Microeconomics Comprehensive Exam

August 2007

Instructions:

(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1.

a. John J. Johnson has preferences over bundles (x, y) in $\boldsymbol{R}_+^2$ given by the utility function $U(x, y) = \max\{x, y\}$.

i. Are John's preferences rational? Prove or disprove.
ii. Show that John's utility function is convex.
iii. Are John's preferences convex? Prove or disprove.

b. (This question is inspired by the alphabetical challenges of the Hungarian language.) Janos J. Janos's preferences over bundles in $\boldsymbol{R}_+^2$ are as follows:

$$(a,b) \succeq (x,y) \Leftrightarrow \begin{cases} \lfloor a \rfloor > \lfloor x \rfloor, \text{ OR} \\ \lfloor a \rfloor = \lfloor x \rfloor \text{ and } b > y, \text{ OR} \\ a = x \text{ and } b = y, \end{cases}$$

where $\lfloor z \rfloor$ denotes the largest integer no greater than z.

iv. Are Janos's preferences continuous? Prove or disprove.
v. Given wealth $w > 0$ and prices $P_1 > 0$ and $P_2 > 0$, derive Janos's Walrasian demand correspondence.

2. The country of Traubisoda is deciding how and whether to attack the country of Lekvaria. Traubisoda has three choices: it can 1) send a well-equipped invasion force, 2) send a poorly-equipped invasion force, or 3) stay home. A well-equipped invasion force is more expensive than a poorly-equipped one, but it is also more likely to win in battle. If Traubisoda chooses to invade, then Lekvaria has two choices: it can either fight or surrender. Crucially, when making its decision, Lekvaria cannot tell whether Traubisoda sent a well-equipped or poorly-equipped invasion force. That fact is common knowledge, as are the following payoffs:

If Traubisoda stays home, its payoff is 900 and Lekvaria's is 600.
If Traubisoda sends a well-equipped force and Lekvaria fights, payoffs are 400 and –600.
If Traubisoda sends a poorly-equipped force and Lekvaria fights, payoffs are –600 and 600.
If Traubisoda sends a well-equipped force and Lekvaria surrenders, payoffs are 1000 and 0.
If Traubisoda sends a poorly-equipped force and Lekvaria surrenders, payoffs are 1200 and 0.

a. Draw the extensive form of this game.
b. Find all the pure-strategy perfect Bayesian equilibria. (HINT: There is at least one.)
c. Find a perfect Bayesian equilibrium in which Traubisoda stays home with probability one and Lekvaria randomizes non-trivially.
d. Find a perfect Bayesian equilibrium in which both countries randomize non-trivially.

3. Consider the following version of the Lucas-tree economy. Time is discrete, and there are only two periods: $t = 0, 1$. At $t = 1$, there are two possible states of nature: $\omega = 1, 2$ with $\pi_\omega = \text{prob}(\omega) \in (0, 1)$, and $\pi_1 + \pi_2 = 1$. There are two trees - an apple (good 1) and orange (good 2) tree. Trees bring fruits (random dividends) at $t = 1$ so that the dividend from tree $j = 1, 2$ is $d^j = (d^{j1}, d^{j2})$, where $d^{j\omega} > 0$ are exogenous parameters. There are no dividends at $t = 0$. There are two consumers, who consume in period $t = 1$ only. Their utility functions are

$$u_i(x_{1i}, x_{2i}) = \sum_{\omega=1,2} \pi_\omega (a_{1i} \log x_{1i}(\omega) + a_{2i} \log x_{2i}(\omega)), \; i = 1, 2,$$

where $x_{j,i}(\omega)$ is consumption of good j by consumer i in state $\omega = 1, 2$, and $a_{ji} > 0$ ($j = 1, 2$), $a_{1i} + a_{2i} = 1$ ($i = 1, 2$). At $t = 0$, consumers are endowed with shares of the apple and orange tree: $\theta_i = (\theta_i^1, \theta_i^2)$ ($i = 1, 2$), where $\theta_i^j \geq 0$ and $\theta_1^j + \theta_2^j = 1$ ($j = 1, 2$). There is no other endowment. Shares of the trees are the only assets which are traded in the asset market.

a. Carefully define the Radner (financial) equilibrium (be specific about objects traded, if any, at each date).
b. Give a complete characterization of the Radner equilibrium.
c. Is the equilibrium allocation Pareto efficient in this model? Why or why not?

4. Consider a duopoly where the two firms in question produce an undifferentiated product. Demand for the product is given by $p = \max\{(A - X), 0\}$, where $A > 0$ is a constant, and X is the total output of the two firms. Each firm has a simple cost function: $C_i(x_i) = c_i x_i$, where the subscript refers to the firm, x_i is the firm's output, and c_i is a constant such that $0 < c_i < A$.

a. The firms compete Cournot style. Assume that $A + c_1 > 2c_2$ and $A + c_2 > 2c_1$. Find the equilibrium. Give the equilibrium price, levels of output and profits.

b. Now suppose that while the two firms are described as above, firm 2 is uncertain about the cost function of firm 1. Specifically, c_1 takes on one of a finite number of values, $c_{11}, c_{12}, \ldots, c_{1n}$ with probabilities $p_1, p_2, \ldots, p_n$ ($\sum_{j=1}^{n} p_j = 1$), and $0 < \min_j(c_{1j}) < A$. Firm 2 holds this probability assessment when it chooses its level of output x_2; firm 1 when it chooses x_1 knows the value of c_1 (and is allowed to condition its level of x_1 on c_1). Otherwise, the two engage in Cournot competition. Define an equilibrium as a quantity $\hat{x}_2$ output of firm 2 and, for each realization of firm 1's costs, a quantity $\hat{x}_1(c_1)$, such that firm 2, believing that firm 1's output will be given by the function $\hat{x}_1$, maximizes its *expected profits* with output level $\hat{x}_2$, and firm 1, believing that firm 2 will have output $\hat{x}_2$ and knowing that its own costs are given by c_1, will choose to produce $\hat{x}_1(c_1)$. Characterize the equilibrium.

c. Suppose the setting as in b., but prior to the selection of x_1 and x_2, firm 1 is able to provide irrefutable evidence to firm 2 concerning its cost structure if it chooses to do so. What do you think will happen in this case?