Comprehensive Exam

Macro
Spring 2014 – Retake

August 22, 2014

You have a total of 180 minutes to complete the exam. If a question seems ambiguous, state why, sharpen it up and answer the sharpened-up question. Good luck!

1 Debt and Taxation With Incomplete Markets, Take Two

Consider an economy with a continuum of households, named $i \in [0,1]$. A household's income each period is *independent from all other households* and is given by $y_{i,t} \in \{y_\ell, y_h\}$, with $y_h \geq y_\ell$. Income is Markovian, so that:

$$\text{Prob}(y_{i,t+1} = y_h | y_{i,t} = y_h) = \pi_{hh} \tag{1}$$
$$\text{Prob}(y_{i,t+1} = y_h | y_{i,t} = y_\ell) = \pi_{lh} \tag{2}$$

The household has a CRRA period-utility function:

$$u(c_t) = \frac{1}{1-\sigma} c_t^{1-\sigma}$$

The household has access to only a risk-free asset and is subject to a borrowing constraint that says:

$$a_{i,t+1} \geq 0$$

The market interest rate is r_t and households discount the future at rate β.

1. Write the household's value function assuming that the equilibrium is stationary (i.e., the interest rate is constant).

2. Define a competitive equilibrium (make sure you give the condition which determines the interest rate). Your equilibrium objects should be recursive functions!

3. Solve explicitly for the equilibrium consumption functions and interest rate.

4. Now consider an equilibrium where each period the government borrows D, repays the gross interest $(1+r)D$ (i.e., they lump-sum tax households by rD in order to pay the net interest). Show that there is some level of D which increases ex-anted expected welfare.

2 Tax Distorted Competitive Equilibria

Consider an infinitely lived representative agent economy with preferences:

$$U = \frac{1}{1-\beta} \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$$

where c_t is consumption and ℓ_t is leisure. Each household has k_0 units of capital to begin and can accumulate subject to $k_{t+1} \leq x_t$, where x_t is the amount invested in period t. The aggregate feasibility constraint is therefore:

$$c_t + x_t \leq F(k_t, 1 - \ell_t)$$

where we have assumed that the endowment of leisure is just one. Assume that the government taxes all income at rate τ, period by period, and then lump-sum rebates it. There are no other taxes and no government spending.

1. Carefully define a TDCE for this economy with this fiscal policy.
2. Suppose that $u(c_t, \ell_t) = \nu(c_t)$, so that leisure is not valued. Show that you can find the equilibrium allocation and prices for the TDCE by solving an altered planner's problem, where the planner takes the fiscal policy as given. (Hint: the planner has a different period utility function than the household).
3. How would your answer change if leisure was valued by the household?
4. What would the planner do if he was allowed to choose τ as well?

3 News Shocks

Let the representative agent maximize

$$E_t \sum_{l=0}^{\infty} \beta^l [\log(C_{t+l} - bC_{t+l-1}) + \psi \log(1 - h_{t+l})]. \tag{3}$$

Here h_t is hours worked, C_t is consumption at time t. The parameter $b \in [0, 1)$ controls the amount of internal habit persistence. The maximisation is subject to the resource constraint and the capital accumulation technology given by

$$\begin{aligned} C_t + I_t \leq Y_t &= A_t K_t^{\alpha} (h_t)^{1-\alpha} \\ I_t &= K_{t+1} - (1 - \delta) K_t \end{aligned}$$

Suppose we have news about future technology. To model that, let us introduce a shock that captures exogenous news about future technology. From the perspective of the information set at time t, technology is expected to follow the stochastic process given by

$$\log(A_{t+1}) = \rho \log(A_t) + e_{t+1} + x_t \tag{4}$$

where the contemporaneous technology shock e_{t+1} is an i.i.d random variable:

$$e_{t+1} \sim N(0, \sigma_e^2)$$

and x_t is news at time t about one period ahead technology. News is an i.i.d random variable:

$$x_t \sim N(0, \sigma_x^2)$$

Thus, the news shock realised at time t affects the expected mean of technology at time $t + 1$:

$$E_t \log(A_{t+1}) = \rho \log(A_t) + x_t$$

Also suppose that process in (4) is highly persistent (for example $\rho = 0.9$). Also normalize $E(A_t) = A = 1$.

1. Derive the FOC wrt h_t and wrt C_t.
2. **Pure news shocks.** For this subquestion assume there is no habit persistence ($b = 0$). Suppose that at time t there is a negative 'news' shock: $x_t = -\overline{x} < 0$.
   (a) What is the qualitative response of the endogenous variables (C_t, h_t, I_t, Y_t) to the shock x_t? You only need to provide intuition.
   (b) Can the model generate a recession today (a drop in Y_t) following negative news about the future? Explain the economics behind your result.
3. **News and habit persistence.** For this subquestion assume there is internal habit persistence, i.e. $b > 0$.

(a) Use the FOCs wrt h_t and C_t to argue that when $b = 0$ labor and consumption would move in opposite directions when a news shock about future technology (irrespective of its sign) hits.

(b) Argue that when $b > 0$, the logic in (a) is broken and that labor and consumption can potentially move in the same direction.

4. **News and investment adjustment costs.** Suppose now that the representative agent still maximises the expected utility with internal habit formation given by (3) but is faced with a different capital accumulation technology:

$$K_{t+1} = (1 - \delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{5}$$

where I_t is period t investment. The function S reflects adjustment costs in investment. The function S is increasing and convex, with steady state values of $S(1) = S'(1) = 0$.

(a) Give an economic justification for the capital accumulation technology in (5).

(b) Derive the FOC wrt I_t. What is the main qualitative difference in the FOC compared to the case without investment adjustment costs?

(c) Take a negative news shock as in (2). Use the FOC and the qualitative difference found in point (b) to argue about the equilibrium forces that drive I_t up or down. (*Hint*: There is no need to linearize the FOC. Simply argue about equilibrium forces that affect the optimal choice of I_t.)

(d) Argue whether the model with internal habit formation and investment adjustment costs can generate contemporaneous positive comovement between Y_t, C_t and I_t (by positive comovement here we mean that Y_t, C_t and I_t move on impact in the same direction).