Macroeconomics Comprehensive Examination

University of Texas at Austin

August 2005

Instructions. There are two parts to this exam. Part A consists of a series of six short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 40 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (One hour: 60 points)

Instructions. Each of the following problems contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each question is worth 10 points for a total of 60 points.

1. In the overlapping generations model without money, if we ignore the initial generation of old agents, then there are always allocations which yield agents higher lifetime utility than autarky.
2. In the stochastic growth model, preferences influence the steady state level of capital but not the transitional dynamics.
3. If the firm has no costs of adjusting its capital stock, then the optimal capital stock it chooses for the future is independent of the capital stock today.
4. A stationary rational expectations equilibrium is the same as a steady state equilibrium.
5. The elasticity of hours with respect to the real wage must be very low to match macroeconomic facts. Unfortunately, microeconomic studies find that this elasticity is very large.
6. If marginal costs of production are decreasing with output, then the firm will increase inventories when sales are low.

B. Multipart Questions (Two hours: 120 points)

Instructions. Do all of the multi-part questions below. Each question is worth 40 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your name and question # on each page.

Question 1.

Consider an infinite horizon overlapping generations model, with the following specification.

- the initial old have capital at the start of time, period $t = 1$
- all agents except the initial old
  1. live for two periods
  2. supply labor **inelastically** in the first period of life
  3. consume in both youth and old age with preferences given by $u(c) = ln(c)$ for each generation in every period of life
  4. capital is the only store of value: there is no money
- each generation has population of 1
- technology is specified by
  1. output is produced from a constant returns to scale technology with capital and labor as inputs.
  2. until part (d) of the question, total factor productivity is constant
  3. the technology is owned by perfectly competitive firms who rent capital from the old agents and hire the young agents as workers
- the capital depreciation rate is 100%
- there is a single good: output produced in a period is split between investment and consumption.
- all markets are competitive

Use this model to answer the following questions. If you need to add additional pieces to the environment, you can do so but be sure to make clear any assumptions you are making.

a. (2.5 points) Write down and describe the optimization problem for a representative household of generation t. What is the household's first order condition for saving? Interpret this condition.

b. (2.5 point) Write down the first order conditions for a firm's demand for labor and capital.

c. (20 points) Find the steady state level of the capital stock. Find the steady state real interest rate. Is the steady state efficient?

d. (15 points) Now suppose that productivity is stochastic. Write down a stochastic difference equation characterizing the evolution of the capital stock. When productivity is high in a period, what happens to the real interest rate?

Question 2.

Consider the following job search model. An unemployed agent receives benefits b in the current period while she searches and draws a future wage offer from a distribution $F(w')$. Wage offers are i.i.d. over time. An agent with a wage offer w can decide whether to accept it and work (thereby being employed) or reject it (thereby becoming unemployed) and search. An agent who chooses to work receives the same wage offer the following period with probability δ but becomes unemployed the following period with probability $1-\delta$. The agent's preferences are $E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$ where c_t denotes consumption, $u(\cdot)$ is an increasing function, and $\beta < 1$. Assume no borrowing or lending.

a. Set up the worker's decision problem as a dynamic program.

b. Show that the solution to the worker's problem is of the reservation wage form (i.e. that there exists a wage w^* such that for all draws $w \geq w^*$ the worker accepts and continues to search otherwise). Prove that the reservation wage is increasing in the level of unemployment benefits.

c. Calculate the probability that an unemployed worker is unemployed for N periods in a row as a function of the reservation wage and show that this probability is decreasing in N. How does this probability vary with unemployment benefits b? Justify your answer.

d. Suppose now that there are a large number of workers who all face the same problem as above. Define the unemployment rate as the fraction of all workers who are unemployed. How does the unemployment rate vary with b?

Question 3.

Consider the following optimal taxation problem. There is an infinitely lived representative agent. Her utility function is given by $\sum_{t=0}^{\infty}\beta^t u(c_t, \ell_t, g_t)$ where c_t is consumption, ℓ_t is leisure, and g_t is government services. Assume that the utility function is strictly increasing, concave, and differentiable in each of its arguments and that $\beta < 1$. The agent has an endowment of 1 unit of time. The production function is given by $F(k_t, n_t)$ which is strictly increasing, concave, and differentiable in each of its arguments. There is no technological change. Investment is given by $i_t = k_{t+1} - (1-\delta)k_t$. Suppose that the government can only levy proportional labor (τ_t^n) and capital (τ_t^k) income taxes in order to finance its expenditure on g_t but can freely borrow and lend at gross real rate of return R_t (so that it faces a present value budget constraint). Assume that the consumer takes prices and government policy as given when making decisions.

a. Set up and define a competitive equilibrium for each fixed sequence $(g_t, \tau_t^n, \tau_t^k)$.

b. Assuming interiority of the equilibrium, give the first order conditions that characterize the equilibrium you defined in (a).

c. Set up and define a Ramsey equilibrium assuming the government acts benevolently in choosing $(g_t, \tau_t^n, \tau_t^k)$.

d. Is it true that $\tau_t^k \to 0$? That is, does the Chamley-Judd characterization of the asymptotic behavior of Ramsey taxes extend to this setting where g_t enters the utility function? If your answer is yes, prove it. If your answer is no, prove it. Assume that in the Ramsey allocation, all quantities converge to constant levels (i.e. $c_t \to \bar{c}$, etc.)