Macroeconomics Comprehensive Examination: August 2011

Instructions. You have three hours to answer four questions for a total of 180 points. The points for each question reflect how many minutes you should spend and their relative weight in the comp. Budget your time carefully. Your exam number (e.g. E3) and question number (e.g. Q2) should appear on the top of every page of your answers.

1. (45 points) Consider an overlapping generations model where individuals live for two periods. Each generation consists of a continuum of agents. The size of each generation is normalized to unity. Population growth rate is zero. Every individual born at $t = 0, 1, 2, ...$ has identical preferences:

$$\ln c_t^t + \ln c_{t+1}^t$$

where c_j^t is time j consumption of an individual born in period t. Each member of the initial old generation is endowed with physical capital k_0. Individuals in subsequent generations are endowed with one unit of labor/leisure in their youth and h_t units of human capital where

$$h_t = \theta h_{t-1}^{\mu}, \ \theta > 0 \text{ and } \mu \in [0, 1].$$

Human capital of generation 0 is assumed to be h_0, which we will take as an initial condition. Each firm in this economy produces output according to

$$y = F(k, nh)$$

where y is the amount of output, k is the physical capital stock, nh is the skill-weighted labor input and $F(\cdot)$ is a concave constant returns to scale production function. Physical capital depreciates at the rate δ.

   (a) (16 points) Formulate the problem faced by an individual born in period t (define your notations clearly). Given the initial k_0 and h_0, define an equilibrium for this economy.

   (b) (16 points) Suppose that $\mu < 1$ and

$$F(k, nh) = Ak^{\alpha}(nh)^{1-\alpha}, \ A > 0, \ \alpha \in (0, 1).$$

   Show that the balanced growth path of this economy cannot exhibit *sustained* growth. This suggests $\mu = 1$ is a necessary condition for sustained growth. Sketch the restrictions on other parameters for the economy to exhibit sustained growth.

   (c) (13 points) Suppose that $\mu = 0$ and

$$F(k, nh) = Ak^{\alpha}(nh)^{1-\alpha} + Bk, \ A > 0, \ \alpha \in (0, 1), \ B > 0.$$

   Show that the balanced growth path of this economy cannot exhibit sustained growth.

2. (45 points) Consider a competitive economy where households have (common) preferences over consumption sequences, given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t - \alpha)^{1-\sigma}}{1-\sigma}$$

where $c_t \geq \alpha$ is consumption in period t, $\alpha > 0$ is the minimum consumption required in each period, $\beta \in (0, 1)$ is the discount factor and $\sigma > 1$. All households have access to a production technology :

$$Y_t = aK_t$$

where $a > 0$, K_t is the stock of capital at the beginning of period t and Y_t is the output. Households also have access to a technology for accumulating capital:

$$K_{t+1} = (1 - \delta)K_t + X_t$$

where $\delta \in (0, 1)$ is the rate of depreciation of capital and X_t is gross investment in period t. Assume $a > \delta$. Households are heterogeneous in their initial endowment of capital. There are N types of households; let i denote the household type. The measure of each type is $\frac{1}{N}$. Let k_0^i denote the initial capital stock (or wealth) of a household of type i. Assume that k_0^i is large enough for each i to ensure the minimum required level of consumption each period. Let the rental rate for one unit of beginning-of-period capital in terms of the single (consumption or investment) good in period t be r_t.

(a) (15 points) Formulate household i's problem. Provide an intuitive argument for why the rental rate would be constant over time and derive the decision rules for the household.

(b) (15 points) Suppose that the mean wealth in period 0 is $\overline{k}_0$. Holding mean wealth constant, show that the heterogeneity in initial wealth does not affect the equilibrium evolution of *per capita* income.

(c) (15 points) Assume $\beta(1 + a - \delta) > 1$. Show that in the long run the wealth inequality (measured by the ratio of the richest household's wealth to the poorest household's wealth) remains constant. Along the transition path, show that the wealth inequality increases over time. Explain why.

3. (45 points) Consider the following matching model. There are two types of households, $i \in \{g, b\}$. In each period, households may switch from type i to type i' with probability $\delta(i'|i)$. The unconditional fraction of type g households in the economy is γ. A household can be employed ($e = 1$) or unemployed ($e = 0$). The distribution measure for type i households who are in employment status e is denoted as μ_i^e. households discount the future at rate β. The utility function for households is $u(c)$ which is strictly increasing and strictly concave in consumption c. Employers discount the future at the same rate of β as households. After a successful match, if a household takes the offer from an employer (i.e. it is employed), he draws his productivity $a \in \{a_H, a_L\}$ where $a_H > a_L \geq 0$. The production function is assumed to be in the form of $y_i = a_i$. Type g households are on average more productive than type b households. In particular, if $f_i = prob(a_i = a_H)$, then $f_g > f_b$. If a household of type i is unemployed, then it receives nonstorable output k_i where $k_g > k_b$ from home production. The matching technology is assumed to have the following form:

$$M(u, v) = \mathtt{u}^{\alpha}\mathtt{v}^{1-\alpha} \tag{1}$$

where $\mathtt{u}$ is the fraction of unemployed households and $\mathtt{v}$ is the number of job vacancies. An employer can post a vacancy at cost κ and be matched with a worker with probability $\frac{M(\mathtt{u},\mathtt{v})}{\mathtt{v}} \equiv \psi$. An unemployed household meets with an employer with probability $\frac{M(\mathtt{u},\mathtt{v})}{\mathtt{u}} \equiv \phi$. If matched, bargaining satisfies take-it-or-leave-it offers by the employer. Specifically, the employer makes a take-it-or-leave-it offer w to the household after the realization of the household's productivity. If the household accepts, it produces and receives the wage payment. If the household rejects, the match is dissolved, the job remains vacant and a household of type i receives home production k_i. The information and timing are such that if an employer and household are matched (which depends on the matching technology), the employer learns the applicant's type and decides whether or not to hire the household. This question asks you to find the parametric conditions under which an equilibrium where only type g households get hired.

(a) (7.5 points) State the household's problem under the assumption that only type g get hired.

(b) (7.5 points) State the employer's problem if only type g get hired.

(c) (5 points) State the laws of motion of employed and unemployed agents under the conjectured equilibrium.

(d) (25 points) What are the conditions on parameters such that this equilibrium exists?

4. (45 points) Consider a closed monetary economy populated by a continuum of households with unit mass. Each household has an initial endowment of M units of money. Each household is also endowed with one unit of a perishable consumption good each period. This endowment cannot be consumed by the household, but should be sold for money in the goods market. Assume also that the money obtained from selling the endowment cannot be used for current consumption. All the purchases of consumption goods are subject to a cash-in-advance constraint. Denote the nominal price of goods in period t by p_t.

The preferences of each household are given by

$$\theta^j \log(c_1^j) + \sum_{t=2}^{\infty} \beta^{t-1} \log(c_t^j),$$

where $\beta \in (0,1)$, $\theta^j \in \{\theta^H, \theta^L\}$ is the household's idiosyncratic taste shock in the first period and c_t^j is the consumption of household $j = L, H$ in period t. Assume that $\theta^H > \theta^L = 1$, and that half of the households receive the high shock θ^H.

Suppose that the government decides to sell B one-period nominal bonds at a price q (in terms of money) in the beginning of the first period after the taste shocks are realized. (Each nominal bond pays off 1 unit of money in the next period.) Assume that money obtained from the bonds sale are right away injected back into the economy as equal lump sum transfers to all the households. After this open market operation is completed, the goods market opens. In the second period the government prints B units of money to pay back its debt on nominal bonds. This is also done before the goods market opens in the second period. Thus, the households can use the cash revenues from bonds to purchase consumption goods in the second period. No more bonds are ever sold and no more money is ever printed in the future.

Denote by m_t^j the amount of money held by household j *at the beginning* of period t and by $b^j \geq 0$ the amount of bonds that a household of type j buys in the first period.

(a) (5 points) Would the households want to borrow from each other in the first period? Why or why not? Give only a short verbal explanation.

(b) (5 points) What monetary policy could the government implement in order to increase aggregate welfare (measured as the sum of all the agents' utilities) if the households are not allowed to borrow and the government is able to observe the individual taste shocks? Give only a short verbal answer.

(c) (5 points) For the parts below, suppose that households are not allowed to borrow money or goods, neither from each other nor from the government and that the realization of θ^j cannot be observed by the government.

Set up the decision problem of the household of type j (it might be useful to write down the constraints for the first and the second periods separately). Define a competitive equilibrium in this economy.

(d) (5 points) Derive the Euler equation which relates c_1^j and c_2^j. Does the Euler equation look different if the borrowing constraint is binding? Explain.

(e) (10 points) Assume that the cash-in-advance constraint always holds with equality. Suppose that the government sets $q \in (\frac{2\beta}{1+\theta^H}, \beta)$. It is possible (but tedious) to verify that for this price level at least one type of the households does not buy any nominal bonds. Taking this result as given, explain which households (H-type or L-type) would demand zero bonds. Provide both formal and intuitive arguments.

Use your answer to determine the equilibrium amount of bonds B that the government will be able to sell at this price and find closed-form solutions for p_t and c_t^j.

(f) (10 points) What would be the equilibrium consumption allocation if the government did not sell any bonds in the first period (e.g., by setting a very high price q)?

Show that if q is sufficiently close to β, the economy with bonds delivers higher social welfare (measured as the sum of all the agents' utilities) than the economy without bonds (Hint: it is convenient to rewrite individual consumption found in part e as $c_t^j = 1 + x_t^j(q)$, where $x_t^j(q)$ is some function of bond price measuring by how much the agent's consumption differs from his/her endowment).

Explain intuitively why selling such high priced nominal bonds might be socially beneficial.

(g) (5 points) Do you think the government should continue selling bonds from period 2 onwards? Give a short verbal explanation.