Macroeconomics Comprehensive Examination: August 2008

Instructions. There are two parts to this exam. Part A consists of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 50 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following statements contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each statement is worth 10 points for a total of 30 points.

1. Increases in uncertainty reduce spending on durable goods.
2. An Euler equation provides only a necessary condition for optimality when an optimization problem has an infinite horizon but the Euler equation is necessary and sufficient for finite horizon optimization problems.
3. According to the stochastic growth model, the Great Depression came about from an increase in the value of leisure for the representative agent.

B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is worth 50 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your exam # and question # on each page.

Question 1.

Consider an economy with a large number of two-period lived agents. Each has an endowment of the single good in youth and an endowment of the single good in old age. The endowment of agent α is α units in youth and $(1-\alpha)$ units in old age. Let the average value over all agents of the endowment in youth be $\bar{\alpha}$.

Agents can borrow and/or lend in a loan market with a real interest rate of R. There is a borrowing constraint, denoted $\bar{b}$. The amount borrowed by an agent cannot exceed $\bar{b}$.

Agents have utility over consumption in youth, c^y, and old age, c^o, given by $u(c^y) + v(c^o)$. Assume that both $u(\cdot)$ and $v(\cdot)$ are strictly increasing, concave. At least one of these functions is strictly concave.

1. (10 points). Write down the optimization problem of an agent. Write down the first-order condition, being careful about the borrowing constraint. Explain both the optimization problem and the first order condition in words. What can you say about which types of agents will face a binding constraint?
2. (15 points). Suppose $u(c^y) = ln(c^y)$ and $v(c^o) = c^o$. Find the market clearing real interest rate when the borrowing constraint is not binding. Find the market clearing real interest rate when the borrowing constraint is binding for some agents. Explain in words the difference between these two rates.
3. (10 points). Does the existence of the borrowing restriction reduce the utility of all agents?
4. (15 points). The government proposes to sell debt to agents and transfer the proceeds to agents in youth in a lump sum manner. The government will then raise the revenue to pay off its debt in the second period. Show that this policy can affect the welfare of agents in this economy. Explain.

Question 2.

2. Consider the following firm decision problem in a competitive market. Productivity shocks can take on two values $z_t \in \{z^L, z^H\}$ with $1 = z^H > z^L = 0$. Productivity shocks follow a markov process (i.e. $prob(z_{t+1} = z^i | z_t = z^i) = \theta$, $i \in \{H, L\}$). Any individual firm's production is given by $y_t = 2z_t n_t^{1/2}$ where n_t is the quantity of labor hired in period t. The price of a firm's good is denoted P and the wage is normalized to 1, both assumed independent of time. Any time a firm chooses to produce, it incurs a fixed cost c_f. Firms discount the future at rate β. Firms can enter the market as well. To enter they must pay a fixed cost $c_e > \frac{c_f}{(1-\beta\theta)}$ and take a draw from a distribution which assigns equal probability to each of the two shocks (this happens to be the ergodic distribution of productivity shocks). Assume that there are perfect capital markets so that if in any period it cannot cover its fixed cost, it can borrow against periods in which it can cover it.

The timing in any given period is as follows:

- A firm's productivity shock z_t is realized at the beginning of the period.
- The firm chooses whether to exit or not. If it exits, it receives zero payoff in the current and all future periods.
- If a firm stays, it chooses how much labor to hire and receives resulting profits

a. (10 points) Write down and solve a firm's (static) profit function. What are its properties with respect to z?

b. (10 points) Write down the firm's dynamic programming problem

c. (15 points) Under what conditions on parameters does an equilibrium where all firms with low productivity shocks exit and all firms with high productivity shocks remain?

d. (15 points) Under what conditions is there firm entry? What does this imply about prices P and ultimately the conditions in part c?

Question 3.

Consider the following risk sharing problem. There is a unit measure of ex-ante identical agents who live for 2 periods. Everyone is endowed with a unit of the storable consumption good at time 1. The gross return on storage R is 1. In the second period, they must expend effort e to produce the consumption good $y_2 = e$. Half the agents will receive a shock to the disutility of effort θ_H and the other half will receive a shock $\theta_L < \theta_H$. Expected utility is given by

$$U(c_1) + \frac{1}{2}\left[U(c_{2H}) - \theta_H V(e_H)\right] + \frac{1}{2}\left[U(c_{2L}) - \theta_L V(e_L)\right]$$

where U is strictly increasing, strictly concave, and differentiable while V is strictly increasing, strictly convex, and differentiable.

a. (15 points) Write down and solve the planner's problem if all consumption and effort is observable. What are the optimal allocations of consumption and effort across states and time. What do the resulting consumption and effort choices imply about transfers across time and states?

b. (10 points) Now suppose that disutility shocks are unobservable. Is the planner's allocation in question (a) incentive feasible? That is, would anyone have an incentive to lie? Write down the incentive compatibility constraints that are necessary for truthtelling. Specifically, if we assume that agents are assigned report contingent allocations of consumption and effort (c_{2H}, c_{2L}, e_H, e_L), what constraint implies truthtelling?

c. (25 points) Write down the information constrained planner's problem. How does second period consumption vary with the state? Show that the solution to the planner's problem can be characterized by an "inverse Euler equation" written as

$$\frac{1}{U'(c_1)} = E\left[\frac{1}{U'(c_2)}\right].$$

What does this imply about consumption in periods 1 and 2 relative to what was in question a?