Macroeconomics Comprehensive Examination: August 2010

Instructions. There are two parts to this exam. Part A consists of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully. Your exam number (e.g. E3) and question number (e.g. QB2) should appear on the top of every page of your answers.

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following statements contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible but keep the answer short (within 10 lines or so). Each statement is worth 10 points for a total of 30 points.

1. Consider a Huggett economy in which individuals can only borrow up to a certain amount $\bar{a}$. Assume that the equilibrium allocation is such that no individuals are at the borrowing limit. As a result, increasing the borrowing limit to $\bar{\bar{a}} > \bar{a}$ would have no impact on the equilibrium allocation.

2. The neoclassical growth model allows for persistent differences in per-capita income across countries.

3. Consider a model with two-sided limited commitment along the lines of Kocherlakota (1996). In this model, the individual rationality constraint is more likely to bind for the individual with a relatively low endowment as this individual is more likely to be unable ro repay his debt.

B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is weighted differently.

Question 1. (30 points)

Each period an unemployed worker receives a wage offer w drawn from the distribution $F(w)$, where $w \in [0, \overline{w}]$ (i.e. $F(\overline{w}) = 1$, $\overline{w} < \infty$). The worker has to choose whether to accept the job -and in that case to work for that wage w forever- or to search for another offer and collect c in unemployment compensation. The worker, endowed every period with 1 unit of time, who decides to accept the job must choose the fraction of his time to work in each period. His preferences are represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \; u(y_t, l_t) \right\}$$

$\beta \in (0,1)$ and $u : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$ satisfies standard conditions. The income corresponding to period t, y_t, consists of c if the worker is unemployed, and $w(1 - l_t)$ if the worker is employed and works $(1 - l_t)$ hours with leisure $l_t \in [0,1]$.

1. **(15 points)** Formulate the Bellman equation for this worker and show that there is a unique value function v solving the corresponding operator.

2. **(15 points)** Analyze the worker's problem. Argue that the optimal strategy has the reservation wage property. Show that the number of hours worked is the same in every period.

Question 2. (50 points)

Consider an environment where agents/workers are matched with jobs. The output of a worker-job match is random: with probability π output is $q = 1$ and with probability $1 - \pi$ output is $q = 0$. A job-worker match is characterized by the probability π and comes in two different types: good matches π_1, and bad matches π_2, with $\pi_1 > \pi_2$. That is, good matches are more likely to generate positive output. The type of a match is not directly observed, but based on the past output performance of a worker, one can make inferences about the type of the match. Let α denote the a-priori probability that a match is of type $i = 1$. Let $\alpha' = \Phi(\alpha, q)$ denote the a-posteriori probability that a match is of type $i = 1$, that is, after observing that output is q.

1. **(10 points)** Use Bayes' law to derive the updating function Φ. *Hint: Recall that Bayes' Law stipulates* $\Pr(A|B) = [\Pr(B|A)\Pr(A)] / \Pr(B)$.

Suppose workers maximize the expected present value of utility from output

$$E_0[\sum_{t=0}^{\infty} \beta^t U(q_t)]$$

with $0 < \beta < 1$ and U a continuous, strictly increasing and strictly concave function with $U(0) = 0$. In any period, a worker can be matched with only one job. If at the beginning of a period a worker decides to leave a job, that worker will be matched with a new job at the end of the period. The new job match is equally likely to be of good or bad quality, that is when a new job match is created $\alpha = 0.5$. During the search period the worker does not receive any income. If a worker decides to stay with a job match, the match may still be exogenously dissolved at the end of the period with probability θ.

2. **(10 points)** Let $v(\alpha)$ denote the value of a match conditional on the a-priori probability α that the match is of good quality. Define the Bellman equation for $v(\alpha)$.

3. **(10 points)** Argue that there is a unique solution to Bellman's functional equation.

4. **(20 points)** Argue that there exists a critical value α^* such that a worker will quit a match if $\alpha < \alpha^*$.

Question 3. (70 points)

Consider an infinite horizon production economy populated by I (types of) agents. Time is discrete where $t = 0, 1, 2..., \infty$. Agents' preferences are represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{(c_{i,t})^{1-\sigma} - 1}{1-\sigma} \right\},$$

where $\beta \in (0,1)$ and $\sigma > 0$. Here, $c_{i,t}$ stands for i's consumption at date t.

There is an aggregate linear technology to produce the consumption good where aggregate output at date t is

$$Y_t = s_t K_t$$

where K_t denotes the aggregate stock of capital at date t and s_t is an i.i.d, s_t TFP shock at date t.

Let K_0 denote the date-0 stock of capital and the law of motion for capital is

$$K_{t+1} = K_t(1-\delta) + I_t,$$

where I_t denotes investment (in units of the consumption good) and $\delta \in (0,1)$ denotes the depreciation rate. Assume that

$$\ln(1 - \delta + s_t) \sim \mathcal{N}(\mu, \xi^2)$$

such that $\beta E(1 - \delta + s_t) < 1$.

1. Pareto Optimal Allocations

Let α_i be the welfare weight corresponding to agent i. Define the Planner's problem recursively for this economy.

1.a. (10 points) Characterize the solution justifying your answer. Show that individual consumption is a constant fraction of aggregate consumption, C. Moreover, argue that then one can solve the Planner's problem for a fictitious representative agent and then backout individual consumption from the previous expression.

1.b. (10 points) Provide an explicit representation for equilibrium $\ln C$ in terms of s, β, δ, μ, ξ^2.

2. Competitive Equilibrium with Sequential Trading

Suppose that every period agents meet in spot real and financial markets to trade the consumption good, a full set of Arrow securities and shares of a representative firm that operates both technologies (that is, one to produce the consumption good and the other to produce new capital; the firm accumulates capital and rents labor). Agents are endowed with $\theta_{i,0} > 0$ shares at date 0.

2.a. (5 points) Define a Recursive Competitive Equilibrium (RCE) for this economy. Be as precise as possible.

2.b. (5 points) Characterize a RCE justifying your answer. *Hint: write all the conditions that the individual (the agents' and the firm) policy functions must satisfy coupled with the consistency conditions.*

3. Decentralization and Equilibrium Portfolios

3.a. (10 points) Show how to find a welfare weight, α_0, that parametrizes a PO allocation that can be decentralized as a RCE. *Hint: Define the candidate price system and construct recursively the transfers needed to decentralize a PO allocation parameterized by α. Find α_0 that makes these transfers equal to zero at date* 0.

3.b. (10 points) Argue that the RCE can be supported with fixed equilibrium portfolios.

4. Asset prices

4.a. (10 points) Find a formula for the gross return on one-period risk-free bonds in this economy in terms of s, β, δ, μ, ξ^2. Then, provide a formula for the expected natural log of the one-period gross return to the firm's stock in this economy in terms of s, β, δ, μ, ξ^2.

4.b. (10 points) Provide a formula for the variance of the natural log of the gross return to the firm's stock in this economy in terms of s, β, δ, μ, ξ^2. The market price of risk is defined to be:

$$\frac{E(\ln(R^{mkt}) - \ln(R^f))}{var(\ln(R^{mkt}))}$$

where R^{mkt} is the gross return to the overall stock market and R^f is the gross return on one-period risk-free bonds. In our economy, R^{mkt} is the return to the firm's stock. Find an expression for the market price of risk in the model economy in terms of s, β, δ, μ, ξ^2.