Macroeconomics Comprehensive Examination
University of Texas at Austin
June 2005

Instructions. There are two parts to this exam. Part A consists of a series of six short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 40 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (One hour: 60 points)

Instructions. Each of the following problems contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending our choice as your success is largely determined by how well you defend your choice. Each question is worth 10 points for a total of 60 points.

1. Government deficits have no effect on the real interest rate.
2. If utility depends on consumption and leisure, then the marginal utility of consumption will not follow a random walk.
3. The stochastic growth model implies a positive correlation between productivity and the nominal interest rate.
4. Inflation is always and everywhere a monetary phenomenon.
5. The government should lower taxes in a recession.
6. After the firm builds a plant, the government will raise the firm's tax rate.

B. Multipart Questions (Two hours: 120 points)

Instructions. Do all of the multi-part questions below. Each question is worth 40 points in total, but the subcomponents are not weighted equally.

Start each of these questions on a new page with your name and question # on each page.

1. Consider a two-sector economy. Sector 1 produces a composite good which can be used either for consumption or investment. Sector 2 produces a single good which is only consumed. The production function for sector i is $A_i f^i(k_i)$ where A_i is a technology shock and $f(\cdot)$ is a strictly increasing, strictly concave function of k_i, the capital allocated to sector i.

   There is no labor. Preferences over the consumption of the two goods are represented by $u(c_1, c_2)$.

   Time is infinite. The economy starts with a given stock of capital and it evolves depending on investment and the rate of depreciation, denoted δ.

   The two shocks A_i for $i = 1, 2$ are known at the start of the period and follow an AR(1) process. Assume the shocks are not correlated across sectors. The capital stock is allocated across sectors after the realization of A_i for $i = 1, 2$.

   a. Write down the planner's problem for this economy as a dynamic programming problem. What are the state variables? What are the control variables? What are the resource constraints? [5 points]

   b. Write down a first-order condition for the problem which characterizes the allocation of capital across sectors within a period. Interpret this first-order condition. Suppose that $u(c_1, c_2) = \tilde{u}(c_1 + c_2)$. If $A_1 > A_2$, what can we conclude about the allocation of capital across the two sectors? What is the intertemporal Euler equation for this problem? [15 points]

   c. Assume now that capital is immobile between sectors: investment is sector specific so that new capital produced in sector 1 in period t becomes either sector 1 or sector 2 capital in period $t + 1$. Investment in sector 1 and sector 2 must be nonnegative.

   Write down the planner's problem for this economy as a dynamic programming problem. What are the state variables? What are the control variables? What are the resource constraints? What are the intertemporal first-order conditions? [15 points]

   d. Suppose you wished to enrich the model to study the sectoral allocation of labor. In particular, you wanted to see conditions under which employment would be positively correlated across sectors. **Explain in words** how you would amend the model to study this fact. [5 points]

2. Consider the behavior of a firm which maximizes profits taking as given exogenously period t sales (equivalently demand), denoted S_t. Assume the price is 1 so that sales equal revenues for the firm. The firm must meet its sales (demand) either by selling from its stock of inventory or producing. Here sales are stochastic and inventories must be nonnegative.

   Assume the cost of producing q units is given by $C(q, A)$ where $C(\cdot)$ is strictly increasing and strictly convex in q. Here A is a random variable and $C_q(q, A)$ is decreasing in A.

   Goods can be stored as inventories. A unit placed in inventory in period t yields R units of the good in period $t+1$. The firm has a fixed discount rate β where $\beta R = 1$.

   a. Write down the infinite horizon optimization problem for the firm. What are the state variables? What are the controls? Write down and interpret the first-order condition for the inventory decision. [20 points]

   b. Under what conditions will this model imply that the variance of production is less than the variance of sales? [10 points]

   c. In actual data though, the variance of production frequently exceeds the variance of sales. Under what conditions will this model imply that the variance of production exceeds the variance of sales? Explain and contrast this finding with part b. [10 points]

3. Consider the following job search model. There is a unit measure of workers and a unit measure of entrepreneurs. Workers can choose to work ($n_t = 1$) or not ($n_t = 0$). Entrepreneurs each own a technology that takes as input one worker's time and yields output $y_t = A_t n_t$. Productivity shocks $A_t \in \{A_H, A_L\}$ are distributed identically and independently across entrepreneurs and follow a symmetric markov process $\pi_{jk} = prob(A_{t+1} = A_j | A_t = A_k)$ where $\pi_{HH} = \pi_{LL} = \pi$ and $A_H > A_L$. Productivity shocks occur whether the entrepreneur is matched with a worker or not and are fully observable. Both workers and entrepreneurs have linear preferences and discount the future at rate $\beta < 1$: $c_t - \gamma n_t$ for workers and c_t for entrepreneurs where c_t denotes consumption. Each period, workers and entrepreneurs are either in a match (employed) or unmatched (unemployed searching for a job opportunity). Each period an unemployed worker is matched with an entrepreneur who has a vacancy. Workers and entrepreneurs can choose to end a match. The timing in a given period is: (i) at the beginning of the period the entrepreneur receives her technology shock; (ii) unmatched workers and entrepreneurs are matched; (iii) workers and entrepreneurs play a Nash bargaining game with equal weights to determine the worker's wage subject to voluntary participation where the choice not to participate means both parties enter the unemployment pool to be matched next period; (iv) production and consumption occurs in active matches at the end of the period (remember this when you think about discounting).

a. Set up the worker's choice problem of whether to accept a job (conditional on the entrepreneur offering it) or continue to search. Let p_i denote the probability of a match between a type i entrepreneur and a worker. [7.5 points]

b. Set up the entrepreneurs choice problem of whether to offer the job (conditional on the worker accepting it) or to continue to search (post a vacancy). Note that an "unemployed" entrepreneur of type i is matched with probability one with a worker.[7.5 points]

c. Set up the Nash wage bargaining problem in an active production match and characterize wages by providing the first order conditions. [5 points]

d. What is the probability that an unemployed entrepreneur has a high productivity project? Explain your answer. [2.5 points]

e. Assume $A_H > \gamma > A_L$. Is it possible to construct an equilibrium where only high productivity matches continue? If so, provide sufficient conditions. (Hint: Given your answer in part d, what is the probability that an unemployed worker matches with a high productivity entrepreneur in this type equilibrium? Is there a way to construct a limiting argument with $\pi \to 1$?) [17.5 points]