MICROECONOMICS COMPREHENSIVE EXAM

JUNE 2014

Instructions:

(1) Please answer each of the four questions on **separate** pieces of paper.
(2) Please write only on **one side** of a sheet of paper
(3) Please write in **pen only**
(4) When finished, please arrange your answers **alphabetically** (in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.)

1. A decision-maker with von Neuman-Morgenstern expected utility is asked to rank the following 3 lotteries:

$L_1 =$

0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0
0	40	50	60	70	120

$L_2 =$

0	$\frac{3}{4}$	0	0	0	$\frac{1}{4}$
0	40	50	60	70	120

$L_3 =$

$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
0	40	50	60	70	120

(a) Assume that the decision-maker's expected utility index u is an increasing and strictly concave function. How would the decision maker rank the above lotteries according to stochastic dominance? Explain carefully why.

(b) How will your answers to the questions in (a) change if the decision-maker's expected utility index u is an increasing linear function? Explain carefully why.

(c) Suppose that lotteries 1 and 2 represent payoffs of risky projects that require a fixed sum investment $I \in (40, 50)$. There is a risk-neutral entrepreneur, who knows the type of a project (i.e., whether it is L_1 or L_2), and has no money of his/her own. There is a risk-neutral lender, who is willing to make the investment, but cannot observe the type of a project (loan applicant).

The lender agrees to invest the capital I in return of R. If profit realization of the project is greater than or equal to R, the lender is paid back, and the entrepreneur gets the difference between the profit and R. If profit realization of the project is less than R, the lender gets the profit, and the entrepreneur gets nothing. The entrepreneur is rational and will only apply for the loan if the expected net gain is non-negative. Let R_j denote the maximal repayment to the lender which makes it rational for entrepreneurs with project of type $j \in \{1, 2\}$ to apply for the loan. Write down the expected net gain for each project type and calculate R_1 and R_2.

(d) Suppose that the lender believes that the probability of facing the entrepreneur with type 2 project is $\gamma \in (0, 1)$, so that the probability distribution over the types is $(1 - \gamma, \gamma)$. What is the compound lottery of the lender's payoffs? What is the lender's expected payoff as a function of R? Which R maximizes the lender's payoff?

(e) Suppose now that the lender has multiple prior beliefs: namely, (s)he thinks that γ can be any number in the interval $(1/3, 2/3)$. Assume that the lender has the Gilboa-Schmeidler preferences, that is the lender chooses γ that minimizes her/his expected payoff. What is the lender's expected payoff as a function of R? Which R maximizes the lender's payoff?

2. Consider a two-period ($t = 0, 1$) state contingent claims economy. There are two possible states of the economy at $t = 1$. There is a single consumption good. There are two consumers: Alice (A) and Bob (B). Alice's endowment is $e_A = (0, 2, 1)$, and Bob's endowment is $e_B = (0, 1, 2)$. In a vector $e_j = (\cdot, \cdot, \cdot)$, the first entry indicates endowment in the consumption good at $t = 0$, and the other two entries indicate endowments in the consumption good in states 1 and 2, respectively, at $t = 1$. Both consumers derive no utility from consumption at $t = 0$.

Let $x = (x_1, x_2)$ denote state contingent consumption at $t = 1$. Assume that both consumers have expected utility of consumption at $t = 1$:

$$V^A(x) = \pi_1^A \ln x_1 + \pi_2^A \ln x_2, \; V^B(x) = \pi_1^B \ln x_1 + \pi_2^B \ln x_2,$$

where $\{\pi_1^A, \pi_2^A\}$ ($\pi_1^A, \pi_2^A > 0, \pi_1^A + \pi_2^A = 1$) are Alice's beliefs about the probability distribution over the future states; and $\{\pi_1^B, \pi_2^B\}$ ($\pi_1^B, \pi_2^B > 0, \pi_1^B + \pi_2^B = 1$) are Bob's beliefs about the probability distribution over the future states. Assume that $\pi^A = \{\frac{1}{2}, \frac{1}{2}\}$, and $\pi^B = \{\frac{1}{3}, \frac{2}{3}\}$.

At $t = 0$, there is a market for state contingent consumption of the good in states 1 and 2. Let (p_1, p_2) be the ($t = 0$) prices of one unit of state contingent consumption in states 1 and 2, respectively. There is no market for the consumption good at $t = 0$, because there is no demand for or supply of this good.

(a) Carefully define a competitive equilibrium in this economy.
(b) Derive an equilibrium. Is it unique or not?
(c) Will the equilibrium allocation be Pareto optimal? Explain why or why not.
(d) Will the consumers bear risk in equilibrium? Discuss.

3. Consider an infinitely repeated game in which, at each stage, player 1 chooses rows, player 2 chooses columns, and player 3 chooses matrices, and the stage-game payoffs are given by:

L:

	l	r
T	$1,1,1$	$0,0,0$
B	$0,0,0$	$0,0,0$

R:

	l	r
T	$0,0,0$	$0,0,0$
B	$0,0,0$	$1,1,1$

(a) Give a definition of the mixed strategy minmax payoff for generic player $i \in \{1, 2, 3\}$. Find the mixed strategy minmax payoff for each player and describe the set of individually rational feasible payoffs in this game.
(b) Show that it is not possible to simultaneously hold all players to their minmax payoff.
(c) Let v^* be the minimum payoff attainable in any subgame-perfect equilibrium, for any player and any discount factor, in this three-player game. Show that this lower bound is at least $1/4$, for every discount factor. To do this, follow the following steps:
  (i) First show that given any specification of stage-game actions, there is at least one player who, conditional on the actions of the other two players, can achieve a payoff of $1/4$ in the stage game.
  (ii) Now fix a discount factor $\delta \in (0, 1)$ and a subgame perfect equilibrium, and let player i be the one who, given the first-period actions of his competitors, can choose an action giving him a payoff of at least $1/4$ in the first period. Argue that doing so ensures player i a repeated-game SPE payoff of at least $(1 - \delta)\frac{1}{4} + \delta v^*$.
  (iii) Conclude therefore that the stated lower bound on v^* is valid.
(d) Can you reconcile these results with the folk theorem?

4. A monopolist who produces a single product at marginal cost $c > 0$ is selling to a continuum population of consumers, where each consumer is either of type θ_H or θ_L, with $c < \theta_L < \theta_H$ and where the proportion of type θ_L consumers is $\lambda \in (0, 1)$. Types are not observable by the monopolist. Any consumer has decreasing marginal utility from the product: the marginal utility of a consumer of type $\theta \in \{\theta_L, \theta_H\}$ from the x^{th} unit consumed is equal to $\theta(1 - x)$. Both types of consumers have quasi-linear preferences, i.e. a type θ's utility from consuming amount x of the good and paying a total of T for it is $u(x, T, \theta) = v(x, \theta) - T$ where we assume $v(0, \theta) = 0$.

(a) Show that $v(x, \theta) = \theta \, \frac{x(2-x)}{2}$.

(b) Consider a non-discriminating monopolist (i.e. a monopolist who charges a single price p per unit purchased). Derive his optimal pricing policy if both types are served, and if only one type is served. (Hint: Show that $p = \frac{\hat{\theta}+c}{2}$ when both types are served, where $\hat{\theta} = \frac{\theta_L \theta_H}{\theta_H \lambda + \theta_L (1-\lambda)}$).

(c) Argue in economic terms that he serves both classes of consumers if either θ_L or λ is "large enough", without solving for the relevant thresholds.

(d) Compute the optimal non-linear tariff, i.e. find the contracts (x_L, T_L) and (x_H, T_H) that the monopolist optimally offers. (Assume that the monopolist serves both types).

(e) The quantities consumed by each type under the full information benchmark satisfy $\theta_L(1 - x_L) = \theta_H(1 - x_H) = c$. Briefly discuss in economic terms how the quantities consumed in (b) and (d) differ from that benchmark.