MACROECONOMICS

Prelim Exam

Austin, June 1, 2012

Instructions

- This is a closed book exam. If you get stuck in one section move to the next one. Do not waste time on sections that you find hard to solve.
- Show all your work! Partial credit will be awarded if it is clear that you were approaching the problem in an essentially correct manner.
- Please hand in the exam promptly at the end of the time allotted. You will have 10 minutes additional minutes at the beginning of the exam so that you can read the text of the questions before the clock starts ticking.
- Each question is worth 25 points. The point total for each section of a question is indicated at the beginning of the section. Look at these "relative prices" when deciding how to allocate your time!!
- If you believe that a question is wrong or poorly worded, please make (and explain) the "minimal" necessary changes to make it "well-posed". Of course, unnecessary changes on a question could result in a lower grade.
- Please remember to put your name in your exam so we know who deserves credit for the work done.
- Good luck!

1 Problem 1 (Fiscal Policy and Interest Rates) - 25 points

Households have preferences over consumption, c_t, and leisure, l_t, given by,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t,l_t\right)\right],\ 0<\beta<1,$$

where,

$$u\left(c_t,l_t\right)=\frac{\left(c_t l_t^{\alpha}\right)^{1-\theta}}{1-\theta},\ \alpha>0,\theta>0.$$

The economy's feasibility constraint is given by,

$$\begin{aligned} c_t &\leq y_t = n_t s_t, \\ n_t + l_t &\leq 1, \end{aligned}$$

where s_t is a random productivity shock, y_t is output and n_t is labor. Let the stochastic process for s_t satisfy,

$$s_{t+1}=\left(s^*\right)^{1-\phi}s_t^{\phi}\epsilon_{t+1},$$

where ϵ_t is a sequence of i.i.d. random variables with mean one and constant variance. Households work for competitive firms that operates the linear technology described above. There is a government that taxes labor income and uses the proceeds to make transfers to the representative household. Thus, if wages at time t are w_t, a household receives an after tax wage of $\left(1-\tau_t\right)w_t$ and a transfer equal to Tr_t. Households take the sequence of tax rates τ_t and transfers Tr_t as given. At time t, households can trade on one-period government bonds b_{t+1}, so the household budget constraint can be written as,

$$s.t.\ c_t+p_t b_{t+1}\leq\left(1-\tau_t\right)\varpi_t n_t+Tr_t+b_t.$$

The government budget constraint is given by,

$$p_t B_{t+1}+\tau_t w_t n_t=Tr_t+B_t,$$

where B_{t+1} is the stock of one-period bonds issued at time t. We assume that the government bonds are in zero-net supply, so $B_{t+1}=0$. Note that by arbitrage the price of a bond p_t corresponds to the inverse of the one-period interest rate $R_t=\frac{1}{p_t}$. Government expenditure is set at zero.

[Hint: The important properties of the log-normal distribution $\epsilon_t\sim Log-N\left(\mu,\sigma^2\right)$ (or $\ln\epsilon_t\sim N\left(\mu,\sigma^2\right)$) for this problem are that $\mathbb{E}_t\left(\epsilon_t\right)=e^{\mu+\frac{\sigma^2}{2}}$ and that $\epsilon_t^a\sim Log-N\left(a\mu,a^2\sigma^2\right)$ for $a\neq 0$.]

1. (10 points) Define a competitive equilibrium (CE) in which, in each period, individuals trade one-period bonds, two-period bonds, and labor.
2. (5 points) Derive the first-order conditions for the problem of the firm and the household.
3. (5 points) Show that the analytic expression for equilibrium employment, n_t, is given by $n_t=\frac{\left(1-\tau_t\right)}{\left(1-\tau_t\right)+\alpha}$.
4. (5 points) Assume $\epsilon_t\sim Log-N\left(-\frac{\sigma^2}{2},\sigma^2\right)$ (or $\ln\epsilon_t\sim N\left(-\frac{\sigma^2}{2},\sigma^2\right)$) in order to match the problem's assumption that ϵ_t is a sequence of i.i.d. random variables with mean one and constant variance. Note that $-1<\phi<1$, otherwise the process for s_t is non-stationary. Assume that $\tau_t=\tau$ for all t and show that the implied interest rate on one-period government bonds R_t decreases with risk σ^2, but it is completely unaffected by the tax rate τ.

Note: You can just assume that the parameters are such that all quantities make sense: that is, $0<n_t<1$ and $0<\tau_t<1$. You do not need to verify these inequalities, simply assume that they are satisfied.

2 Problem 2 (Cash-in-Advance and Money Transfers) - 25 points

Let us assume an infinitely-lived, representative household with a per-period endowment e_t. There are two types of consumption goods in this economy: "cash-goods" c_{1t} and "credit-goods" c_{2t}. Households can trade in zero-net supply bonds b_{t+1}, and receive a money transfer that is proportional (instead of lump-sum) in the amount of $\tau_t m_t$. Thus, the representative household solves the following optimization problem,

$$
\begin{aligned}
&\max_{\{c_{1t},c_{2t},b_{t+1},m_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u\left(c_{1t}, c_{2t}\right), \\
&s.t.\ p_t c_{1t} + p_t c_{2t} = p_t e_t + \left(1+\tau_t\right) m_t - m_{t+1} + p_t R_t b_t - p_t b_{t+1}, \\
&p_t c_{1t} \leq \left(1+\tau_t\right) m_t + p_t R_t b_t - p_t b_{t+1}, \\
&m_0 > 0, b_0 = 0 \text{ given.}
\end{aligned}
$$

The individual endowment sequence $\{e_t\}_{t=0}^{\infty}$ and the aggregate money supply sequence $\{M_t\}_{t=1}^{\infty}$ are exogenously given. There is a government who decides the transfer rate sequence $\{\tau_t\}_{t=0}^{\infty}$ and the aggregate bond sequence $\{B_t\}_{t=1}^{\infty}$ to balance its budget every period. Assume government bonds are in zero-net supply so that $B_t = 0$ for all $t \geq 1$.

1. (5 points) Re-write the representative household's problem in recursive form with the corresponding Bellman's equation. Set-up the government's consolidated budget constraint.

2. (10 points) Define a recursive competitive equilibrium (RCE) for this economy.

3. (5 points) Derive the FOCs, Envelope conditions and Euler equations corresponding to this economy. Show that the restriction on an equilibrium price sequence $\{p_t, R_t\}_{t=0}^{\infty}$ that ensures the Lagrange multiplier on the cash-in-advance constraint is non-negative can be expressed as,

$$R'\pi \geq \left(1+\tau'\right).$$

Notice that if this Lagrange multiplier is strictly positive, then the cash-in-advance constraint is always binding. [Hint: you may want to define gross inflation as $\pi = \frac{p'}{p}$].

4. (5 points) Assume that the per-period endowment $e_t = e$ is constant and the money supply follows this exogenously given rule,

$$M' = \mu M, \mu \geq 1,$$

where μ is the growth rate of the money supply (the key policy parameter). Let us look at the steady state where $\frac{M'}{p'} = \frac{M}{p} \Rightarrow \pi = \frac{p'}{p} = \frac{M'}{M} = \mu$ and $c_2' = c_2$. We know that the inequality with **lump-sum money transfers** is $\pi \geq \beta$ (or simply $\mu \geq \beta$) Hence, Friedman's rule in that case merely sets $\mu = \beta$ to ensure that the Lagrange multiplier is zero and that the cash-in-advance distortion is neutralized. Show that with the **proportional money transfers** postulated by the problem, the constraint does not depend on the policy parameter μ and, moreover, show that under the standard assumption that $0 < \beta < 1$ it is not possible to eliminate the cash-in-advance distortion (that is, the Lagrange multiplier is always going to be strictly positive).

3 Problem 3 (Consumption and Commitment) - 25 points

This question is designed to make you think about insurance and consumption smoothing. The economy is familiar. There is a measure 1/2 of agents who live forever and receive endowment stream $\omega_{1,t} = 1+\epsilon(-1)^{t+2}$ for $t = 0, 1...$ and an equal measure who receive $\omega_{2,t} = 1 - \epsilon(-1)^{t+1}$. They share the same preferences over sequences of consumption given by $\sum_{t=0}^{\infty} \beta^t u(c_{i,t})$ where $u'(c) > 0, u''(c) < 0$.

1. 2pts Define a competitive equilibrium with time zero trading. Characterize it and derive the correlation between income and consumption.
2. 2pts Characterize the set of pareto efficient allocations. Show that the CE allocation from [1] is in the set of PO allocations and that any PO allocation can be decentralized as a CE using lump sum transfers.
3. 7pts Now imagine a planner who wishes to maximize the equally weighted utility of each agent, but cannot force them to remain in the formal economy. That is, at any point in time, either agent can choose between the planner's sequence of consumption or leave immediately and live in Autarky, thereby receiving utility $V_{i,t}^{AUT} = \sum_{j=0}^{\infty} \beta^j u(\omega_{i,t+j})$. Write the planner's problem with the additional constraint that each agent must willingly choose the planner's allocation over autarky at every point in time.
4. 4pts Show that your solution to [2] (with equal weights) cannot be a solution to [3]. When would agents wish to take the Autarkic consumption sequence over the planner's?
5. 3pts The esteemed neo-classical economist Piston Honda says that this is a model of default. Use your insight from part [4] to critique his statement.
6. 4pts Characterize the solution to [3]. (Hint: Guess that consumption oscillates between two values). Now what is the correlation between income and consumption?
7. 3pts The esteemed behavioral economist Soda Popinksi observes data from [6] and claims that it means individuals lack foresight since they fail to smooth consumption in the face of forecastable income fluctuations. Do you agree with his conclusion?

4 Short Answer Questions (15 pts, 5 each)

1. Show that the utility function over consumption and hours, $u(c) = \nu(c) - \phi h^{\epsilon}$ allows for balanced growth if and only if $\nu(c) = \log(c)$.

2. In the Ramsey problem of optimal taxation, why do we rule out lump sum taxes as a potential fiscal instrument?

3. Let B and β be parameters and $u(x)$ be an increasing, strictly concave function. State conditions and show that the functional operator $Tf(x) = \max\{x + \beta f(x), B\}$ has a fixed point in the space of continuous and bounded functions. Is it in the space of concave functions?