MICROECONOMICS COMPREHENSIVE EXAM

JUNE 2013

Instructions:

(1) Please answer each of the four questions on **separate** pieces of paper.
(2) Please write only on **one side** of a sheet of paper
(3) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.).

1. Consider a two-good economy and a consumer whose utility of consumption of a bundle (x_1, x_2) (where x_i, $i \in \{1, 2\}$ denotes the quantity of good i) is given by a quasi-linear function

$$u(x_1, x_2) = x_1 + \phi(x_2).$$

Here good 1 is the numeraire commodity, its price is normalized to one; $\phi : \mathbb{R}_+ \to \mathbb{R}$ satisfies the Inada conditions. Namely, ϕ is increasing, strictly concave, continuously differentiable, $\phi(0) = 0$, $\lim_{x \downarrow 0} \phi'(x) = +\infty$, and $\lim_{x \to +\infty} \phi'(x) = 0$. Let $p > 0$ be the price of the second good, $w > 0$ be the consumer's wealth.

(a) Let the consumption set be $X = \mathbb{R} \times \mathbb{R}_+$, where $x_1 < 0$ can be interpreted as borrowing of the numeraire good. Carefully write down the utility maximization problem for the consumer. Carefully indicate which of the constraints bind and which do not bind and explain why. Derive the consumer's Walrasian demand functions. Show that the demand function for the second good is independent of the consumer's wealth.

(b) Demonstrate that the consumer's indirect utility function is quasi-linear in wealth, i.e., it can be represented as

$$V(p, w) = w + \psi(p)$$

for some function ψ.

(c) Now suppose that the consumption set is $X = \mathbb{R}^2_+$ (i.e., borrowing is prohibited). Derive the consumer's Walrasian demand functions. How does the consumer's demand for each good vary with w? When does the non-negativity constraint on the numeraire good become relevant?

2. Consider an investor with the initial wealth normalized to one, who contemplates investing a fraction α of the wealth in a risky asset and the remaining fraction $1 - \alpha$ in a riskless asset. Observe, that there is no restriction on the value of α: $\alpha > 0$ means that the investor is buying the risky asset; $\alpha < 0$ indicates that the investor is short-selling (or borrowing) the risky asset. If $\alpha = 0$, the investor does not trade the risky asset. Values $\alpha < 1$, $\alpha > 1$, and $\alpha = 1$ can be interpreted in the same manner in terms of trading in the riskless asset. The rate of return on the riskless asset is $r > 0$ (in other words, each \$1 invested into the riskless asset generates $\$(1+r)$). The rate of return on the risky asset is a random variable $\rho \in \{\rho_0, \rho_0 + \Delta\rho\}$, where $\rho_0 < r < \rho_0 + \Delta\rho$. Assume that the investor is strictly risk averse. Her Bernoulli utility function u is such that $u' > 0$, and $u'' < 0$. Let w denote the future wealth.

(a) Let the distribution of ρ be given by a number $\pi \in (0, 1)$, so that $\text{prob}(\rho = \rho_0 + \Delta\rho) = \pi$, and $\text{prob}(\rho = \rho_0) = 1 - \pi$. Assume that the investor chooses α so as to maximize the expected utility of wealth:

$$\max_{\alpha} E^{\pi}\left[u(w)\right],$$

where E^{π} denotes the expectations operator under the probability measure defined by π. Given r, ρ_0, and π, specify restrictions on $\Delta\rho$ which are consistent with the choice (i) $\alpha > 0$; (ii) $\alpha < 0$; and (iii) $\alpha = 0$.

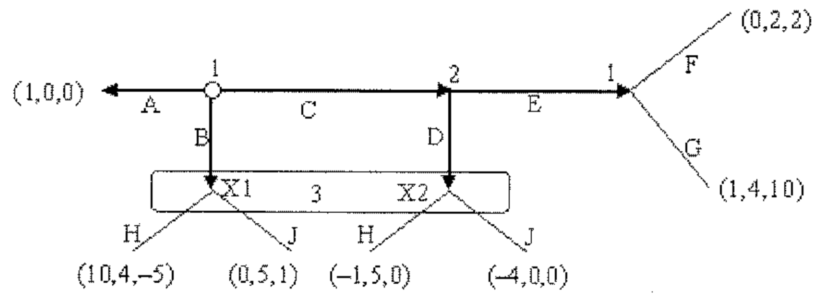
(b) Now, suppose that the investor is not sure about the value of π, which defines the distribution of ρ. Namely, assume that the investor knows only that $\pi \in [\underline{\pi}, \bar{\pi}] \subset [0, 1]$. The investor has the Gilboa-Schmeidler maximin expected utility, that is, the investor chooses α so as to

$$\max_{\alpha} \min_{\pi \in [\underline{\pi}, \bar{\pi}]} E^{\pi}\left[u(w)\right].$$

Given r, $\rho_0, \underline{\pi}$, and $\bar{\pi}$, specify restrictions on $\Delta\rho$ which are consistent with the choice (i) $\alpha > 0$; (ii) $\alpha < 0$; and (iii) $\alpha = 0$.

(c) How does the ambiguity over the distribution of ρ affect the investor's participation in the market for the risky asset?

3. Consider the following extensive form game, where the initial node is marked with an open circle:



a) Find a pure strategy Nash equilibrium that is not subgame perfect.

b) Find a pure strategy subgame perfect equilibrium that is not part of a perfect Bayesian equilibrium.

c) Find all of the perfect Bayesian equilibria, including those in mixed strategies.

4. Consider a sealed-bid auction with the following rules: the item is awarded to the highest bidder. (Ties are broken uniformly.) The price that the winner pays is equal to the constant $a > 0$ times the amount of the second-highest bid. There are $N > 1$ bidders. Values are independent and are distributed uniformly on $[0, 1]$. Each bidder i observes only his own valuation v_i. Bidders may bid any nonnegative real number.

a) Describe the strategy space for a bidder.

b) Derive a symmetric pure strategy Bayes-Nash equilibrium of this auction. (HINT: look for an equilibrium in which bidder i's bid $B(v_i)$ is a linear function of his valuation, so that $B(v_i) = bv_i$ for some scalar b.)

c) Argue that the bidding function that you derived in part c is a weakly dominant strategy.

d) Calculate the seller's expected revenue from the equilibrium that you derived in part c, and compare it to the expected revenue from a standard second-price auction.