Microeconomics Comprehensive Exam

August 2006

Instructions:

(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1. Hungry Henry Hippo and Ravenous Rachel Raccoon live in different parts of the forest, but they happen to have the exact same preferences over three goods: apples, bananas, and cherries. The prices facing Henry are $(\tilde{p}_A, \tilde{p}_B, \tilde{p}_C) = (1,1,1)$, and the prices facing Rachel are $(\hat{p}_A, \hat{p}_B, \hat{p}_C) = (3,1,2)$. You observe that Henry chooses consumption vector $(\tilde{a}, \tilde{b}, \tilde{c})$ = (6, 24, 6), and you observe that Rachel chooses $\hat{c} = 8$. Unfortunately, you were distracted by a lion, so you did not observe Rachel's choices $\hat{a}$ or $\hat{b}$. You also do not directly observe Henry's wealth $\tilde{w}$ or Rachel's wealth $\hat{w}$. You do know that their preferences are locally non-satiated.

a. Find the set of values of $(\hat{a}, \hat{b})$ that are consistent with the weak axiom of revealed preference. Draw a graph of that set.

b. Now suppose that the lion tells you that Henry and Rachel's preferences can be represented by a Cobb-Douglas utility function $u(a,b,c) = a^{\alpha} b^{\beta} c^{\gamma}$, where $\alpha, \beta, \gamma > 0$. Find $(\hat{a}, \hat{b})$. Does it lie in the set that you found in part a?

2. Two people may contribute money to a public project in periods $t = 1, \ldots, T \leq \infty$. At the end of period T, the project is "activated" and the players get utility as a function of the total contributions. Players also derive utility from money. If player i's initial wealth is $w_i > 0$, his sequence of contributions is $(z_i(1), z_i(2), \ldots, z_i(T))$, and the other player's sequence of contributions is $(z_j(1), z_j(2), \ldots, z_j(T))$, then player i's utility is

$$\ln\left(\sum_{t=1}^{T}\left(z_i(t)+z_j(t)\right)\right)+\alpha\left(w_i-\sum_{t=1}^{T}z_i(t)\right).$$

Note that both players have the same $\alpha > 0$. All contributions must be non-negative.

a. Suppose that $T = 1$. Find all the pure strategy Nash equilibria of this simultaneous-move game. Do not forget about budget constraints.

b. Now suppose that $T = 2$. Also suppose that player 1 can contribute only in period 1, and that player 2 can contribute only in period 2 (after observing player 1's contribution). Find all the pure strategy subgame perfect equilibria of this extensive-form game. Do not forget about budget constraints. If $w_1 = w_2$, which player gets higher utility in equilibrium?

c. Continue to suppose, as in part b, that $T = 2$, that player 1 can contribute only in period 1, and that player 2 can contribute only in period 2 (after observing player 1's contribution). However, now the players have different α's in their utility functions. In particular, $\alpha_2 > \alpha_1 > 0$. Find all the pure strategy subgame perfect equilibria of this new game. You may assume that both w_1 and w_2 are very large, so that budget constraints do not bind, but do not forget about non-negativity constraints.

3. Consider an economy with a single good and two individuals whose preferences can be represented by von Neumann-Morgenstern expected utility functions. Let u_1 and u_2 be their Bernoulli utility functions over quantities of this good. Suppose that the individuals are strictly risk averse, that is $u_i' > 0$ and $u_i'' < 0$. Suppose that there are two states of the world, s_1 and s_2, and the probability of state s_1 is π. The individuals have endowments that are state dependent. Specifically, agent 1 has initial endowment $e_1 = (1, 0)$ and agent 2 has initial endowment $e_2 = (0, 1)$. (A commodity bundle (x, y) denotes x units of the good in state s_1 and y units of the good in state s_2.) A feasible allocation in this economy is a pair of commodity bundles (x_1, y_1) and (x_2, y_2) ((x_i, y_i) represents the amount that agent i gets in each of the two states of the world) such that the sum of the amounts the two agents consume in each state of the world is not more than the amount available, i.e., the sum of the agents' endowments.

a. Show that for any Pareto optimal allocation neither agent bears any risk, that is, for $i = 1, 2$, $x_i = y_i$.
b. What are the agents' (common) marginal rates of substitution at an interior Pareto optimal allocation?
c. What is the competitive equilibrium price in this economy?
d. Suppose now that agent 1's initial endowment is $(2, 0)$, while agent 2's endowment is not changed, and both agents' utility functions are $u(m) = \ln m$. Describe the Pareto efficient allocations.

4. There are two dates: at date 1 there are three states; at date 0 there is trade in assets. There are two basic assets whose return vectors in current dollars are

$$d_1 = (64, 16, 4) \text{ and } d_2 = (0, 0, 1).$$

The market prices of these assets are $S_1 = 32$ and $S_2 = 1$ respectively. In the following you are asked to price by no arbitrage a variety of derived assets.

a. Suppose that one unit of a derived asset is described as "One unit of this asset confers the right to buy one unit of asset 1 at 75% of its spot value in period 1 (after the sate of the world occurs)." Write the return vector of this asset and price it.

b. The situation is the same as in (a) except that the asset is modified to read 'One unit of this asset confers the right to buy one unit of asset 1 at 75% of its spot value in period 1 (after the sate of the world occurs) provided the spot value is at least 10."

c. Suppose that the asset is as in (b) except that "at least 10" is replaced with "at least 19". Write down the return vector and argue that this asset cannot be priced by no arbitrage with the available primary assets.

d. How would the analysis in (c) differ if we had in addition a riskless asset $d_3 = (1, 1, 1)$ with the price $S_3 = 1$?

e. Suppose that now the asset is further complicated to read "One unit of this asset confers, at the choice of the holder, either \$1 in period 1 or the right to buy one unit of asset 1 at 75% of its spot value in period 1 (after the sate of the world occurs)." Write the return vector of this asset and price it.