Macroeconomics Comprehensive Examination: May 2010

Instructions. There are two parts to this exam. Part A consists of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 50 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully. Your exam number (e.g. E3) and question number (e.g. QB2) should appear on the top of every page of your answers.

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following statements contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each statement is worth 10 points for a total of 30 points.

1. We typically choose ℓ_∞ as the commodity space because it makes it possible to prove the first welfare theorem.
2. In a standard neoclassical growth model, it is optimal to tax initial holdings of capital at confiscatory rates because individuals do not consume capital.
3. The first welfare theorem fails in overlapping generations models even if the economy ends at some date T.

B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is weighted differently.

Question 1.

Consider the following optimization problem

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln c_t$$

$$\text{s.t.} \quad q a_{t+1} + c_t = a_t; \quad t = 0, 1, \ldots,$$

$$a_0 > 0 \text{ given.}$$

where $q > 0$ is a given parameter.

1. **(7.5 points)** Write down Bellman's functional equation (FE) as it applies to this problem.
2. **(7.5 points)** Verify that, for some number B, the function

   $$v(a) = \frac{1}{1-\beta} \ln a + B$$

   satisfies (FE).
3. **(7.5 points)** Show that the candidate solution defined via $c_t = (1-\beta)a_t$ is not optimal by describing a feasible alternative that yields higher utility.
4. **(7.5 points)** The problem, as stated, has no solution. How would you modify it to ensure that it does?

Question 2.

Consider an economy populated by a large number of identical worker with preferences represented by

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t c_t \right]$$

where c_t stands for consumption at t and $\beta \in (0, 1)$.

The worker does not suffer disutility from working; i.e., labor supply is inelastic. There is no financial market (for instance, agents can't save) and thus income must be consumed. Assume that if an individual is employed at wage w, his income net of taxes is $y = w - T(w)$, where

$$T(w) = \begin{cases} \tau w \text{ if } w < w_0 \\ \tau w_0 \text{ if } w \geq w_0 \end{cases}$$

where $\tau \in (0, 1)$.

Each period an unemployed worker draws one, and only one, offer to work at wage w. Wages are i.i.d. draws from the continuous c.d.f. F, where $F(0) = 0$ and $F(w^{\max}) = 1$ for some $w^{\max} > 0$. Unemployed agents decide whether to accept the offer or not. If he accepts the offer w, he gets w until his job is exogenously destroyed with probability θ (i.e., the separation rate). If he doesn't accept the wage offer, the unemployed worker is entitled to unemployment compensation, $\gamma > 0$, and pays no taxes. Also, he has the opportunity of a new wage offer draw next period. Assume that $w_0 < w^{\max}$ and $\gamma < w^{\max} - \tau w_0$.

1. **(10 points)** Provide a *recursive characterization of the worker's problem.* Argue that there exists a unique value function that is continuous, bounded and increasing.

2. **(20 points)** Prove that the worker's optimal policy has the *reservation wage property.* Derive an expression for this reservation wage w^*. *Hint: Use $\gamma < w^{\max} - \tau w_0$ to prove the existence.*

3. **(20 points)** Assume that $\theta = 0$ (i.e. workers keep their jobs forever). Go as far as you can analyzing the impact of changes in τ and w_0 on the before and after tax reservation wages. *Hint: If you denote the reservation wage by w^*, consider the two cases $w^* > w_0$ and $w^* < w_0$.*

Question 3.

Consider an infinite horizon economy with two types of agents indexed by $i \in \{1,2\}$. Time is discrete where $t = 0, 1, 2..., \infty$. Agents' preferences are represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\ln c_{i,t} - \frac{l_{i,t}^{\phi}}{\phi} \right) \right\},$$

where $\beta \in (0,1)$ and $\phi > 1$. Here, $c_{i,t}$ stands for i's consumption at date t and $l_{i,t}$ is his corresponding labor supply.

The process $\{s_t\} \in \{1,2\}$ is i.i.d with probability $\pi(s_t) > 0$ for all s_t, where s_0 is known at date 0. Let S^t be the set of partial histories up to date t, with typical element $s^t = (s_0, ..., s_t)$. These histories are observed by all the agents. The probability of s^t is constructed from π in the standard way and denoted by $\pi(s^t)$.

Agents are endowed with no financial wealth at date 0, and in each period they are endowed with labor productivity $e_{i,t}$ given by

$$e_i(s_t) = \epsilon_i + \tau_i(s_t)$$

where $\epsilon_i > 0$, $\epsilon_1 + \epsilon_2 = \Theta$, and $\tau_1(s_t) + \tau_2(s_t) = 0$ for all s_t.

There is an aggregate linear technology to produce the consumption good where aggregate output at date t is

$$Y(s^t) = B[e_1(s_t)l_1(s^t) + e_2(s_t)l_2(s^t)]$$

for some constant $B > 0$.

1. **(10 points) Pareto Optimal Allocations.** Let α be the welfare weight corresponding to agent 1. Define the Planner's problem recursively for this economy. Characterize the solution justifying your answer. *Hint: optimal policy functions will depend on welfare weights.*

2. **(20 points) Competitive Equilibrium with Sequential Trading.** Suppose that every period agents meet in spot real and financial markets to trade the consumption good and a full set of Arrow securities. Also, they participate in the labor market to sell their effective labor, $e_i l_i$, to competitive firms which operates the constant returns to scale technology. Agents are endowed with no financial wealth at date 0. Define a Recursive Competitive Equilibrium (RCE) for this economy. Be as precise as possible. Characterize a RCE justifying your answer. *Hint: write all the conditions that the individual (for the agents' and the firm) policy functions must satisfy coupled with the consistency conditions.*

3. **(20 points) Decentralization.** Argue how to find a welfare weight, α_0, that parametrizes a PO allocation that can be decentralized as a RCE. *Hint: Define the candidate price system and construct recursively the transfers needed to decentralized a PO allocation parameterized by α. Argue (no formal proof is required) that there exists some α_0 that makes these transfers equal to zero at date 0.*

4. **(20 points) Equilibrium Portfolios and Wealth Distribution.** Argue that equilibrium portfolios that support any RCE are kept constant. Define individual human wealth (i.e., the present discounted value individual labor income) and discuss how the distribution of total (i.e. financial plus human) individual wealth may or may not vary across time and state.