Macroeconomics Comprehensive Examination:
May 2011

Instructions. You have three hours to answer four questions for a total of 180 points. The points for each question reflect how many minutes you should spend and their relative weight in the comp. Budget your time carefully. Your exam number (e.g. E3) and question number (e.g. Q2) should appear on the top of every page of your answers.

1. (20 minutes) Consider the following two period decision problem. A household enters the first period with an exogenous amount of housing h which is valued at the period 1 relative price of housing in terms of consumption goods denoted p. At the beginning of the first period they receive earnings (in terms of consumption goods) of y and at the beginning of the second period they receive earnings $y' > y$. Preferences are given by $\log(c) + \beta [\log(c') + Ah']$ where h' is a quantity of housing chosen in the second period which is valued at p' and $\beta \in (0, 1)$. After receiving their first period income, the household can borrow or lend in a risk free asset market; if a household borrows $b' > 0$ units of goods in period 1 they must repay $(1 + r)b'$ in period 2. Finally, there is a borrowing constraint which says that an agent cannot borrow more than the value of their housing collateral γph where γ reflects the severity of financial frictions (i.e. $\gamma = 0$ means the agent cannot borrow against housing collateral).

   (a) (5 points) State the decision problem of a household.

   (b) (15 points) Characterize the household decision problem. When is the borrowing constraint more likely to bind? How do housing prices affect a household's ability to consumption smooth?

2. (50 minutes) Consider an infinite-horizon economy inhabited by a representative agent who consumes computer services. At the beginning of each period t, there is a stock of computers s_t available in the economy. Some of these are used for computer consumption, c_t, and the rest are used as inputs, x_t, into the production of new computers. The new computers are produced according to $f(x_t, n_t)$, which has constant returns to scale, and n_t is labor input. Old computers depreciate at the rate $\delta \in [0, 1]$. The consumer has 1 unit of labor available every period and supplies it inelastically. The consumer's utility function is

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

(a) (15 points) Using a dynamic programming formulation, specify a planning problem for this economy. Identify clearly the state variable(s) and the choice variable(s).

(b) (10 points) Now, consider a decentralized economy (that is, with markets). Let the consumer own the stock of computers, part of which he decides to keep at home and part of which he rents to a firm at a market price. The consumer also sells his labor services to a firm at a market price. He uses his income to purchase new computers from the firm. The firm, which is competitive, thus rents computer and labor from the consumer and produces computers to be available next period. Let the price of new computers be the numeraire. Carefully define a recursive competitive equilibrium for this economy.

(c) (15 points) Obtain first-order and envelope conditions for the consumer's problem. Use these conditions to obtain the Euler equation and interpret it.

(d) (10 points) Define a steady state equilibrium for this economy. Specify the conditions on preferences and technology such that this steady state exists and is unique. Justify your answer using a graph.

3. (50 minutes) Consider an economy with a continuum of identical agents who have access to a home technology that produces services using labor:

$$c_h = g(n)$$

where c_h is the quantity of services and n is the amount of labor. Each agent is initially endowed with 1 tree that produces fruits. The stochastic process for fruits is two-state Markov. Each agent is also endowed with 1 unit of time each period that may be split between labor and leisure. The preferences of the agents are described by:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, c_{ht}, l_t\right)$$

where c_t is amount of fruits consumed in period t, l_t is the amount of leisure and $\beta \in (0,1)$ is the discount factor. Trees and fruits are traded in competitive markets, but home services are not tradeable.

(a) (20 points) Formulate the representative agent's problem (describe your notation clearly). Derive and interpret the Euler equation. Define the competitive equilibrium for this economy.

(b) (13 points) Suppose that $u(c, c_h, l) = \ln(c) + \ln(c_h) + l$ and $g(n) = n^{\alpha}, \alpha \in (0,1)$. Derive the equilibrium asset pricing function. Compare the equilibrium asset price sequence of this model with that of the Lucas asset pricing model with similar preferences.

(c) (17 points) Suppose that $u(c, c_h, l) = c^{1-\alpha} c_h^{\alpha} + l$ and $g(n) = n$. Determine the equilibrium asset pricing function. Describe the response of home services to fluctuations in the aggregate state of the economy.

4. (60 minutes) Consider the following matching model. There is a unit measure of borrowers and a unit measure of "lending machines" which have an arbitrarily large amount of funds available to them. At any date t, there are two types of borrowers $i_t \in \{g, b\}$ where type determines the distribution of earnings shocks that the borrower receives. A period is comprised of two subperiods. In the first subperiod a borrower is matched with a lending machine but the borrower has no earnings. In the second subperiod the borrower receives earnings $e_{2,t} \in \{e_H, e_L\}$ where $e_H > e_L$ drawn from a discrete density function with $\pi^i = prob(e_H)$ and $\pi^g > \pi^b$ (so that a "good" type is more likely to receive high earnings). While the earnings draws are iid over time and individuals, the draws do depend on type. An agent's type changes over time according to a Markov process: (1) if $i_t = g$, the agent becomes "bad" at the beginning of next period (i.e. $i_{t+1} = b$) with probability θ^g; and (2) if $i_t = b$, the agent becomes "good" at the begining of next period (i.e. $i_{t+1} = g$) with probability θ^b. We will assume there is type persistence (i.e. $\theta^i < 1/2$) so that earnings realizations can be persistent as in the data.

At any date t, borrowers have strictly increasing, concave preferences over consumption in the two subperiods $u(c_{1,t})$ and $u(c_{2,t})$ with Inada condition $u'(0) = \infty$. A borrower in period t discounts the future (i.e. $t+1$) at rate $\beta < 1$ but there is no discounting between subperiod 1 and 2; hence utility within period t can be written $u(c_{1,t}) + E[u(c_{2,t})]$.

A borrower's type i_t and earnings $e_{2,t}$ are unobservable to anyone but the borrower.

A lending machine is programmed to offer a fixed amount ℓ_t of a loan to a borrower at a price q_t in subperiod 1 in order to consume $c_{1,t}$. If a household borrows in subperiod 1 (which throughout this problem we will assume occurs given the Inada condition), they *can* repay ℓ_t in subperiod 2 after the borrower's realization of $e_{2,t}$ since we assume that $\ell_t < e_L$. A borrower can, however, default. Let $d^i_{2,t}$ denote the default decision of a borrower of type i in subperiod 2 where $d^i_{2,t} = 1$ means default and $d^i_{2,t} = 0$ means no default. Specifically, a borrower receives $q_t \ell_t$ units of the good in subperiod 1 and pays back ℓ_t units in subperiod 2 provided he does not default. Lending machines do not allow borrowing across periods (i.e. from t to $t+1$).

There is a record keeping technology in this economy. At the beginning of subperiod 1, the technology has a record $\delta_t \in \{0, 1\}$ of whether a borrower defaulted in the previous period (once again assuming they borrow); that is, $\delta_t = d_{2,t-1}$. Using this information and a prior given by the unconditional probability any given borrower is a good type, the lending machine forms an assessment that the borrower is a good type at the time he must repay in subperiod 2. The assessment is formed using Bayes rule conditioned on observing δ_t.

Further, the lending machine can condition the price on whether the borrower has a default flag on his record (i.e. $q_t(\delta_t)$ so that it posts a price $q_t(0)$ to a "prime" borrower without a default flag or $q_t(1)$ to a "subprime" borrower with a default flag. The lending machine is programmed to make zero expected profits in any given period.

(a) (5 points) Given the stochastic process for type, what is the steady state proportion of good types in the economy (which is also the prior stated above)?

(b) (10 points) Let $V^i(\delta)$ denote the value function of a borrower with default record δ at the beginning of any subperiod 1 and let $d^i(e, \delta)$ denote the optimal default decision of a borrower of type i with default record δ in earnings state e. Write the borrower's value function.

(c) (10 points) Conditional on having observed default record δ, what is the Bayesian assessment that a person is a good type at the beginning of subperiod 2 when they must make their default decision? Hint: Recall that in the two type case, if $\Pr(g)$ is the prior, then the Bayesian posterior that a persion is a good type is given by

$$\Pr(i = g|\delta) = \frac{\Pr(\delta|g)\Pr(g)}{\Pr(\delta|g)\Pr(g) + \Pr(\delta|b)\left[1 - \Pr(g)\right]}.$$

As a further hint, note that timing matters for this calculation. The above posterior is what the person's type is at the end of last period and the question asks for their type at the beginning of subperiod 2 in the current period. So to correctly answer the question, you must also account for the type shock.

(d) (10 points)What is the price (i.e. $q(\delta)$) of a risky loan made to a person with default record δ? Hint: The lending machine must consider the borrower's type when considering his likelihood of default and the machine receives nothing in the case of default.

(e) (10 points) Suppose that both types only default with a bad earnings realization. Is the price of a "prime" $q(0)$ loan higher than a "subprime" $q(1)$ loan?

(f) (15 points) What incentive conditions must hold for an equilibrium like that in part (e) - where both types only default with a bad earnings realization - to exist? Discuss the reasoning why or why not such an equilibrium exists.