Macroeconomics Comprehensive Examination
University of Texas at Austin
August 2004

Instructions. There are two parts to this exam. Part A consists of a series of six short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 40 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (One hour: 60 points)

Instructions. Each of the following problems contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending our choice as your success is largely determined by how well you defend your choice. Each question is worth 10 points for a total of 60 points.

1. In the stochastic growth model, the correlation of employment and productivity is negative due to diminishing returns to scale.

2. If a household faces a constraint on how much it can borrow, then consumption allocations depend on more than just the discounted present value of income.

3. Due to the presence of durable goods, Hall's theory that consumption follows a random walk is false.

4. Private debt cannot have value in a finite horizon economy without commitment.

5. A one period reduction in distortionary labor taxes will have a larger impact on labor supply than will a permanent tax cut.

6. The observation that a country has a positive inflation rate indicates that the monetary authority suffers from a commitment problem.

B. Multipart Questions (Two hours: 120 points)

Instructions. Do all of the multi-part questions below. Each question is worth 40 points in total, but the subcomponents are not weighted equally.

Start each of these questions on a new page with your name and question # on each page.

1. Consider an overlapping-generations model with a single perishable good. The representative agent of generation t has preferences over consumption in old age c_t^o and employment in youth n_t given by: $u(c_t^o)) - \varepsilon g(n_t)$ where ε is a random shock to preferences known to generation t agents before they decide on their labor supply. Assume:

   - ε is common to all agents in a given generation
   - ε takes on two values, $\varepsilon_h, \varepsilon_l$ with $\varepsilon_h > \varepsilon_l$ and follows a first-order Markov process with a known transition matrix.
   - $u(\cdot)$ is strictly increasing and strictly concave
   - $g(\cdot)$ is strictly increasing and strictly convex
   - the size of each generation is equal to N
   - output $y_t = f(n_t)$ where $f(\cdot)$ is strictly increasing and strictly concave
   - there is a fixed stock of fiat money endowed only to the old in the initial period.

   a. Describe the optimization problem of a representation generation t agent. Write down this problem and the first-order conditions. (5 points)

   b. Write down the system of equations characterizing the employment levels in a stationary rational expectations equilibrium (SREE) with valued fiat money. (10 points)

   c. Assume ε is iid: i.e., the distribution of ε_{t+1} is independent of the period t realization of ε. In this case, prove that employment is higher and prices are lower when $\varepsilon_t = \varepsilon_l$ than when $\varepsilon_t = \varepsilon_h$. (15 points)

   d. Now consider the solution to the planner's problem. Write down the planner's objective function, constraints and the necessary conditions for a social optimum. Compare this allocation with the SREE. Show that if $u(c) = ln(c)$, then the equilibrium and planner's solution coincide. (10 points)

2. Consider an economy with a unit measure of agents. Each agent can choose to engage in one of two activities: crime or work. There is a government which chooses taxes $\tau_w \in \{0, 1\}$ and $\tau_c \in \{0, 1\}$ subject to the constraint $0 \leq \tau_c + \tau_w \leq 1$. Suppose the fraction of agents who work is given by μ and the fraction of agents who engage in crime is $(1 - \mu)$. Assume that any agent who engages in crime receives payoff $(1 - \tau_c)(1 - \mu)$ and any agent who chooses to work receives payoff $2(1 - \tau_w)\mu$. The government is benevolent and so it receives a payoff of $2\mu^2(1 - \tau_w) + (1 - \mu)^2(1 - \tau_c)$ (which is the average payoff of all agents).

   In what follows, you may restrict attention to pure strategy, symmetric equilibria (in which all agents make the same choices).

a. Suppose first that the timing is that the government moves first. Explicitly, the government first chooses (τ_c, τ_w) from the set $\{(0,0),(1,0),(0,1)\}$. The agents observe the government's choice. Then agents simultaneously choose whether to work or to engage in crime. What is the set of Ramsey equilibrium outcomes? Justify your answers. (17.5 points)

b. Now suppose that the government chooses second. Explicitly, the agents first simultaneously choose whether to work or engage in crime. The government observes μ (but not the choices of individual agents). Then the government chooses (τ_c, τ_w) from the set $\{(0,0),(1,0),(0,1)\}$. What is the set of time consistent equilibrium outcomes? Justify your answers. (17.5 points)

c. Does the government have a higher payoff in the time consistent equilibrium or the Ramsey equilibrium? Explain why or why not. (5 points)

3. Consider the following model with infinitely lived agents who have identical preferences but differ in their endowment profiles. Preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $u(c_t) = \log(c_t)$. Endowment profiles for the two different types of agents are given by

$$\begin{aligned} y_t^1 &= 1, t = 0, 1, \dots. \\ y_t^2 &= \begin{cases} 0 & t \text{ even} \\ w & t \text{ odd} \end{cases} \end{aligned}$$

where y_t^i is the endowment of agent type $i = 1, 2$. There is a unit measure of each type. Assume that $1 > \beta > \frac{1}{1+w}$.

a. Assume there is no money. Write down the decision problem faced by a type i agent who can buy or sell private one-period claims b_{t+1}^i to $t+1$ goods at price q_t at the beginning of any period t.Assume these claims are enforceable. (2.5 points)

b. State and solve for a competitive equilibrium allocation and prices. Hint: Since agents face the same environment every two periods, try constructing a periodic equilibrium. (15 points)

c. Using your results in (a) and (b), compute interest rates on one-period bonds in an economy where agent types' endowments follow:

$$\begin{aligned} y_t^3 &= 1, t = 0, 1, \dots. \\ y_t^4 &= \begin{cases} w & t \text{ even} \\ 0 & t \text{ odd} \end{cases}. \end{aligned}$$

(2.5 points)

d. Suppose there is a turnpike with a countably infinite number of rest stops. At each rest stop there are large numbers of the two types of agents in part A heading east who meet large numbers of the two types of agents in part B heading west (the large numbers assumption means you can continue to think of markets at the rest stops as being competitive). Private claims are enforceable between agents who meet each other at rest stops. Given the interest rates you computed in parts A and B, are there arbitrage opportunities? Who would like to trade with whom? Who trades with whom? (5 points)

e. Endow each agent type 2 with $\overline{M}$ units of fiat money at $t = 0$ and assume that there is no money supply growth. Write down the decision problem faced by a type i agent who can continue to buy or sell private one-period claims to $t+1$ goods b^i_{t+1} at current consumption price q_t and hold money M^i_{t+1},where the nominal price of consumption goods is given by P_t. Continue to assume that private claims are enforceable between agents who meet each other at rest stops. (2.5 points)

f. Is it possible to construct a competitive equilibrium where P_t is constant and money is held despite the fact that some private bonds are being offered at higher return? If so, construct it. If not, explain why not. (12.5 points)