MICROECONOMICS COMPREHENSIVE EXAM

AUGUST 2014

Instructions:

(1) Please answer each of the four questions on **separate** pieces of paper.
(2) Please write only on **one side** of a sheet of paper
(3) Please write in **pen only**
(4) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.)

1. Consider the Standard model of a competitive firm with input-output vector $y \in \mathbb{R}^l$ $(2 \leq l < \infty)$ and technology represented by a transformation function $G : \mathbb{R}^l \to \mathbb{R}$. Let $p \in \mathbb{R}^l_{++}$ be the vector of market prices.

(a) Define the firm's production set and carefully specify the assumptions of the Standard model of the firm.

(b) Show that, given the underlying assumptions, for each $p \in \mathbb{R}^l_{++}$,

$$\begin{aligned} y(p) &= \arg\max p \cdot y \\ & \quad \text{s.t. } G(y) \geq 0 \end{aligned}$$

is well-defined, continuous, homogeneous of degree zero, and satisfies $p \cdot y(p) \geq 0$.

(c) Show that, given the underlying assumptions, for each $p \in \mathbb{R}^l_{++}$,

$$\begin{aligned} \pi(p) &= \max p \cdot y \\ & \quad \text{s.t. } G(y) \geq 0 \end{aligned}$$

is well-defined, continuous, homogeneous of degree one and convex.

(d) Suppose now that market prices are uncertain. Let $f(p)$ denote the density function of p on $\mathbb{R}^l_{++}$. Assume that the expected value

$$E[p] = \int_{\mathbb{R}^l_{++}} p f(p) dp$$

is finite.

The firm is risk neutral with respect to profits. The firm faces a choice between two alternatives: (i) make production decisions after the uncertainty is resolved; (ii) make production decisions "now", given the deterministic market price vector $\bar{p} = E[p]$. There is no discounting between "now" and the moment when the uncertainty will be resolved.

Which of the two alternatives will the firm prefer? Carefully explain why.

2. Consider an example of a pure exchange economy with two consumers and two goods. Let $x_i^j \geq 0$ $(i, j = 1, 2)$ denote the quantity of good j in a consumption bundle of consumer i. Suppose that Mr. 1's utility function is

$$u_1(x_1^1, x_1^2) = A - (x_1^1 - 1)^2 - (x_1^2 - 3)^2,$$

and the bliss point $(1, 3)$ is an interior point of Mr. 1's budget set. Ms. 2's utility function $u_2(x_2^1, x_2^2)$ is differentiable, increasing and strictly quasi-concave. The initial endowments in goods are $e_1 = (3, 1)$ and $e_2 = (1, 3)$. Let $p \in \mathbb{R}^2_{++}$ be the vector of market prices.

(a) Carefully define a competitive equilibrium in this economy. Distinguish cases when free disposal is allowed and not allowed.

(b) Using diagrams, describe

 (i) a competitive equilibrium without free disposal;

 (ii) a competitive equilibrium if Mr. 1 has access to free disposal.

(c) What can you say about optimality of equilibrium allocations that you obtained in (b)? Discuss.

3. A risk neutral firm hires a risk averse worker, whose utility function is given by

$$v(w, e) = u(w) - c(e),$$

where $w \geq 0$ denotes the wage, $e \in \{e_L, e_H\}$ denotes the effort level; $u(\cdot)$ is the utility function, and $c(\cdot)$ is the cost of effort. The utility function is strictly increasing and strictly concave, and $c(e_H) > c(e_L) \geq 0$. The worker's reservation utility is $\bar{u}$. The output produced by the worker can take three distinct values, $0, 500$ or 1000. The probability distribution over output signals, y, depends upon the worker's effort and is given by:

	0	500	1000
e_H	1/4	1/2	1/4
e_L	1/2	1/4	1/4

The worker's effort is not observed by the principal whereas the output level is.

(a) Explain what is meant by incentive compatibility in the context of contract design.

(b) Suppose that the firm wants the worker to supply effort e_L. Solve for the optimal contract.

(c) Assume henceforth that the firm wants the worker to supply effort e_H.

  (i) Set out the firm's constrained maximization problem, and the first order conditions for an optimum.

  (ii) Why must the incentive constraint bind in an optimal contract?

  (iii) Why must the participation constraint bind in an optimal contract?

  (iv) At what output level is the worker paid the most, and at what output level is she paid the least? (You do not have to solve for the optimal contract to answer this question).

  (v) Suppose now that the problem is modified so that the firm must pay identical wages at $y = 500$ and $y = 1000$. Solve for the *utilities* of the worker, $u(w_0)$ and $u(w_{500})$ in the optimal contract, given that $\bar{u} = 10, c(e_H) = 2$; $c(e_L) = 0$.

  (vi) Provide one reason why the restriction on wages in (v) (i.e. that they must be equal after $y = 500$ and $y = 1000$) may arise.

4.

(a) Explain the difference between a private value auction and a common value auction.

For the rest of the question, consider a first price sealed bid *all pay auction* for a single object with two bidders. That is, both bidders must submit sealed bids, and the object goes to the highest bidder. A bidder has to pay his bid regardless of whether he wins or loses the object. The private value of the object to each bidder is independently and identically distributed on $[0, 1]$ according to the cumulative distribution function $F(v) = v^m$ where $m \geq 1$.

(b) Suppose that bidder i assumes that his opponent uses a bid function $b(v)$ that is strictly increasing. Write down the expected payoff to i to bidding $b(w)$, $w \in [0, 1]$, when his valuation is in fact equal to v.

(c) What are the first order conditions that must be satisfied if bidder i is to find it locally optimal also to use the strategy $b(v)$?

(d) Solve the resulting differential equation and establish global optimality to find a symmetric equilibrium of this Bayesian game.

(e) How does equilibrium behavior change as m increases, and tends to infinity? Describe in economic terms.