MACROECONOMICS

Prelim Exam

Austin, Aug 20, 2012

Instructions

• This is a closed book exam. If you get stuck in one section move to the next one. Do not waste time on sections that you find hard to solve.

• Show all your work! Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.

• Please hand in the exam promptly at the end of the time allotted. You will have 10 minutes additional minutes at the beginning of the exam so that you can read the text of the questions before the clock starts ticking.

• Each question is worth 25 points. The point total for each section of a question is indicated at the beginning of the section. Look at these "relative prices" when deciding how to allocate your time!!

• If you believe that a question is wrong or poorly worded, please make (and explain) the "minimal" necessary changes to make it "well-posed". Of course, unnecessary changes on a question could result in a lower grade.

• Good luck!

1 Problem 1 (Fiscal Policy and Interest Rates) - 25 points

Households have preferences over consumption, c_t, and leisure, l_t, given by,

$$\mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty}\beta^t u\left(c_t, l_t\right)\right],\ 0<\beta<1,$$

where,

$$u\left(c_t, l_t\right)=\frac{\left(c_t l_t^{\alpha}\right)^{1-\theta}}{1-\theta},\ \alpha>0, \theta>0.$$

The economy's feasibility constraint is given by,

$$\begin{aligned} c_t &\leq y_t = n_t s_t, \\ n_t + l_t &\leq 1, \end{aligned}$$

where s_t is a random productivity shock, y_t is output and n_t is labor. Let the stochastic process for s_t satisfy,

$$s_{t+1}=\left(s^*\right)^{1-\phi} s_t^{\phi}\epsilon_{t+1},$$

where ϵ_t is a sequence of i.i.d. random variables with mean one and constant variance. Households work for competitive firms that operate the linear technology described above. There is a government that taxes labor income and uses the proceeds to make transfers to the representative household. Thus, if wages at time t are w_t, a household receives an after tax wage of $\left(1-\tau_t\right) w_t$ and a transfer equal to T_t. Households take the sequence of tax rates τ_t and transfers T_t as given. At time t, households can trade on one-period government bonds b_{t+1}, so the household budget constraint can be written as,

$$s.t.\ c_t + p_t b_{t+1} \leq \left(1-\tau_t\right) w_t n_t + T_t + b_t.$$

The government budget constraint is given by,

$$p_t B_{t+1} + \tau_t w_t n_t = T_t + B_t,$$

where B_{t+1} is the stock of one-period bonds issued at time t. We assume that the government bonds are in zero-net supply, so $B_{t+1}=0$. Note that by arbitrage the price of a bond p_t corresponds to the inverse of the one-period interest rate $R_t=\frac{1}{p_t}$. There is no government expenditure.

[Hint: The important properties of the log-normal distribution $\epsilon_t \sim Log-N\left(\mu, \sigma^2\right)$ (or $\ln\epsilon_t \sim N\left(\mu, \sigma^2\right)$) for this problem are that $\mathbb{E}_t\left(\epsilon_t\right)=e^{\mu+\frac{\sigma^2}{2}}$ and that $\epsilon_t^a \sim Log-N\left(a\mu, a^2\sigma^2\right)$ for $a \neq 0$.]

1. (10 points) Define a competitive equilibrium (CE) in which, in each period, individuals trade one-period bonds, consumption and labor.

2. (5 points) Derive the first-order conditions for the problem of the firm and the household.

3. (5 points) Show that the analytic expression for equilibrium employment, n_t, is given by $n_t=\frac{(1-\tau_t)}{(1-\tau_t)+\alpha}$.

4. (5 points) Assume $\epsilon_t \sim Log-N\left(-\frac{\sigma^2}{2}, \sigma^2\right)$ (or $\ln\epsilon_t \sim N\left(-\frac{\sigma^2}{2}, \sigma^2\right)$) in order to match the problem's assumption that ϵ_t is a sequence of i.i.d. random variables with mean one and constant variance. Note that $-1<\phi<1$, otherwise the process for s_t is non-stationary. Assume that $\tau_t=\tau$ for all t and show that the implied interest rate on one-period government bonds R_t decreases with risk σ^2, but it is completely unaffected by the tax rate τ.

Note: You can just assume that the parameters are such that all quantities make sense: that is, $0<n_t<1$ and $0<\tau_t<1$. You do not need to verify these inequalities, simply assume that they are satisfied.

2 Problem 2 (Dynamic Programming in the Growth Model) - 25 points

Consider an economy with a representative consumer who lives for T periods. There is a good in each period that can be consumed or saved as capital. There is also labor. The consumer's utility function is given by:

$$\sum_{t=0}^{T} \beta^t log(c_t)$$

Where $0 < \beta < 1$. The consumer is endowed with one unit of labor in each period and k_0 units of capital at $t = 0$. The depreciation rate is 100% and output is given by the function $y_t = \theta k_t^\alpha n_t^{1-\alpha}$, where n_t is hours of market work.

a. 5pt Assume that $T < \infty$ and solve for the optimal savings and labor supply policy for a given period t.

b. 5pt Now assume that $T = \infty$. Write the Bellman's equation and use it to solve for the stationary policy rules for savings and labor supply.

c. 2pt Take the limit of your answer in part a. as $T \to \infty$. Does it approach the answer from part b. ?

d. 5pt Define an Arrow-Debreu equilibrium for this economy with $T = \infty$ and time zero trading. Use your answer from part b. to calculate the Arrow-Debreu equilibrium.

e. 5pt Now suppose that there are two types of consumers in the economy, indexed by $i = 1, 2$. They are identical except that type i has a period-utility function given by $\log(c_t - \underline{c}^i)$ and their initial endowments of capital differ. Define an Arrow-Debreu equilibrium for this economy.

f. 3pt Can you aggregate the economy in part e. to a representative consumer? Show how or why you can't do so.

3 Problem 3 (Ramsey Taxation with Human Capital Accumulation) - 25 points

Take the utility function from question 1 part b. For production, add the following tweak: instead of just physical capital, there is now human capital with initial stock h_0. Assume that that capital now depreciates at some rate δ_k, not necessarily equal to 1, and human capital at rate $\delta_h \geq k$. Output is now generated by the function $y_t = \theta k_t^\alpha (h_t n_t)^{1-\alpha}$. The household owns the capital stock, which firms rent at price r_t. Firms hire workers in spot markets and pay them w_t per hour. The government has to finance an exogenous sequence of spending given by g_t at a period in time and can only use taxes on capital and labor income to finance it, τ_{kt} and τ_{nt}.

a. 5pt Define a Tax Distorted Competitive Equilibrium for this economy.

b. 10pt What is the Ramsey problem for this economy? In particular, carefully derive and explain the implementability constraint.

c. 5pt Assume that $\delta_k = \delta_h$. What is the relationship between the optimal tax rates τ_{kt} and τ_{nt}?

d. 5pt If $\delta_k = \delta_h$ then what can your say about $\lim_{t\to\infty} \tau_{kt}$? What does that mean for the limiting value of τ_{nt}?