Microeconomics Comprehensive Exam

August 2009

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1. There is a risk-neutral pirate, and there are three sealed treasure chests, A, B, and C. The amount of treasure X_i in each chest $i \in \{A, B, C\}$ is a random variable. Each X_i is independent of the amount of treasure in the other chests, and is distributed uniformly on the interval [0, 1]. The pirate may choose the treasure from a single chest by searching the chests without replacement. That is, first he opens chest A and observes X_A. He may then either throw the other two chests overboard and consume X_A, or throw chest A overboard, open chest B, and observe X_B. If he opens chest B, then he may either throw chest C overboard and consume X_B, or throw chest B overboard, open chest C, and consumer X_C.

a. Describe the pirate's optimal strategy (or strategies) and calculate his expected utility.

b. Now suppose that chest C has either 1 or 0 units of treasure, with equal probabilities. Chests A and B are as before. Describe the pirate's optimal strategy (or strategies) and calculate his expected utility. (NOTE: since the chests are no longer identical, the pirate's strategy must now specify the order in which he searches the chests.)

c. Note that the distribution of X_C described in part b is a mean-preserving spread of its original (uniform) distribution. Is the pirate's expected utility higher, lower, or the same after this mean-preserving spread? Relate your answer to the definition of risk neutrality. Is there a contradiction?

d. Repeat part a under the assumption that the pirate is not risk-neutral, but instead is an expected-utility maximizer with Bernoulli utility function $u(x) = x^{1/2}$.

2. Otto McBeal derives utility from driving and from money. In particular, his utility function is given by

$$U(m, w) = \alpha_V m^{1/2} + w,$$

where $m \geq 0$ represents the number of miles driven and $w \in \mathbf{R}$ represents the amount of money that Otto has left. (Note that w is allowed to be negative; there is no budget constraint.) The coefficient α_V depends on the type of vehicle that Otto buys: a small car ($V = S$) or a large car ($V = L$). Assume that $\alpha_L > \alpha_S > 0$. (We can interpret α_V as a measure of how much fun it is to drive a vehicle of type V.) The distance driven depends on the amount of gasoline purchased g and the vehicle V: $m = \beta_V g$. Assume that $\beta_S > \beta_L > 0$. (We can interpret β_V as the fuel efficiency of a vehicle of type V.) The price of a small car is $c_S > 0$, and the price of a large car is $c_L > c_S$. The price of gasoline is $p > 0$, and Otto has initial wealth I.

a. Suppose that Otto has purchased a car of type $V \in \{S, L\}$. How much gasoline will he buy? Will he buy more gas if he has a small car or a large car?

b. Derive Otto's Walrasian demand for small cars, large cars, and gasoline.

c. Suppose that the government wants to reduce the total *amount of gasoline* that Otto uses by raising the price of gasoline p. Is it possible that the plan will backfire? That is, can an increase in p ever result in an increase in g? Provide an example, or show that this outcome is not possible.

d. Now suppose that the government wants to reduce the total *distance* that Otto drives by raising the price of gasoline p. Is it possible that the plan will backfire? That is, can an increase in p ever result in an increase in m? Provide an example, or show that this outcome is not possible.

3. Alice's house is worthless to her if she cannot sell it. Bob and Carol are the potential buyers. Bob and Carol know their own valuations of the house, but this information is not known to anybody else. It is common knowledge that their valuations are independent and identically distributed and can take only two values: high, $v_H = \$4$(mln.), and low, $v_L = \$3$(mln.). The probability of low valuation is $p \in (0, 1)$. All agents are risk neutral. Focus only on symmetric equilibria. Consider a direct mechanism to sell the house. Recall that in a direct mechanism, players are asked to state their type. Suppose that an agent who announces *High*, provided the other agent tells the truth, expects to pay H for the house and wins the auction with probability h. An agent who announces *Low*, provided the other agent tells the truth, expects to pay L for the house and wins the auction with probability l.

a. What is Alice's objective function?
b. What are Bob's and Carol's constraints?
c. What additional constraints are due to the symmetry of the auction?
d. What are optimal values of h, H, l, and L? What is Alice's expected payoff?

4. After receiving your PhD at UT, you join the Economics department at the University of ... The department is in the process of electing a new chairperson. There are three potential candidates: Alice, Bob and Carol. The executive committee (EC) of the department contemplates two possible election procedures: majority rule (by majority rule, we mean sequential, pairwise majority votes; a candidate who gets more than half of the votes wins) and plurality rule (a candidate who receives the largest number of votes wins). Each faculty member is allowed to vote for only one candidate.

a. Since you still remember your Micro Theory class, you present to the EC the list of at least 3 desirable properties of a social welfare function as well as examples (one for each voting rule) when at least one of these properties may not hold.

b. Having heard your arguments, the EC tells you that the faculty feel strong about Carol: for each voter, Carol is either his or her favorite candidate or his or her least favorite. After that, you recommend them to use which of the two rules?