Macroeconomics Comprehensive Examination: June 2008

Instructions. There are two parts to this exam. Part A consists of three short answer questions, each are worth 10 points. Part B consists of three multi-part questions: each multi-part question is worth 50 points. The exam is designed to take three hours to complete and the total points on the exam are 180, so that there is one point per minute. Budget your time carefully.

A. Short Answer Questions (30 minutes: 30 points)

Instructions. Each of the following statements contains one or more assertions that are either True, False or Uncertain. Classify each assertion as either True, False or Uncertain and **defend** your choice. Be as specific (this means support your words with at least the outline of a model) as possible in defending your choice as your success is largely determined by how well you defend your choice. Each statement is worth 10 points for a total of 30 points.

1. If agents are finitely lived and are searching for a job, then their reservation wage will rise as they become older.

2. In the stochastic growth model, the correlation of real interest rates and investment is positive.

3. If households can borrow and lend, there is no gain to tax smoothing.

B. Multipart Questions (2.5 hours: 150 points)

Instructions. Do all of the multi-part questions below. Each question is worth 50 points in total, but the subcomponents are not weighted equally. Start each of these questions on a new page with your name and question # on each page.

Question 1.

Consider an infinite horizon overlapping generations model with two period lived agents. Each agent produces when young and consumes when young and old. There is a single good produced each period which can be consumed or stored. Preferences of a generation t young agent are given by:

$$u(c_t^y - g(n_t)) + v(c_{t+1}^o). \tag{1}$$

Here c_t^y is consumption when young, c_{t+1}^o is consumption when old and n_t is hours worked in youth of a generation t agent. Both $u(\cdot)$ and $v(\cdot)$ are increasing and concave. The function $g(\cdot)$ is increasing and convex.

Output comes from the labor input of a household according to the production function $y_t = f(n_t)$, where $f(\cdot)$ is strictly increasing and strictly concave. There are no firms, just household production of the single good.

The good can either be consumed by the household, stored (negative storage is not feasible) or sold for fiat money. If a household places s_t units into storage in period t, then the household has $z_{t+1}h(s_t)$ units of the good in the following period. Here $h(\cdot)$ is strictly increasing and strictly concave and z_{t+1} is an iid random variable, common across households.

1. (10 points). Write down the optimization problem of a representative generation t agent. Assume that the realization of z_{t+1}, the shock on storage, is **not** known when the storage decision is made.

2. (15 points). Characterize a stationary rational expectations equilibrium in which money is valued and storage is positive. How do equilibrium prices and quantities (storage, real money demand, consumption, hours worked) vary with the shock on storage? What is the equilibrium relationship between the return on storage and the return on fiat money? What would that relationship be if $v(\cdot)$ was linear instead of strictly concave?

3. (25 points). Now characterize a stationary rational expectations equilibrium for the case in which the return on storage for generation t agents **is known** before they make their storage decision. To do so, write down and discuss the individual optimization problem, (10 points) and then characterize the stationary rational expectations equilibrium (15 points) with valued fiat money. Is there always an equilibrium with valued fiat money and positive storage? How do employment, consumption and savings decisions depend on the realization of z? Is there an equilibrium in which prices, storage and real money demand are independent of the current realization of z?

Question 2.

Consider an economy with a unit measure of identical, infinitely lived individuals. Each maximizes the discounted present value of utility given by $\sum_t \beta^t u(c_t)$ where $u(\cdot)$ is strictly increasing and strictly concave and $\beta \in (0, 1)$.

Each individual has access to a backyard technology which produces output from capital and aggregate output given by:

$$y_t = Ak_t^{\alpha} Y_t^{\gamma}. \tag{2}$$

In this expression, y_t is the output of an individual producer, k_t is the capital of an individual producer, $A > 0$ is a constant and Y_t is average (aggregate) output in the economy (i.e. $Y_t = \int y_t d\mu$ where μ is the unit measure). Assume that both α and γ lie between zero and one and $\alpha + \gamma < 1$. Output is either consumed or invested to accumulate capital through $k_{t+1} = k_t(1 - \delta) + i_t$ where $\delta \in (0, 1)$. All production is done by individual households (there are no firms in this economy).

1. (5 points). Write down and explain the optimization problem of an individual taking Y_t as given. What is the individual's Euler equation? Explain it in words.
2. (10 points). Given there are many identical households, what are the conditions for equilibrium of this decentralized economy? What is the steady state equilibrium level of the capital stock and of output?
3. (15 points). Suppose there is a single agent (a planner) who allocated economywide resources in this economy. What is the planner's optimization problem? The planner's Euler equation? Compare the steady state of the planner's problem with the steady state of the decentralized problem. Are the steady state levels of the capital stock different? Explain why or why not.
4. (20 points). Suppose the government can commit to a balanced budget sequence of production taxes/subsidies $s_t y_t$ along with a lump-sum tax/subsidy T_t in the decentralized economy. What should the government do if it wants to maximize the welfare of its citizens? Suppose there isn't a government commitment technology. Is there a commitment problem?

Question 3.

Consider the following version of the original Kiyotaki-Wright model of indivisible, nonstorable goods and indivisible money $m \in \{0, 1\}$. The key difference is that agents can choose to go to a bank to ensure they can consume (achieving utility U) but have to pay a fixed disutility cost $b < U$ to do so. Assume that the feasibility of this allocation at the bank is always guaranteed to any agent (for example, the bank can borrow abroad, etc.). If an agent chooses not to go to the bank, then she is randomly matched to another agent. If the agent has no money, then she is matched with a buyer who likes her good and has money with probability M. If the agent has money, then she is matched with a seller without money who can produce the good she likes with probability $(1 - M)$. In any match, the agent also receives utility U if she consumes but receives disutility $e < b$ if she produces in a match. The unit measure of agents who are initially endowed with money is given by the fraction M. Agents discount the future at rate $1/(1 + r)$. The timing in any given period is: decide whether to go to the bank or not, if not match in pairs, consume/produce at the end of the period.

1. (10 points). Let V_m^d denote the value function of an agent with $m \in \{0, 1\}$ units of money (in a steady state equilibrium) who decides to take action $d \in \{0, 1\}$ where $d = 1$ means she goes to the bank and $d = 0$ means she does not go to the bank. Let $V_{m'}$ denote the value after she has made the decision whether or not to go to the bank. Let δ_m denote the probability that an agent with money holdings m takes action $d = 1$. Further, when not at a bank, let π_0 denote the probability that a person without money chooses to produce in exchange for a unit of money and Π_0 denote the probability that all other agents choose to produce. Similarly, let π_1 denote the probability that a person with unit of money buys a good that she likes and Π_1 denote the probability that all other agents choose to buy the good that they like. Write the value functions.
2. (5 points). Characterize decision rules d_m and π_m in terms of value functions.
3. (5 points). Does there always exist an autarkic equilibrium (where no one consumes)?
4. (15 points). Is there a pure strategy equilibrium where no one goes to banks? If so, how does it depend upon the supply of money M in the economy? On the costs of going to the bank?
5. (15 Points). Can money and banks co-exist? That is, is there an equilibrium where there is active trade in money and at banks? If so, what are the conditions for existence?