Microeconomics Comprehensive Exam

June 2010

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) Please write only on one side of a sheet of paper
(3) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 (a), 1 (b), etc.).

1. Consider a pure exchange economy with two consumers, Alphanse and Betatrix, and two goods, Perrier and Brie. Alphanse and Betatrix have the following utility functions respectively:

$$u_\alpha = \min\{x_{p\alpha}, x_{b\alpha}\}, \quad u_\beta = \min\{x_{p\beta}, \sqrt{x_{b\beta}}\},$$

where $x_{p\alpha}$ is Alphanse's consumption of Perrrier, $x_{b\alpha}$ is Alphanse's consumption of Brie, $x_{p\beta}$ is Betatrix's consumption of Perrrier, and $x_{b\beta}$ is Betatrix's consumption of Brie. Neither Alphanse, nor Betatrix can consume negative amounts of a good. Both of them are price takers.

(a) Alphanse has an endowment of 30 units of Perrier and none of Brie. Betatrix starts with 20 units of Brie and none of Perrier. Find equilibria in this economy. Illustrate your solution with the Egdeworth box diagram. [Hint: Start with the offer curves of the agents. Don't forget that the price vector $p = (p_p, p_b) > 0$, but it is not always the case $p \gg 0$.]

(b) Now suppose that Alphanse starts with 5 units of Perrier and none of Brie. Betatrix starts with 20 units of Brie and none of Perrier. Find equilibria in this economy. Illustrate your solution with the Egdeworth box diagram.

2. Consider an investor with an increasing and strictly concave Bernoulli utility function $u(w)$, where w is wealth. There are two assets: a safe asset with a (gross) return of \$1 per dollar invested and a risky asset with a (gross) return of \$$z$ per dollar invested. The random return z has a distribution function $F(z)$ on $\mathbb{R}$ such that $\int_{\mathbb{R}} z dF(z) > 1$; that is the mean return on the risky asset exceeds the return on the safe asset.

The investor wants to invest her initial wealth, $w_0 > 0$. The wealth can be divided in any way between the two assets. Let $\gamma \in [0, 1]$ denote the fraction of wealth invested in the risky asset.

(a) Characterize the investor's optimal choice of the portfolio in terms of γ. Pay particular attention to when (or if) corner solutions exist.

(b) Let $r_R(w)$ be the investor's coefficient of relative risk aversion. Show that if $r_R(\cdot)$ is increasing, then the fraction, γ, of wealth invested in the risky asset is decreasing in w. Similarly, if $r_R(\cdot)$ is decreasing, then γ is increasing in w.

3. There are two firms on the market, an Incumbent and an Entrant. The quality of the Incumbent's product is drawn randomly from a distribution: it has a high quality with probability $\lambda > \frac{1}{2}$ and low quality with probability $1-\lambda$. The Incumbent knows the quality of its product, but the Entrant does not know.
The Incumbent chooses how much to advertise its own product. The cost of advertisement level $a \geq 0$ is equal to ca, where $c > 0$ is a constant.
After observing the level of advertisement a, the Entrant decides whether to enter (*Enter*) or to stay out (*Out*). If the Entrant enters, his profits are equal to 1 if the Incumbent's quality is low and -1 if the Incumbent's quality is high. If the Entrant stays out, his profits are equal to 0.
Incumbent's payoffs depend on the level of advertisement a, the decision of the Entrant, the type of the Incumbent, and they are given in the table:

	High	Low
Enter	$-ca+\sqrt{a}$,	$-ca$,
Out	$\pi - ca + \sqrt{a}$,	$\pi - ca$,

where $\pi > 0$ is a constant. In particular, the advertisement raises only the payoffs of the high type of the Incumbent.

(a) Suppose first that the Incumbent type is known to be high, i.e., $\lambda = 1$. Show that for each a such that
$$\pi - ca + \sqrt{a} \geq \frac{1}{4c}, \tag{1}$$
the game has a Nash equilibrium in which the Incumbent plays a. On the other hand, show that there exists only one subgame perfect equilibrium, in which the Incumbent chooses $a^* = \frac{1}{2c}$.
In the rest of the question, assume that $\lambda \in (1/2, 1)$. Unless it is otherwise specified, assume that the solution concept is pure-strategy Weak Perfect Bayesian Equilibrium.

(b) In a pooling equilibrium, the two types of the Incumbent choose the same level of advertisement a. Show that in each pooling equilibrium, $a \leq a_0 = \frac{\pi}{c}$ and a satisfies (1). Describe the behavior and the beliefs in such equilibria.

(c) In a separating equilibrium, the two types choose different levels of advertisement a_L and a_H. Describe all separating equilibria.

(d) Assume that $a^* > a_0$ (i.e., that $\pi < \frac{1}{2}$). Argue that there exists a unique equilibrium that satisfies the Intuitive Criterion.

4. (a) Consider the problem of locating a public facility on a real line $\mathbb{R}$. There are two individuals, a and b. Let $\Theta_a = \mathbb{R}$ be the set of a's types and $\Theta_b = \mathbb{R}$ be the set of b's types, which describe their preferred locations respectively.
Preferences are assumed to be quasi-linear in the following form,

$$u_a(g, t_a; \theta_a) = -(\theta_a - g)^2 + t_a,$$
$$u_b(g, t_b; \theta_b) = -(\theta_b - g)^2 + t_b,$$

where g denotes the location and t_a and t_b denote income transfers to a and b respectively. Let $f = (g, t_a, t_b)$ be a social choice function, where $g : \Theta_a \times \Theta_b \to \mathbb{R}$ and $t_a, t_b : \Theta_a \times \Theta_b \to \mathbb{R}$ satisfy $t_a(\theta_a, \theta_b) + t_b(\theta_a, \theta_b) \leq 0$.

(a)-(i) Calculate the social choice function which is prescribed by the pivotal mechanism.
(a)-(ii) Verify that it is always a (weakly) dominant strategy for each individual to submit his true type.

(b) Consider auctioning a single object in an environment with symmetrically and independently distributed private values. There are n risk-neutral bidders, whose valuations are drawn from $[0, \overline{\theta}]$ according to an i.i.d. differentiable distribution F with density f.

Consider the following auction rule, which has no reservation price: (i) bidders simultaneously submit sealed bids; (ii) the highest bidder wins and pays the winning bid; (iii) **plus**, every bidder including the winner pays λ times her own bid, where $\lambda \geq 0$.

(b)-(i) Establish the first-order condition for the bidding function in symmetric Bayesian Nash equilibrium.
(b)-(ii) Describe the conditions under which the revenue equivalence theorem holds, and explain whether or not those conditions are satisfied here.