Microeconomics Comprehensive Exam

June 2011

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1. Consider a consumer in a world with l goods. The consumer's preferences over goods are represented by a utility function u that satisfies the standard properties. The consumer is endowed with a bundle of goods $e \in \mathbb{R}^l_{++}$. Market prices are given by a vector $p \in \mathbb{R}^l_{++}$. The consumer's budget set is

$$B(p, e) = \{x \in \mathbb{R}^l_+ | \, p \cdot x \leq p \cdot e\}.$$

Define

$$V(p, e) = \max_{x \in B(p,e)} u(x).$$

(a) (5 points) Explain the meaning of the function V and show that, for any $t > 0$,

$$V(tp, e) = V(p, e).$$

(b) (10 points) Show that, for every bundle e and a constant v^*, the set of vectors p, such that

$$V(p, e) \leq v^*,$$

is convex.

(c) (10 points) Fix all prices but p_i, and all quantities in the initial endowment but e_i. Show that the slope of the indifference curve of V in the two dimensional space, where the parameters on the axes are p_i and e_i, is $(x_i(p, e) - e_i)/p_i$, where $x_i(p, e)$ is the consumer's demand for good i.

2. Alice (A) and Bob (B) are two consumers in a world with uncertainty. There are two states of the world, ω_1 and ω_2, and one consumption good in each state, so that (x_h^1, x_h^2) is the state contingent consumption bundle of consumer $h \in \{A, B\}$. Denote the probability of state ω_1 by $\pi \in (0, 1)$ (then the probability of state ω_2 is $1 - \pi$). Suppose that Alice's Bernoulli utility function is $u_A(x)$, and Bob's one is $u_B(x)$. Assume that the utility functions are increasing, strictly concave and continuously differentiable. Alice's endowment is $e_A = (1, 0)$, and Bob's one is $e_B = (0, 2)$.

(a) (10 points) Give programming characterization of Pareto optimal allocations and prove that the Pareto set is independent of π.

For the remainder of this question, suppose that $u_A(x) = 1 - e^{-\alpha x}$ and $u_B(x) = 1 - e^{-\beta x}$ ($\alpha > 0$, $\beta > 0$).

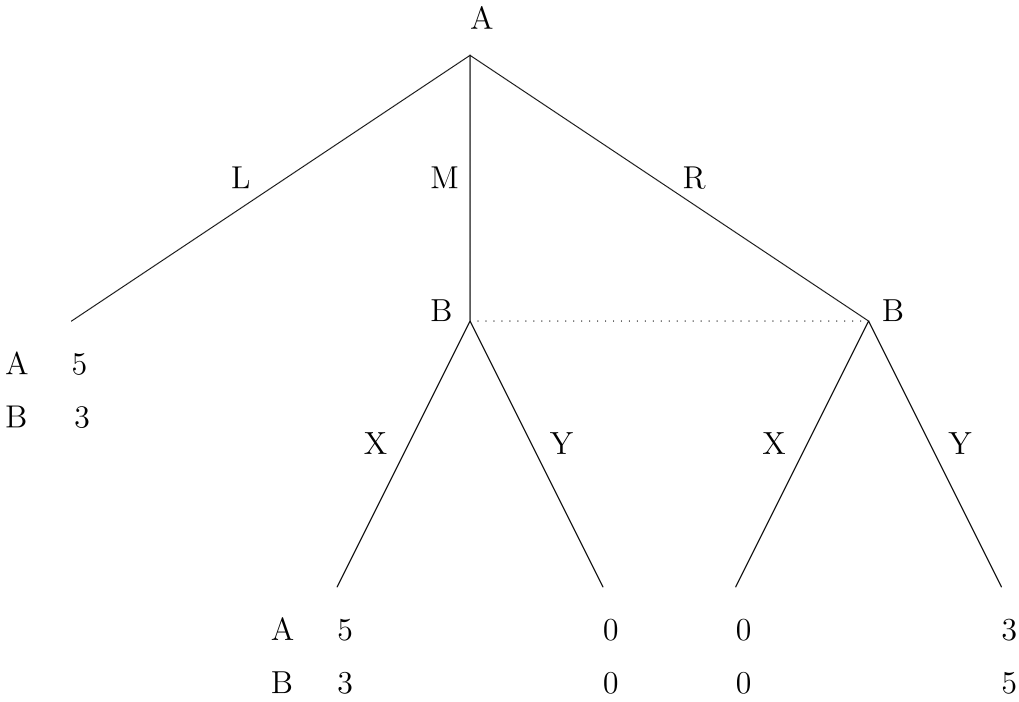
(b) (5 points) What is the equation describing the Pareto set for interior allocations?

(c) (3 points) How do consumers' marginal rates of substitution change along the Pareto set, again for interior allocations?

(d) (4 points) Find the Walrasian equilibrium prices.

(e) (3 points) Draw the Edgeworth box for this economy and show how the Walrasian equilibrium allocation changes as π varies?

3. Consider the following extensive-form game.



The dotted information set says that B cannot distinguish between M and R, although she knows either one is chosen.
(a) (8 points) Find all (pure or mixed-strategy) Nash equilibria for the normal-form expression.
(b) (5 points) Find all trembling-hand perfect Nash equilibria for the normal-form expression.
(c) (8 points) Find all weak perfect Bayesian equilibria.
(d) (4 points) Find all sequential equilibria.

4. There are one seller and one buyer, both are risk neutral. The seller has one object. The seller's value (cost) c is distributed uniformly over $[0, 1]$. The buyer's value v is uniformly distributed over $[0, 1]$, independently of c and vice versa.
(a) Consider the following double auction game: the seller and the buyer bid simultaneously; let b_1 denote the seller's bid, and b_2 denote the buyer's bid. If $b_1 \leqq b_2$, trade occurs and the price is $\frac{b_1+b_2}{2}$. Otherwise trade does not occur.
(a-1) (8 points) Consider the bidding functions β_1 and β_2 for the seller and the buyer, respectively, which form Bayesian Nash equilibrium. Assuming interior solution, derive a system of equations for β_1 and β_2.
(a-2) (3 points) Also, find the bidding functions assuming that they are affine (i.e., the seller's bidding function is of the form $\beta_1(c) = \alpha_1 c + \gamma_1$ and the bidder's one is of the form $\beta_2(v) = \alpha_2 v + \gamma_2$).
(b) (6 points) What is the prescription of the expected externality mechanism?
(c) (8 points) Characterize the implication of Bayesian incentive compatibility. **Briefly demonstrate the derivation.**