_{\{e_t=1\}}$$

Where  $\delta$  is a depreciation term and  $e_t$  is the current employment status, equal to one if the person has a job and zero if not and  $\mathbb{I}$  is an indicator function.

Workers search for jobs which are a good match, given by  $m$ , and receive a wage equal to  $w = hm$  from that match. Once a worker has a job, she keeps it forever. The match quality at a new job is drawn from cdf  $F(m)$ . An unemployed worker receives a constant benefit,  $b$ .

- 1 Write the value functions that characterize the value of being unemployed and employed at a job with a given match quality. What is the state variable?
- 2 Formally prove the existence of the above functions. Characterize the job acceptance policy.
- 3 What is true about the reservation match quality for high skilled workers relative to low skilled (human capital = skill)?
- 4 What happens to the total skill of the workforce if employment benefits become more generous?