Comprehensive Exam

Macro
Spring 2014

June 6, 2014

You have a total of 180 minutes to complete the exam. If a question seems ambiguous, state why, sharpen it up and answer the sharpened-up question. Good luck!

1 Merry Cows, Mad Cows, Moody Cows and Mesmerising Milk

Consider an economy where the representative agent is infinitely lived and has utility according to

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\theta}}{1-\theta} \qquad \beta < 1, \theta > 0.$$

The only endowment in the economy is a cow which produces D_t units of milk in every period t, $D_0 > 0$ given. This cow is owned by a representative agent who can trade Δ_t amount of shares of the cow in a market at price P_t. Think of shares as ownership rights that entitle the owner to his share in the cow's milk. Milk is not storable and it's the only type of resource in the economy (so everything is denominated in units of milk).

1. Suppose initially that the cow's milk yield grows as follows: $D_t/D_{t-1} = g > 1 \ \forall t$. State the problem of the representative agent. Are there any parameter constraints required for a well-defined equilibrium?

2. Solve for the price of a cow, P_t, and explain how it relates to β and θ.

Suppose that there is a constant probability λ per period that the cow contracts mad cow disease and dies (which means no more milk from that point on). As long as the cow survives, its milk output grows at a constant rate g.

3. Solve for the price of a cow and compare it to the one above. Then compute the one-period *ex post* return, i.e. the return of the cow conditional that it did not contract mad cow disease

$$R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t}.$$

   Explain how the risk of mad cow disease affects the return on owning shares of the cow compared to the economy without mad cow disease.

4. What is the "natural rate" in this economy? Call this rate R^f and compute the equity premium R/R^f. How does the equity premium depend on risk (λ) and agents' treatment of risk (β, θ) and why?

5. Consider an econometrician who knows g but is unaware of mad cow disease, i.e. who assumed $\lambda = 0$. If he attempts to infer the risk aversion of the agents, θ, from a time series of the stock returns, would he think the agent is more or less risk-averse than the agent is in reality? Could this observation be used to resolve the equity premium and volatility puzzles and if so how?

From now on assume there is no more mad cow disease risk, but assume the cow has mood swings so that the growth rate of milk is

$$\frac{D_t}{D_{t-1}} \equiv g_t = \begin{cases} g_h & \text{with probability } \pi \\ g_l & \text{with probability } 1-\pi \end{cases} \qquad \text{where } g_h > g_l > 1 \text{ and } \pi \in (0,1).$$

Also, the milk is slightly addictive. Whenever milk growth is low ($g_t = g_l$), the agent is forced to "go cold turkey" which causes a utility loss $\chi / C_t^{1+\theta}$ in that period. His preferences are now described as

$$\sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\theta}}{1-\theta} - \frac{\chi}{C_t^{1+\theta}} \mathbf{I}\left\{g_t = g_l\right\} \right].$$

6. Illustrate graphically a typical realisation for $\log(D_t)$ over time. Starting from the agent's first-order condition, solve for P_t in this economy assuming that milk growth was high in period t, i.e. $g_t = g_h$.
Hint: Milk growth is iid over time, so $E_t\left[\beta^\tau \prod_{i=1}^{\tau} g_{t+i}^z\right] = \left[\beta\pi g_h^z + \beta(1-\pi)g_l^z\right]^\tau \;\; \forall z \in \mathbf{R}$.

7. Consider an econometrician who knows the dynamics of g_t but is unaware of the addictive nature of milk, i.e. he assumes $\chi = 0$. If he attempts to infer the risk aversion of the agents, θ, from a time series of cow share prices, would he think the agent is more or less risk-averse than the agent is in reality? Could this observation be used to resolve the equity premium and volatility puzzles and if so how?

2 Dynamic Programming

Congratulations! News of your expertise at dynamic programming has spread to Hollywood and famed producer Cecil D. S. Mills would like to hire you. In particular, he would like to know how often his studio should reboot film franchises (rebooting is when they take an existing movie, recast it, and tell the story from the beginning... like they did with Batman and Spiderman recently).

He doesn't have the details right now, but tells you that his team of cinema-metricians have modeled the revenue from a movie as (this revenue is only received once, when the movie is released, and the movie can only be rebooted once):

$$R_t = \bar{R} - \gamma^t \Gamma$$

For $t \geq 1$, where R_t is the revenue after t years have passed since the last version of the movie was made, $\bar{R}$ is the revenue from the previous edition of the movie, γ is a parameter in $(0, 1)$, and $\Gamma > 0$.

1. Show that the revenue process can be written recursively (hint: a specific R_1 would have to be given to make sure that the time series of revenues are the same as the original formation).

2. Write this as a dynamic program assuming that Mr. Mills discounts the future using a gross risk free interest rate of R. Make sure to tell him the state variables, he is a stickler for that kind of thing.

3. He is skeptical, so would like you to prove that the functional equation you wrote above exists and is unique.

4. He now wants to know a formula for how long to wait between reboots and how the waiting time depends on the initial revenue, $\bar{R}$.

3 Debt and Taxation With Incomplete Markets

Consider an economy with a large number of households. A household's income each period is normally distributed:

$$y_t \sim N(1, \sqrt{\frac{2}{\gamma}})$$

The household has a CARA period-utility function:

$$u(c_t) = -\frac{1}{\gamma} e^{-\gamma c_t}$$

The household has access to only a risk-free asset, a_t, and is subject to a borrowing constraint that says:

$$a_{t+1} \geq 0$$

The market interest rate is r_t and households discount the future at rate β.

1. Write the household's value function assuming that the equilibrium is stationary.
2. Define a competitive equilibrium (make sure you give the condition which determines the interest rate).
3. Solve explicitly for the equilibrium consumption functions and interest rate. (Hint: if $\log x_t \sim N(\mu, \sigma^2)$ then $\mathbb{E}x_t = e^{\mu + 0.5\sigma^2}$).
4. Now consider an equilibrium where each period the government borrows D, repays the gross interest $(1+r)D$ (i.e., they lump-sum tax households by rD in order to pay the net interest). Explain why this policy must increase expected utility if $\gamma > -\log\beta$ and D is sufficiently small.