Comprehensive Exam

Macroeconomics

June 3, 2013

You have a total of 180 minutes to complete the exam. If a question seems ambiguous, state why, sharpen it up and answer the sharpened-up question. Good luck!

1 Government Debt and Austerity in an OLG Model

Consider a two period OLG endowment economy in which incomes are given by $w_t^t = w_y > w_o = w_t^{t-1}$ for all t. Households have utility over consumption during the two periods of their life given by $u(c, c') = \log c + \log c'$.

1. Define a competitive equilibrium with one period debt.
2. Characterize your equilibrium from 1. What is the interest rate in equilibrium?
3. Now suppose that the government starts borrowing $\bar{B}$ units of debt in each period and pays the interest using lump sum taxes. Define the competitive equilibrium in this economy.
4. Characterize the new equilibrium. What happens to the interest rate?
5. What is the optimal level of $\bar{B}$?
6. Comment on the effect of an austerity program using this model. That is, what is the welfare effect of eliminating public debt? What change to the model could make this a good policy?

2 Ramsey Taxation

Consider a Ramsey problem in which there are two types of workers, high skill and low skill. Time is infinite, workers have utility over consumption and leisure given by $\log(c_t)+\psi\log(l_t)$ and discount the future at rate β. The production function is constant returns to scale in capital and labor (each skill type enters as a distinct factor). There is a measure of high skill workers μ_g and a measure of low skill workers μ_b. There are no markets open before a worker's skill is realized at $t = 0$ (i.e., no ability to insure against skill risk before birth).

1. Write a social planner's problem for this economy. Characterize the solution when the planner weighs each type by their population measure.
2. Define a TDCE for this economy in which labor income taxes can depend on a worker's type (but capital, consumption, or investment taxes cannot). Could lump sum taxes decentralize the solution to the above planner's problem? Why or why not?
3. Derive an implementability condition (or conditions).
4. Are capital taxes positive in the steady state?
5. Now suppose that political pressures make it impossible to tax labor income progressively, so that the two rates must be the same across skill types. Does this impose additional implementability conditions on the Ramsey problem? If so, then derive them.
6. Now are capital taxes positive in the steady state (hint - your answer should depend on the complementarity of each type of labor will capital)?

3 Cash Constraints

Consider an economy in discrete time where a representative household lives infinitely and maximises the expected net present value of utility discounting future periods at rate $\beta \in (0,1)$. The household is endowed with one unit of time each period and derives utility from consumption c_t and leisure $1-l_t$ according to the per-period utility function:

$$\log c_t + \gamma \log(1-l_t), \quad \gamma > 0.$$

Capital fully depreciates every period, so households' investment equals tomorrow's capital stock. Thus, households choose consumption c_t, leisure $1-l_t$, investment k_{t+1}, the level of nominal government bonds b_{t+1}, and the level of money m_{t+1} that they wish to hold between period t and period $t+1$. The price level is denoted by p_t. Households also pay sales taxes that are proportional to output at rate τ_t. The household's purchases of consumption and investment goods are subject to the cash constraint:

$$(1-l_t)^{\alpha}(c_t + k_{t+1}) \leq \frac{m_t}{p_t}$$

with α such that an equilibrium exists given γ. The production technology is given by

$$y_t = f(k_t) = k_t^{\sigma}, \sigma \in (0,1).$$

At time t, the government issues one-period bonds, B_{t+1} that pay net nominal interest of i_t, when the bonds mature at $t+1$, and new money, $M_{t+1} - M_t$, and levies taxes $T_t = \tau_t f(k_t)$ to finance purchases in the amount g_t. It is convenient to express government purchases as a fraction of output: $s_t^g \equiv \frac{g_t}{f(k_t)}$.

1. Set up the household problem, state the government budget constraint and carefully define a competitive equilibrium.

2. Let μ_t be the Lagrange multiplier associated with the budget constraint and λ_t be the Lagrange multiplier associated with the financing constraint. Derive the first-order necessary conditions and explain the economic intuition behind the first-order conditions for k_{t+1} and m_{t+1}.

3. Private agents take as given the sequence of policy variables $\{x_t, \tau_t, s_t^g\}$, where $x_t \equiv M_{t+1}/M_t > \beta$. Assume the economy is in a stationary equilibrium in dates $t+j, j \geq 1$, but starts from some other position at time t. Specifically, assume that:

   $$x_t = x, \tau_t = \tau, s_t^g = s^g$$

   and

   $$x_{t+j} = x_F, \tau_{t+j} = \tau_F, s_{t+j}^g = s_F^g, \quad j \geq 1$$

   Show that the decision rule for capital is given by:

   $$k_{t+1} = \left[1 - \frac{1 - \beta\sigma\frac{1-\tau_F}{1-s_F^g}}{1 - \frac{\beta\gamma\sigma}{\alpha}\frac{1-\tau_F}{1-s_F^g}}\right](1-s^g)f(k_t) \quad \forall t \geq 0$$

   Hint: Note that at time $t+j$ the economy is at the stationary equilibrium for all $j \geq 1$. Combine the government and household budget constraints with the FOCs to obtain a first-order difference equation in c_t which you can solve by guess & verify. In this log utility, full depreciation model, guess that c_t is a linear function in output.

4. Derive how k_{t+1} depends on future taxes and government spending shares, (τ_F, s_F^g), holding all other current policies fixed. Explain your results, especially the result that non-monetary policies have monetary effects. (Hint: Why is money non-neutral in the first place?)

5. Show that part of the equilibrium is a decision rule for leisure that solves the difference equation:

$$(1-l_t)^\alpha \left(\frac{c_t + k_{t+1}}{c_t} - \frac{\gamma}{\alpha}\right) = \frac{\beta}{x_t}\left[(1-l_{t+1})^\alpha \left(\frac{c_{t+1}+k_{t+2}}{c_{t+1}} - \frac{\gamma}{\alpha}\right) + \frac{\gamma}{\alpha}\right]$$

Hint: Use the financing constraint in to substitute out $(1-l_t)^\alpha$ in the expression you have found for λ_t. Then think how you can use other FOCs to express $R_t^m = p_t$ in terms of consumption and p_{t+1} capital.

6. Solve the expression in the previous subquestion and use it together with the capital rule to characterise how the price level p_t depends on the future monetary and fiscal policy variables, (x_F, τ_F, s_F^g), holding current policies fixed. Explain your result and state any transversality condition you might use.

4 Hiring and Firing Costs

Consider a monopolistic firm that faces the following stochastic demand function

$$Y_t = Z_t P_t^{-1/\mu}, \quad \mu \in (0,1).$$

Z_t is a demand shifter that follows a geometric Brownian motion $dZ_t = \theta Z_t dt + \sigma Z_t dW_t$ where dW_t is an increment in a Wiener process. Both $L_0, Z_0 \in (0,\infty)$ are given. The production technology is linear in labour: $Y_t = L_t$.

For each employed unit L_t, the monopolist has to pay flow cost w. If he wants to hire additional workers, he has to pay a hiring cost c_h per newly hired worker; similarly, the monopolist has to pay a firing cost of c_f per fired worker. Additionally, workers quit at exogenous rate δ; obviously, the monopolist does not occur any costs for quitting workers.

1. What is the firm's revenue function $R_t(Z_t, L_t)$? Formulate the firm's dynamic problem assuming that the firm discounts future revenues at constant rate r.

2. State the monopolist's value function $V(Z_t, L_t)$ with demand level Z_t and employment L_t. Without a formal proof explain what assumption guarantees that the value function is concave. Show that $V(Z_t, L_t)$ can be written as $v(\ell_t)Z_t$ where $\ell \equiv L_t/Z_t$.

3. Write down the asset equation characterising the solution to the firm's problem. Use Itō's Lemma for two variables to replace any terms involving expectations. Then, derive an asset equation for $v(\ell_t)$.

   Hint: Itō's Lemma for two-dimensional processes with two Brownian motions

$$dx = \mu_x dt + \sigma_x dW$$
$$dz = \mu_z dt + \sigma_z dW.$$

The diffusion process $y = f(x, z)$ where $f(\cdot)$ is a function that is twice differentiable with respect to x and z. Then,

$$dy = \left[f_x(x,z)\mu_x + f_z(x,z)\mu_z + \frac{1}{2} f_{xx}(x,z)(\sigma_x)^2 + \frac{1}{2} f_{zz}(x,z)(\sigma_z)^2 + f_{xz}(x,z)\sigma_x\sigma_z \right] dt$$
$$+ \left[f_x(x,z)\sigma_x + f_z(x,z)\sigma_z \right] dW$$

4. Argue that the monopolist will not allow ℓ to get outside bounds (ℓ_*, ℓ^*). Derive first-order conditions that pin down ℓ_* and ℓ^*. Write down a differential equation which $v(\ell)$ needs to satisfy $\forall \ell \in [\ell_*, \ell^*]$.

5. Solve the differential equation in the previous subquestion. (Hint: For the homogeneous solution guess $v^h = \ell^\beta$, for the particular solution guess $v^p = m_1 \ell^{1-\mu} + m_2 \ell$. Make sure to compute β, m_1 and m_2.)

6. How many unknowns do you have remaining? For each of them write down a condition that one would use to determine these unknowns. Do not solve these conditions, just write them down. Given the information you have about $v(\cdot)$ at this point, draw a schematic diagram of $v(\ell)$.

7. Discuss how the presence of hiring and firing costs in this model affects the dynamics of the firm's labour force.

8. Next, consider a time t at which the firm is hiring. Let d be the the time span between t and when the firm starts firing workers. Argue that d is a stoppage time. Express the value function of the firm in two terms; one involving the expected discounted revenues between t and $t + d$ and the other one being the expected discounted continuation value in $t + d$. What is the marginal value of labour in t? Substitute the optimality conditions to derive an expression that related d to c_h and c_f and interpret the condition.

9. What happens to the expected duration of a job if the hiring cost increase? Explain.

10. What happens to the expected duration of a job if the firing cost increase? Explain.