MICROECONOMICS COMPREHENSIVE EXAM

JUNE 2012

INSTRUCTIONS:

(1) Please answer each of the four questions on **separate** pieces of paper.
(2) Please write only on **one side** of a sheet of paper
(3) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.).

As usual, answers without supporting arguments will receive little credit.

1. Alice, the decision maker, can choose among three possible actions: going to a soccer game (S), going to a movie (M), or going for a walk (W). Her preferences over these alternatives depend on the state of nature: "rain' or "no rain." If it rains, she prefers M to S, S to W, and, by transitivity, M to W. If it does not rain, she prefers S to W, W to M, and by transitivity, S to M. Alice has to decide which action to take before she finds out whether it will rain or not. If prob(rain)$= \frac{1}{3}$, she is indifferent between M and W, and if prob(rain)$> \frac{1}{3}$, she strictly prefers M to W. Further, if prob(rain)$= \frac{3}{5}$, she is indifferent between M and S, and if prob(rain)$> \frac{3}{5}$, she strictly prefers M to S.

(a) Without knowing the probability of rain, point out an action that Alice will definitely not take if she is rational.

(b) Give an example of a utility function that might be a von Neumann-Morgenstern utility representation of Alice's preferences in the sense that it does not contradict any of the above statements.

(c) Construct another utility function that represents the same preferences under certainty, but is not a von Neumann-Morgenstern representation of these preferences.

(d) Alice's brother, Bob, has the same preferences under certainty as Alice does. However, if prob(rain)$= \frac{4}{5}$, he prefers S to M; and if prob(rain)$= \frac{1}{4}$, he prefers M to S. Are these preferences consistent with Bob being an expected utility maximizer? Explain.

2. Consider the following two good-two consumer pure exchange economy. Mr. 1 has endowment $e_1 = (e_1^1, e_1^2) \gg 0$ and Ms. 2 has endowment $e_2 = (e_2^1, e_2^2) \gg 0$. Their respective utility functions are

$$u_1(x^1, x^2) = \phi_1(x^1) + x^2, \text{ and } u_2(x^1, x^2) = \phi_2(x^1) + x^2,$$

where, for $i = 1, 2$, $\phi_i'(x) > 0$ for $x > 0$, $\lim_{x \to 0} \phi_i'(x) = +\infty$, and $\phi_i''(x) < 0$ for $x > 0$.

(a) Show that any two Pareto optimal allocations in the interior of the Edgeworth box involve the same consumption of the first good. In this case, what is the reason for different levels of the consumers' utility in different Pareto optimal allocations?

(b) Suppose from now on that $\phi_i(x) = a_i \ln x$, where $a_i > 0$, $i = 1, 2$, and $a_1 + a_2 = 1$. What can you say about the sharing rule for the first good for Pareto optimal allocations in the interior of the Edgeworth box? Illustrate your answer with a diagram.

(c) Let the second good be the numeraire. Find all competitive equilibria for this economy.

3. You control a growing asset, perhaps trees growing in a patch of forest, or wool growing on a flock of sheep, or experience accumulating in an office full of software engineers learning their trade. If you liquidate the asset after it has been growing between time 0 and time t, it has a market value $P(t) > 0$. $P(\cdot)$ has the usual "S" shaped properties of a growth curve: $P'(t) > 0$ for $t > 0$; $P''(t) > 0$ for $t \in (0, T)$; and $P''(t) < 0$ for $t \in (T, \infty)$. After liquidating the growing asset, you may, or may not, decide to sell, at a price v, the infrastructure that makes the asset productive (the acreage, flock, or office in the examples), and exit the business.

(a) How does the solution to the problem $\max_{t \geq 0}(P(t) + v)e^{-rt}$ depend on r? On v? Explain both mathematically and intuitively. [Sometimes, but not always, it is easier to take logarithms before going to work.]

(b) Now suppose that in addition to the sale value, you receive a flow utility, $b(t) \geq 0$, from the asset (picnic time in the forest, amusement watching the sheep, amusement from watching the engineers swear when you make their programs input themselves). How does the solution to the problem $\max_{t \geq 0}\left[(P(t) + v)e^{-rt} + \int_0^t b(s)e^{-rs}\,ds\right]$ depend on the function $b(\cdot)$? On r?

(c) Instead of selling the infrastructure, you may decide to keep it and regrow the asset. If so, the question is how often to liquidate the asset. Show that the solution to the problem

$$\max_{t \geq 0} P(t)[e^{-rt} + e^{-r2t} + e^{-r3t} + \cdots], \tag{1}$$

is smaller than the solution to the problem $\max_{t \geq 0} P(t)e^{-rt}$. What can you say about the v that makes the solutions to $\max_{t \geq 0}(P(t) + v)e^{-rt}$ and the problem in (1) equal?

(d) If the asset starts growing at time 0, the probability that it is destroyed by the actions of a malevolent universe at or before time t is $F(t)$ (the malevolent universe takes the form of forest fires, wolves, and the ebola virus in the examples). The "waiting time" until destruction happens has a hazard rate, $h(t)$, so that $F(t) = 1 - e^{-\int_0^t h(s)ds}$.

   i. Show that if two waiting times, F_1 and F_2, are hazard rate ordered, i.e. $h_1(\cdot) > h_2(\cdot)$. Show that F_2 first order stochastically dominates F_1, and that the decision maker is better off with F_2.

   ii. Suppose that you plan to sell the growing asset and liquidate the infrastructure at some time t in the future, but that the asset may be destroyed at a random time. What happens to the optimal planned time to sell if the random time suffers an upward move in the hazard rate?

4. [A fable about farmers and bandits] Group 1 are farmers, they live in a fixed location, and allocate their time budget of 1 between x_1, farming activities, and y_1 defensive activities. Group 2 are bandits, they roam about the forest, and allocate their time budget of 1 between x_2, hunting and gathering in the forest, and z_2, banditry. If the farmers allocate x_1 to farming, they have x_1^α to show for it, if the bandits allocate x_2 to hunting and gathering, they have x_2^α to show for it, $\alpha \in (0, 1)$. If (y_1, z_2) are the defensive-banditry allocations, then $\Phi(y_1, z_2 : \theta)$ is the successful defense probability, that is, the probability that the farmers prevent the bandits from taking x_1^α from the farmers. We assume that $\Phi(\cdot, \cdot : \cdot)$ is continuous, increasing in y_1, decreasing in z_2, and for each value of $\theta \in (0, 1)$, $\Phi(\cdot, \cdot : \theta)$ has increasing differences in y_1 and z_2. If you prefer algebra, you can do this problem with $\Phi(y_1, z_2 : \theta) = (y_1^\beta)/(y_1^\beta + \theta z_2^\beta)$, $\beta \in (0, 1)$. The utilities are $u_1 = x_1^\alpha \Phi(y_1, z_2)$ and $u_2 = x_2^\alpha + x_1^\alpha(1 - \Phi(y_1, z_2))$.

(a) Suppose that (x_1, y_1) and (x_2, z_2) are picked simultaneously. Show that the game has a pure strategy equilibrium.

(b) How does the equilibrium set depend on θ? [If you are doing the general functional form, your job is to give and interpret assumptions on Φ and then justify your answer.]

(c) Suppose that the farmers allocate their efforts first, the bandits observe it, and then choose their effort levels. Under what conditions do the farmer's weakly prefer this leader-follower equilibrium to the previous one? What about strictly prefer?

(d) Give the efficient allocation(s) of activities. Can they be supported as a subgame perfect equilibrium outcome of an infinitely repeated version of the game?