Microeconomics Comprehensive Exam

August 2011

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) Please write only on one side of a sheet of paper
(3) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.).

1. (25 points) Let Z denote a (finite) set of monetary prizes. Let $\mathcal{L}(Z)$ denote the (infinite) set of all lotteries on Z, and $\succeq$ denote a rational preference relation on $\mathcal{L}(Z)$.

(a) (5 points) Spell out the axioms which ensure that $\succeq$ admits the von Neuman-Morgenstern utility representation?

(b) (10 points) For any $p \in \mathcal{L}(Z)$, define

$$U(p) = E(p) - \frac{1}{4} var(p),$$

where $E(p)$ is the expected value of the prize of lottery p, and $var(p)$ is the variance of of the prize p. Show that U defined above induces a preference relation which is not consistent with the assumptions of the von Neuman-Morgenstern theorem.

Hint: Consider, for example, lottery p, which gives a deterministic payoff of \$1 and "coin" lottery q, which pays 0 in the event of "heads" and \$4 in the event of "tails", and their convex combinations with coin lottery r, which pays 0 in the event of "heads" and \$2 in the event of "tails".

(c) (10 points) Define

$$U(p) = E(p) - (E(p))^2 - var(p).$$

Show that U defined above induces a preference relation which is consistent with the assumptions of the von Neuman-Morgenstern theorem.

2. (25 points) Consider an asset structure given by the following payoff matrix:

$$D = \begin{pmatrix} \delta^1 & \delta^2 & \delta^3 \\ \alpha & 0 & \beta \\ 1+r & 1+r & 1+r \end{pmatrix},$$

where $\delta^1 > \delta^2 > \delta^3 > 0$, $\alpha > \beta > 0$, and $r > 0$. Let S^1, S^2, S^3 be asset prices.

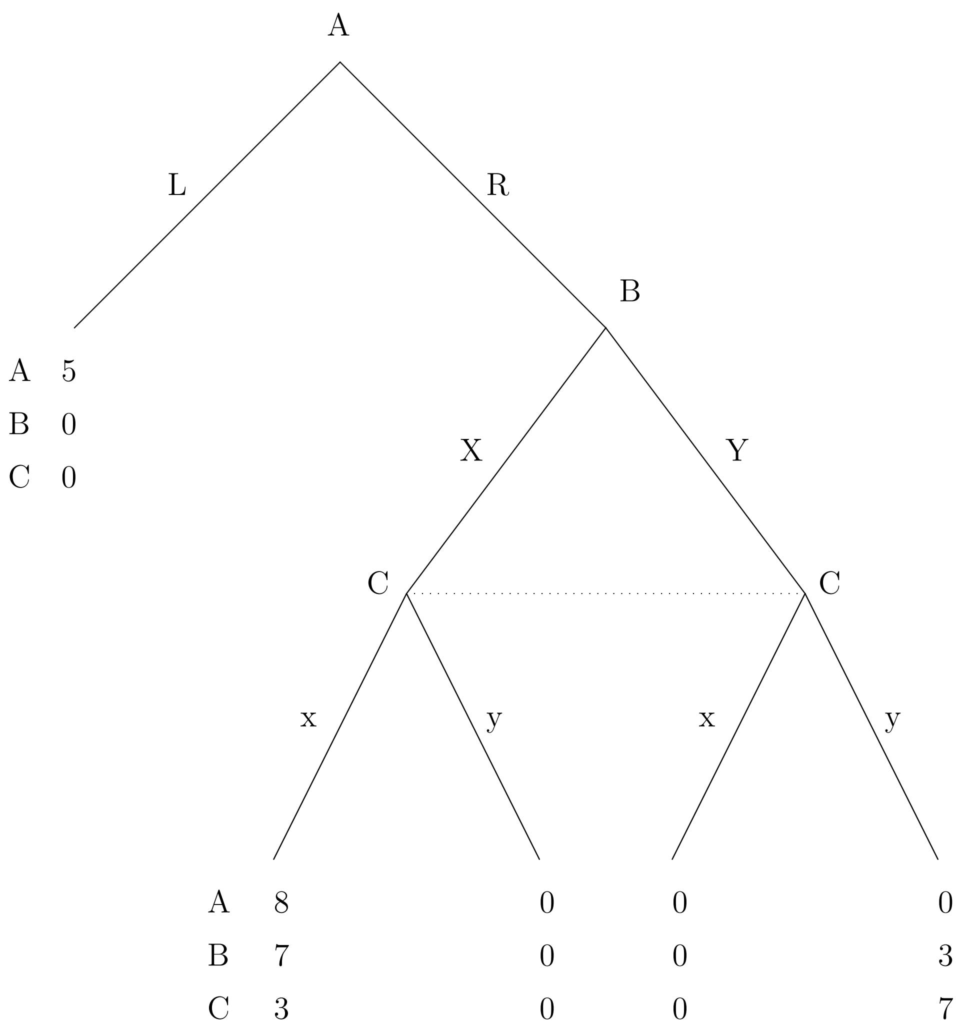
(a) Critically analyze the following propositions:

(1) (10 points) If $S^1, S^2, S^3 > 0$, then the asset structure D permits no arbitrage.

(2) (10 points) If the asset structure D permits no arbitrage, then D exhibits no redundancy. Is this true for a general asset structure specified by an $n \times m$ payoff matrix D?

(b) (5 points) Let $\delta^3 < (1+r)S^1 < \delta^2$, and $S^3 = 1$. Derive the no-arbitrage boundaries for S^2.

3. (25 points) Consider the following extensive-form game.



The dotted information set says that C cannot distinguish between X and Y.
(a) (9 points) Find all (pure or mixed-strategy) Nash equilibria for the normal-form expression.
(b) (7 points) Find all (pure or mixed-strategy) subgame-perfect Nash equilibria.
(c) (9 points) Find all (pure or mixed-strategy) weak perfect Bayesian equilibria.

4. (25 points) There are two individuals. Producing a public good costs 1. Assume that the individuals have quasi-linear preference over the project decision and transfers. Also assume that they are risk neutral. Denote individual 1's valuation of the public good by v_1, and denote 2's by v_2.

(a) Consider a game-form as follows. Let them submit bids, b_1 and b_2 respectively. If $b_1 + b_2 \geq 1$, do the project and make 1 pay b_1 and 2 pay b_2. Otherwise, do nothing (no payment either).

(a-1) (7 points) Consider a complete information case with $0 < v_1, v_2 < 1$ and $1 < v_1 + v_2$. Find all the pure-strategy Nash equilibria.

(a-2) (6 points) Consider an incomplete information case in which v_1 and v_2 are drawn from $\{1/3, 2/3\}$ with even chance independently. Show that the following bidding function

$$\beta(v) = \begin{cases} 0 & \text{if } v = 1/3 \\ 1/2 & \text{if } v = 2/3 \end{cases}$$

forms **a** symmetric Bayesian Nash equilibrium.

(b) (6 points) Assume that v_1 and v_2 are drawn from $[0, 1]$ according to the uniform distribution independently. What is the prescription by the expected externality mechanism?

(c) (6 points) What is the prescription by the Clarke mechanism to this problem? Hint: For (b) and (c), you may formulate the net welfare terms first by assuming equal division as the default point for cost sharing.