MICROECONOMICS COMPREHENSIVE EXAM

AUGUST 2013

INSTRUCTIONS:

(1) Please answer each of the four questions on **separate** pieces of paper.
(2) Please write only on **one side** of a sheet of paper
(3) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.).

1. Consider the Robinson Crusoe (RC) economy with a single consumption good. RC, who lives on an isolated island, derives utility from the consumption good (c) and leisure (l). His utility function, $u(c, l)$ satisfies the standard properties. RC has an endowment of $\bar{l}$ units of leisure (e.g., 24 hours in a day) and no endowment of the consumption good.

There is a single firm (owned by RC) that produces the consumption good using labor as the single input according to the increasing and strictly concave production function $f(z)$, where z is the labor input. Thus, to produce output, the firm must hire RC, effectively purchasing some leisure from him. Let p be the unit price of the consumption good, and w be the unit price of labor. Let $q(p, w)$ denote the firm's output and $\pi(p, w)$ denote the firm's profit.

(a) Carefully define a competitive equilibrium in the RC economy.

(b) Let u satisfy the assumptions of the smooth model, and $f \in C^2$. Set $p = 1$ and define the excess demand function for labor as

$$h(w) = l(1, w) + z(1, w) - \bar{l},$$

where $l(1, w)$ is RC's demand for leisure, and $z(1, w)$ is the firm's demand for labor. Show that in a neighborhood of an equilibrium, $dh(w)/dw < 0$.

(c) Show that the equilibrium in the RC economy with preferences and technology as specified earlier is unique.

2. Alice's house is worthless to her if she cannot sell it. Bob and Carol are the potential buyers. Bob and Carol know their own valuations of the house, but this information is not known to anybody else. It is common knowledge that their valuations are independent and identically distributed and can take only two values: high, $v_H = \$4(\text{mln.})$, and low, $v_L = \$3(\text{mln.})$. The probability of low valuation is $p \in (0,1)$. All agents are risk neutral. Focus only on symmetric equilibria.

(a) What is Alice's first best outcome? What is her expected payoff in the first best outcome? Can Alice achieve the first best outcome?

(b) What is Alice's expected payoff if she simply posts a take-it-or-leave-it price?

(c) What is Alice's expected payoff in the Vickrey auction with no reserve?

(d) Consider the following modification of the Vickrey auction: Bob and Carol are restricted to bid only 3 or 4, and the winner pays the *average* of the winning and losing bids. Will Bob and Carol bid their true valuations? Why or why not? What is Alice's expected payoff?

(e) Which of the above mechanisms will Alice choose to sell her house?

3. (The confusing memo) Consider the following game: there are two workers (*A*dams and *B*aker, modeled as strategic agents) and one boss (modeled as a force of nature). First, the boss randomly (with equal probability) picks a worker $i \in \{A, B\}$, and then decides that she wants worker i to do a project, and she wants the other worker $-i$ to supervise. So the boss uses the company mail system to send a memo to worker $-i$ describing what she wants. Unfortunately, the memo is confusing. It reads, "Make sure that the other worker does the project," and it is unsigned.

When worker $-i$ receives the memo, he can either *Do* the project himself or *Resend* the memo to the other worker through the company mail system. If she *Resends* it, then worker i has the same choice. If both workers choose to *Resend* the memo, then the game ends (because the mailroom reports to the boss what has happened). When he receives the memo, *a worker cannot tell whether it was sent to him directly by the boss or whether it was resent by the other worker.*

If worker $-i$ (the one who receives the memo directly from the boss) chooses to *Do* the project, then the boss gets angry with worker i (for not doing the project) and is mildly pleased with worker $-i$ (who didn't do a good job supervising the other worker but at least got the project done). In that case, worker $-i$ gets a payoff of 1 (measured in units of utility) and worker i gets a payoff of -1.

If worker $-i$ chooses to *Resend* the memo, then payoffs depend on worker i's action. If worker i choose to *Do* the project, then the boss is pleased with both of them, especially worker $-i$ (who showed good supervisory skills). In that case, worker $-i$ gets a payoff of 2 and worker i gets a payoff of 1. If, on the other hand, worker i also chooses to *Resend* the memo, then the boss is angry with both of them, and both get a payoff of -1.

a) Draw the extensive form of this game. Be sure to indicate information sets clearly.

b) What is the strategy space for a worker?

c) Find a Nash equilibrium in pure strategies, or show that none exists.

d) Find all the symmetric perfect Bayesian equilibria.

4. Simone has preferences over consumption bundles in $\mathbb{R}^N_+$. Consider her Walrasian demand correspondence $x(p, w)$, where $p \in \mathbb{R}^N_{++}$ represents the vector of strictly positive prices of the consumption goods, and $w > 0$ denotes Simone's wealth. Are the statements below true or false? For each statement, either prove carefully that it is true or give a counterexample.

a) If Simone's preferences are locally nonsatiated, then $x(p, w)$ satisfies Walras' law (that is, she spends all her money).

b) If Simone's preferences are locally nonsatiated, then $x(p, w)$ is single-valued.

c) If Simone's preferences are strictly convex, then $x(p, w)$ satisfies Walras' law.

d) If Simone's preferences are strictly convex, then $x(p, w)$ is single-valued.