

# MICROECONOMICS COMPREHENSIVE EXAM

AUGUST 2012

## INSTRUCTIONS:

- (1) Please answer each of the four questions on **separate** pieces of paper.
- (2) Please write only on **one side** of a sheet of paper
- (3) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1.(a), 1.(b), etc.).

1. In a certain economy, there are two commodities, education,  $E$ , and food,  $F$ , produced by using labor,  $L$ , and land,  $T$ , according to the production functions

$$E = (\min\{L, T\})^2 \quad \text{and} \quad F = (LT)^{1/2}.$$

There is a single consumer with the utility function

$$u(E, F) = (EF)^{1/2},$$

and the endowment  $(e_L, e_T)$ . To ease the calculations, take  $e_L = e_T = 1$ .

- (a) Find the optimal allocation of the endowments to their productive uses.
- (b) Suppose that the consumer can choose any amount of education and food desired at the going prices (which she takes as given). Both the food and education industries are competitive, i.e., they take output and input prices as given and make zero profits. The food industry maximizes its profit. The education industry minimizes its cost. Let food be the numeraire good. Find the equilibrium prices and allocation in this economy. Compare the equilibrium allocation and the optimal allocation in part (a).
- (c) Recognizing that the production of education entails increasing returns to scale, the government of this economy decides to control the education industry by setting the price for education and finance its operation with a lump-sum tax on the consumer. Suppose that the government sets the price of education to equal the marginal cost of education at the optimal level (derived in part (a)). What is the lump-sum tax necessary to decentralize the optimal allocation in part (a)?

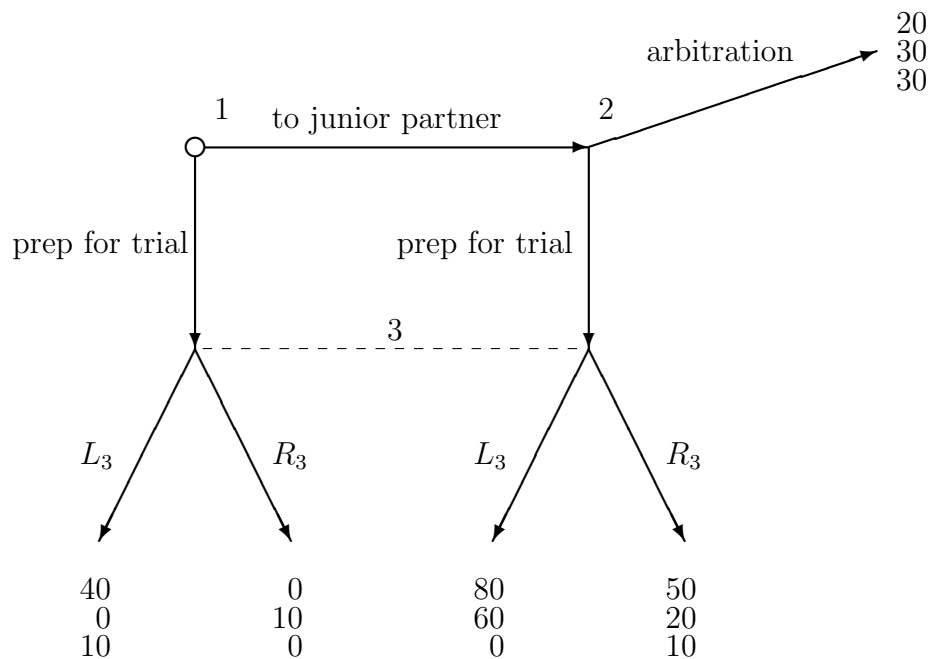
2. Alice's house is worthless to her if she cannot sell it. Bob and Carol are the potential buyers. Bob and Carol know their own valuations of the house, but this information is not known to anybody else. It is common knowledge that their valuations are independent and identically distributed and can take only two values: high,  $v_H = \$4(\text{mln.})$ , and low,  $v_L = \$3(\text{mln.})$ . The probability of low valuation is  $p \in (0, 1)$ . All agents are risk neutral. Focus only on symmetric equilibria.
- (a) What is Alice's first best outcome? What is her expected payoff in the first best outcome? Can Alice achieve the first best outcome?
  - (b) What is Alice's expected payoff if she simply posts a take-it-or-leave-it price?
  - (c) What is Alice's expected payoff in the Vickrey auction?
  - (d) Consider the following modification of the Vickrey auction: Bob and Carol are restricted to bid only 3 or 4, and the winner pays the *average* of the winning and losing bids. Will Bob and Carol bid their true valuations? Why or why not? What is Alice's expected payoff?
  - (e) Which of the above mechanisms will Alice choose to sell her house?

3. Until the invention of limited liability corporations (LLCs), someone who invested  $x > 0$  in a firm might have been liable for more than  $x$  if, say, the firm went bankrupt.

Let  $S$  be a random variable with  $ES > 1$  but  $P(S < 0) > 0$ . Having a wealth  $w$ , one can invest any  $x \in [0, w]$ . If one invests  $x \geq 0$  in an LLC, the payoffs are  $x \cdot \max\{0, S\}$ , which leads to final wealth  $W_{LLC}(x) = [(w - x) + x \cdot \max\{0, S\}]$ . By contrast, without limited liability protections, final wealth after investment of  $x \geq 0$  would be  $W(x) = \max\{0, [(w - x) + x \cdot S]\}$ .

- (a) Give the definition of first and second order dominance for random variables. Show that for any  $x \geq 0$ ,  $W_{LLC}(x)$  both first and second order dominates  $W(x)$ .
- (b) Give conditions on the distribution of  $S$  under which, for any  $w \geq x' > x \geq 0$ ,  $W_{LLC}(x')$  first order dominates  $W_{LLC}(x)$ . For the rest of the problem, assume that  $x' > x$  does **NOT** imply that  $W_{LLC}(x')$  first order dominates  $W_{LLC}(x)$ .
- (c) For a risk averse, expected utility maximizer, and a fixed distribution for  $S$ , show that the optimal investment is larger under the LLC regime.
- (d) If a firm raises  $x_i$  for a group of investors,  $I$ , it has  $X := \sum_{i \in I} x_i$  to work with. Knowing that  $X$  has been raised in an LLC framework means that the creditors know that there is less chance that they will be repaid if things go badly for the firm. Specify, being as explicit as possible, how and why this might affect the distribution of  $S$  so as to push the  $x_i$  lower.

4. Two law firms,  $A$  and  $B$ , are preparing to go to court. Firm  $A$  is a two-lawyer firm, firm  $B$  has only one lawyer. Not knowing how firm  $B$  will present the case, the senior partner in  $A$  decides whether or not to prepare the arguments that will be used in court. If she decides not to prepare the arguments, it will be her junior partner's job to prepare the arguments or to accept arbitration. The lawyer in firm  $B$  does not know who is going to prepare the arguments, and must decide on his strategy before the trial. With payoffs, one extensive formulation of the game is given below, with the senior partner labeled as player 1, the junior partner as 2, and the  $B$  lawyer as 3.



- Give a normal form representation of this game.
- Show that arbitration is a perfect equilibrium outcome.
- Find the equilibrium components for this game.
- Show that arbitration is not part of any stable outcome and give the stable outcome or outcomes for this game.