Macroeconomics Comprehensive Examination
University of Texas at Austin
August 2006

Instructions. There are two parts to this exam. Part A consists of a series of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 50 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following statements contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each statement is worth 10 points for a total of 30 points.

1. If the technology exhibits increasing returns to scale, then there is a unique steady state in the growth model if and only if there is enough curvature in the utility function of the representative agent.

2. In the Overlapping Generations Model only the stationary rational expectations equilibrium is efficient while in the stochastic growth model all equilibria are efficient.

3. Since all agents are identical in the stochastic growth model, there is, in equilibrium, no borrowing and no lending and thus no consumption smoothing.

B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is worth 50 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your name and question # on each page.

Question 1.

Consider the choice problem of an individual who gains utility from the consumption of nondurable goods, c, and the service flow, s, from a durable good, a car. Assume that utility is linear and additive in c and s. Assume each individual has only one car. The service flow from a working car is either good, $s = s^g$, or bad, $s = s^b$, $s^g > s^b > 0$. A car which does not function generates a zero service flow, $s = 0$.

A new car costs p units of the nondurable good and yields service flow s^g. Over time, the service flow of the car changes stochastically. With probability π^g, a car with good service flow in the present period is a car with good service flow in the next period. With probability $(1 - \pi^g)$, a car with good service flow in the present period becomes a car with bad service.

With probability π^b, a car with bad service flow in the present period is a car with bad service flow in the next period. With probability $(1 - \pi^b)$, a car with bad service flow in the present period becomes a car which does not function.

Suppose the individual, after buying a car, can either keep the car or scrap it (at zero value) and buy a new one. Take this statement as given at this point; we relax it below.

a. (2.5 points) Write down a transition matrix characterizing the stochastic structure of car service flow.

b. (2.5 points) What are the state and control variables for the individual's optimization problem?

c. (10) Assume the individual always prefers to buy a new car rather than be without a car. Let V^g be the value for the agents problem if the car is good and let V^b be the value of having a bad car. Write down and explain the equations for V^g and V^b. Is there a solution? Is it unique?

d. (5) Show that as s^g increases, both V^g and V^b increase as well. Explain why this is true.

e. (5) In the statement of part c, it was assumed "... the individual always prefers to buy a new car rather than be without a car". Under what conditions is this statement true?

f. (10) Suppose that an individual can sell a car with bad service flow at a price z. How does this additional choice change V^b? Show that this additional choice will not reduce V^g. When will the individual choose to sell a car with bad service rather than holding on to it until it stops functioning?

g. (15) A researcher is interested in understanding the impact of changes to income on the purchase decisions on cars. Explain to the researcher why this model cannot address that issue. Then explain how you would modify the model to study the impact of income variations on the demand for cars.

Question 2.

Consider an economy inhabited by a unit measure of identical agents whose lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

where c_t is consumption, n_t is labor supply, and $\beta \in (0,1)$. The production technology is $y_t = n_t$ (there is no capital in this economy). Firms hire labor at a competitive wage rate. Each period the government purchases an exogenous, possibly variable amount $g_t < 1$ of consumption goods, which must be financed using a linear consumption tax of rate τ_t. The government starts in period $t = 0$ with no assets or liabilities.

a. (2.5 points). Write down the forward market (or time 0) Arrow-Debreu budget constraint of a household letting p_t denote the price of a unit of consumption at time t in terms of time zero consumption goods with p_0 normalized to 1. Do the same for the government.

b. (2.5 points). Assuming taxes are taken as given, define a competitive equilibrium.

c. (5 points). Write down the first order conditions for the household problem, given arbitrary taxes τ_t and intertemporal prices p_t. Let θ denote the multiplier on the constraint.

For the remainder of the question, assume

$$u(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - n_t.$$

d. (15 points). Assuming that the solution to the household problem is at an interior point, use part c to characterize the equilibrium price sequence. Furthermore, characterize tax revenues (i.e. $\tau_t c_t$) solely in terms of the consumption sequence.

e. (2.5 points). Use the expressions obtained in part d for tax revenues and prices to obtain an expression for the budget constraint of the government that depends only on the sequences $\{g_t, c_t\}_{t=0}^{\infty}$ and any other relevant parameters.

f. (2.5 points). Use the economy-wide resource constraint to obtain an expression for the agent's utility function that depends on the sequences $\{g_t, c_t\}_{t=0}^{\infty}$ and any other relevant parameters.

Now consider the Ramsey problem. That is, consider a benevolent government who wishes to find the sequence of optimal consumption $\{c_t\}_{t=0}^{\infty}$ that maximizes the lifetime utility of the agent (your answer to part f) subject to being able to finance its exogenous purchases $\{g_t\}_{t=0}^{\infty}$ (your answer to question e).

g. (15 points). Let μ denote the Lagrange multiplier of the budget constraint of the government. Write down the FOC associated with the government's problem and show that the optimal level of consumption, c^*, is constant over time. What is the implication of this result in terms of the best tax policy?

h. (5 points). Compute c^* as $\mu \to \infty$. Briefly interpret your result. [Hint: Compare your results to the one that results from maximizing total tax revenues, $c_t^{1-\sigma} - c_t$].

Question 3.

Consider the following matching model. Time is discrete and infinite. There are $G \geq 3$ distinct, divisible, and perishable goods and a unit measure of people who specialize in each of the goods. A person who specializes in good g consumes only good $g+1$ (modulo G).[1] Utility in any period is given by $u(x) - y$ where x is the amount consumed and y is the amount produced and u is a strictly increasing, concave function with $u(0) = 0$ and $u'(0) = \infty$. Agents discount the future at rate $\beta < 1$. There is a record keeping technology which publicly discloses the actions of a fraction $B \in [0,1]$ of each type of agents. We call those agents bankers (denoted b) and the other fraction $1 - B$ nonbankers (denoted n). Each banker can costlessly create storable, indivisible notes. There is a storage technology restriction that an agent can only hold one note of any variety at a time. The matching technology permits only bilateral meetings between agents which given the above specification of preferences and technology means there cannot be double coincidence meetings. Consumption and production takes place at the end of the period.

In what follows we choose the following state variables. For a nonbanker, it is simply whether he is holding a note or not (i.e. $\{0,1\}$). For a banker, the relevant state is not whether she is holding a note or not (since she could always costless produce one) but whether she has deviated from producing in any past or current meeting in a single coincidence where another banker wants what she produces or a nonbanker wants what she produces and has a banknote. Thus, let the state space for bankers be $\{0,1\}$ where 0 denotes that the banker has not deviated and 1 denote that she has deviated. All bankers start in state 0 at the beginning of time.

The following notation will prove useful. Let $y_{i,j}^{k,\ell} \in \mathbb{R}_+$ denote the output produced by a type $k \in \{b,n\}$ agent for a type $\ell \in \{b,n\}$ where the producer is in state i and the consumer is in state j. Further, let x_i^k denote the fraction of $k \in \{b,n\}$ agents who specialize in each type of good who are in state $i \in \{0,1\}$. Consistency requires $\sum_{i\in\{0,1\}} x_i^b = B$ and $\sum_{i\in\{0,1\}} x_i^n = 1 - B$. Using this notation, for instance, the probability that a banker meets another banker who wants his good is $\frac{x_0^b}{G}$.

We will consider a steady state equilibrium with the following strategies. If there has ever been an instance where a banker has not produced for another banker or a nonbanker with a note who likes his good, the strategy specifies that no one produces. Provided there has never been an instance where a banker has not produced for another banker or a nonbanker with a note, then

(i) a banker produces $y_{0,0}^{b,b} = y^* > 0$ for another banker who likes his good (and consumes it),

(ii) a banker produces $y_{0,1}^{b,n} = y^* > 0$ for a nonbanker who has a note and who likes his good. The nonbanker consumes y^* and gives the note to the banker who takes the note out of circulation (i.e. destroys it),

(iii) a nonbanker without a note produces $y_{0,1}^{n,n} = y^* > 0$ for another nonbanker who has a note and likes his good. An exchange of goods for a note occurs and consumption is undertaken,

(iv) a nonbanker without a note produces $y_{0,0}^{n,b} = y^* > 0$ for a banker who likes his good. An exchange of goods for a note occurs and consumption is undertaken,

(v) in all other states and matches, nothing happens (for instance, a banker does not produce $y_{0,0}^{b,n} = 0$ for a nonbanker who does not have a note but who likes his good).

a. (15 points) Under the above strategy, write the value functions v_i^k for $k \in \{b,n\}$ and $i \in \{0,1\}$.

b. (5 points) In order for the above strategy to be optimal, what incentive constraints must be satisfied?

c. (10 points) Under the above strategy and incentive constraints, what is the steady state distribution $\{x_i^k\}_{k\in\{b,n\},i\in\{0,1\}}$ of agents?

[1] All modulo means is that if $G = 3$, then an agent who produces good 3 consumes only good 1.

d. (20 points) Under what restrictions on parameters is a steady state allocation with the above strategies achievable? Explain why these restrictions make sense.