Microeconomics Comprehensive Exam

August 2004

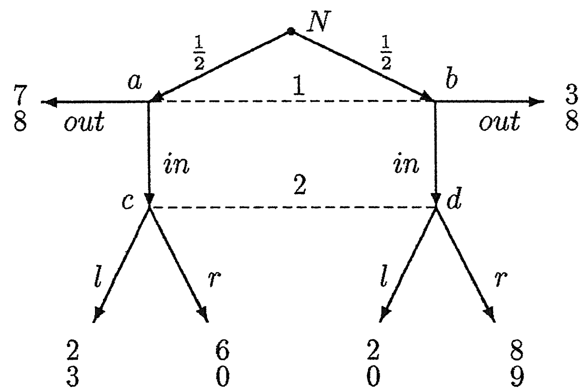
Instructions:

(1) The 4 problems on the exam are equally weighted. Use this information to help you maximize your score.

(2) Please answer each question on separate pieces of paper.

(3) When finished, please arrange your answers alphabetically.

A: The problem concerns the following extensive form game.



The node, •, starts the game. It is where Nature, N, moves. The non-terminal nodes are labelled a, b, c, and d. In this game, N plays to arrive at nodes a and b with probability $\frac{1}{2}$ each, and has no strategic interests.

The utilities of players 1 and 2 are given in the columns of numbers, with player 1's utility on top and 2's on the bottom.

1. Give the normal form for this game.
2. Find all of the equilibria, pure and mixed, for the normal form you just gave.
3. Show that all of the normal form equilibria you just gave correspond to perfect Bayesian equilibria of the extensive form game given above.
4. Show that only the equilibrium involving 1 playing *in* is sequential.

B: Suppose that the pure exchange economy $\mathcal{E}$ has two people, $I = \{1, 2\}$, each has consumption set $X_i = \mathbb{R}^2_+$, the endowments are $\omega_1 = (2, 6)$, $\omega_2 = (8, 19)$, that preferences $\precsim_1$ can be represented by the utility function $u_1(x_1, y_1) = \min\{x_1, \sqrt{y_1}\}$, and that $\precsim_2$ can be represented by the utility function $u_2(x_2, y_2) = 3x_2 + y_2$ where x_i is person i's consumption of the first good and y_i their consumption of the second good.

1. Carefully give the definition of a competitive (or Walrasian) equilibrium for general exchange economies.

2. Carefully state the First Fundamental Theorem of welfare economics.

3. Find and graph all of the competitive equilibria for the specific economy given above, if there are any.

4. Find and graph all of the core allocations for the specific exchange economy given above, if there are any.

5. Define what it means for an allocation to be Pareto optimal. Using your definition, find and graph all of the Pareto Optimal allocations for the specific exchange economy given above, if there are any. [Hint: person 1's utility function is not differentiable. Using derivative based techniques, including Lagrangeans, will make this problem very easy to grade, and completely waste your time.]

C: Background: Farmers in the Mississippi basin put nitrogen based fertilizer on their soil before they know how much rain there will be during the growing season. The fertilizer is especially effective when there is a great deal of rain, and is wasted (it ends up in the river) if there is not a great deal of rain. Since it is too late to apply the fertilizer when the rains come, there is a great deal of "precautionary" fertilizing.

When there is not a great deal of rain, the extra fertilizer ends up in the Mississippi river, flows into the Gulf of Mexico, encourages algæ growth, the algæ use up almost all of the oxygen in the water, and this kills all other sea life.[1] This has a negative effect on both the fishing and tourism industries.

Recently, in an effort to cut back on the precautionary use of fertilizer, the following kind of insurance program has been put in place: farmers designate one quarter of their acreage as "High," and the rest as "Low;" they put different amounts of fertilizer on each area; if there is heavy rain, the yield of the (presumably more heavily fertilized) High quarter of their acreage is measured, and farmers are paid for the extra that would have grown if they had had the High results on all of their acreage.

Farmer Francine owns A acres in the Mississippi basin. In Francine's part of the basin, there is a Hi amount of rain with probability π, $P(X = Hi) = \pi$, and a Lo amount of rain with probability $1 - \pi$, $P(X = Lo) = 1 - \pi$, $0 < Lo < Hi$. The *per acre* production function is given by $G(n, X)$ where: n measures the tons of nitrates on an acre; X measures the inches of rain on an acre; and $G(\cdot, \cdot)$ is an increasing, continuous differentiable, concave function satisfying $\partial^2 G/\partial n \partial X > 0$. Francine takes the price of her output, p, the price of fertilizer, c, and the terms of the insurance program as given. Further, since Francine's part of the Mississippi basin is small relative to the grain market, the prices are not affected by the weather in her region.

1. Write out Francine's optimization problem before the insurance program was put into place. Give the first order conditions. Show that her optimal fertilizer level: increases with p; increases with π; and decreases with c.

2. Write out Francine's optimization problem if she takes part in the insurance program. Give the first order conditions. Show that the amount of fertilizer on the quarter of her acreage is higher than it is without the insurance program.

3. Give conditions on the insurance program that make Francine's participation voluntary.

4. Is it possible for the insurance program to produce a socially optimal level of fertilizer use? Explain.

[1]The area of the dead zone that has resulted is bigger than many of the states that were in existence when the U.S. became an independent country.

D: Each person in a large population has an expected value of medical care, R. The proportion of the population with $R \leq r$ is given by $F(r) = 1 - e^{-\lambda r}$ for some $\lambda > 0$. A complete insurance contract collects an amount p up front and then pays for all medical expenses. As a function of R, the maximal willingness to pay to for a complete insurance contract is $p(R) = w(1 - \frac{1}{R+1})$ for some $w > 0$.

For parts of the following, you might want the formulæ: $\int_0^\infty \lambda r e^{-\lambda r}\, dr = \frac{1}{\lambda}$; and an equation $ax^2 + bx + c = 0$ has two roots, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Note that there are ways of getting the all of the answers below without using either of these.

1. If everyone having $R \geq r$ buys a complete insurance plan, then the expected payout for the insurance organization is $g(r) = E(R|R \geq r)$. Show that $g(r) = r + \frac{1}{\lambda}$.
2. Show that if w is large enough, then $p(r) = g(r)$ for two different positive values of r.

For the rest of this problem, assume that w is large enough that there are two positive roots, $0 < r_1 < r_2$ for the equation $p(r) = g(r)$.

3. Show that a monopolist charging a price p for complete insurance that is either in the interval $[0, r_1)$ or in the interval (r_2, ∞) would make negative profits.
4. Write out the monopolist's profit maximization problem. [You need not solve it.]
5. Suppose now that complete insurance is offered through a cooperative pool: a price p is charged, anyone who wants can buy complete insurance at p, and the medical expenses are paid using the collected monies. There are two prices at which the cooperative breaks even. What are they? Which is Pareto superior?