Microeconomics Comprehensive Exam

August 2008

Instructions:

(1) Please answer each of the four questions on separate pieces of paper.
(2) When you finish, please arrange your answers in the order in which they appear in the questions, i.e., 1 a., 1 b., etc.

1. Stripes the zebra gets utility from consuming Peruvian carrots (whose quantity is denoted by x). His preferences are quasilinear in wealth. (His wealth is $w > 0$.) The marginal cost of a Peruvian carrot depends on how good a job Stripes does in searching for the best deal, and in particular on the quantity y of gasoline that he uses in driving to different stores and on the quantity z of newspapers that he subscribes to (in order to clip coupons). The prices of gasoline and newspapers are p_y and p_z, respectively. His overall utility is given by

$$u(x) + w - c(y,z)x - p_y y - p_z z,$$

where $u(x)$ is strictly increasing and strictly concave; $c(y, z)$ is strictly decreasing in both its arguments and strictly convex, and also satisfies $c_{yz}(y,z) < 0$; and $u(x) - c(y,z)x$ is strictly concave.

a. Write down Stripes's utility maximization problem and the Kuhn-Tucker conditions that characterize the solutions $x(w, p_y, p_z)$, $y(w, p_y, p_z)$, and $z(w, p_y, p_z)$. Are the second-order conditions satisfied?

Now suppose that Stripes cannot adjust the quantity of newspapers; it is fixed at $\bar{z}$. Let $x(w, p_y, p_z, \bar{z})$ and $y(w, p_y, p_z, \bar{z})$ denote the optimal levels of Peruvian carrots and gasoline in this case.

b. Give conditions on the optimal levels of Peruvian carrots and gasoline under which small changes in the price of newspapers do not effect those optimal levels (that is, under which

$$\frac{\partial x(w, p_y, p_z, \bar{z})}{\partial p_z} = \frac{\partial y(w, p_y, p_z, \bar{z})}{\partial p_z} = 0),$$

and provide an intuitive explanation for your answer. You may assume that the optimal levels of Peruvian carrots and gasoline are positive.

c. Show carefully that $x(w, p_y, p_z, \bar{z}) > x(w, p_y, p_z)$ if and only if $\bar{z} > z(w, p_y, p_z)$. You may assume that the solutions to both utility maximization problems are interior, and that the budget constraint does not bind in either case. (HINT: one approach is to think about $\frac{\partial x(w, p_y, p_z, \bar{z})}{\partial \bar{z}}$.)

2. Two expected-utility-maximizing gangsters, Arnold and Betty, want to kill each other. If a gangster is killed, he or she gets utility 0. If neither gangster is killed, both get utility 1. If gangster i kills gangster j without being killed, gangster i gets utility 2.

Each gangster has a gun with one bullet. The gangsters are initially one kilometer apart, and are moving toward each other at a constant rate. Each gangster must choose the distance $x \in [0, 1]$ at which to fire. The probability of hitting (and killing) the other gangster from distance x is $1 - x^2$. A gangster cannot fire after being killed.

For parts a - c, suppose that the guns are very quiet, so that gangster i cannot distinguish between gangster j's having fired and missed and gangster j's not having fired yet.

a. Describe the strategy space for a gangster.
b. Is this situation a zero-sum game? Explain.
c. Suppose that Arnold randomizes over the distance x_A at which he shoots (conditional on not having been killed yet) according to the continuous and differentiable cumulative distribution F. What is Betty's expected utility if she decides to fire at distance $x_B \in [0, 1]$ (conditional on not having been killed yet)?

For parts d - f, suppose that the guns are very noisy, so that gangster i notices immediately if gangster j fires and misses.

d. Describe the strategy space for a gangster.
e. Does a symmetric pure-strategy subgame perfect Nash equilibrium exist? Explain.
f. Suppose that Arnold randomizes over the distance x_A at which he shoots (conditional on Betty's not having fired yet) according to the continuous and differentiable cumulative distribution F. What is Betty's expected utility if she decides to fire at distance $x_B \in [0, 1]$ (conditional on Arnold's not having fired yet)?

3. Consider a two-period economy under uncertainty. Suppose that 2 securities are traded at date 0: a riskless bond and a forward contract for delivery of a barrel of oil at date 1, with the delivery price K. The price of oil at date 1 is the only source of uncertainty in the economy. Assume that $r > 0$ is the interest rate. Notice that since it costs nothing to enter a forward contract, its price at date 0 is zero.

(a) Suppose that the price of oil at $t = 1$ can be either $p_1 = \$150$ or $p_2 = \$120$ per barrel. Find the set of delivery prices, K, which are arbitrage free. Find all risk neutral probability measures.

(b) Suppose that K does not satisfy conditions which you found in (a). Give an example of trading which makes money out of nothing.

(c) Suppose that the price of oil at $t = 1$ can be $p_1 = \$150$, $p_2 = \$120$ or $p_3 = \$130$ per barrel. Find the set of delivery prices, K, which are arbitrage free. Find all risk neutral probability measures.

4. Consider a two-period pure exchange economy. At each period of time, there are $2 \leq L < \infty$ consumption goods. There are $2 \leq I < \infty$ consumers whose preferences are given by

$$u_i(x_i^0, x_i^1) = \sum_{l=1}^{L} a^l \log x_i^{l0} + \delta \sum_{l=1}^{L} b^l \log x_i^{l1},$$

where x_i^0 and x_i^1 are consumption bundles at time 0 and 1 respectively, $a^l > 0$, $b^l > 0$, $\sum_{l=1}^{L} a^l = 1$, $\sum_{l=1}^{L} b^l = 1$, and $0 < \delta < 1$ is the discount factor.

Consumer i's income, w_i^0, w_i^1, is strictly positive at each date, and the income at time 0 can be spent on date 0 consumption or saved for the future. Let s_i denote consumer i's savings. Notice that consumers may also borrow at date 0; in this case, savings will be negative. Saving and borrowing are interest free.

(a) Let $p^0, p^1 \in \mathbb{R}_{++}^L$ be the prices at $t = 0$ and $t = 1$ respectively. Describe consumer i's behavior regarding choices of $(x_i^0,\, x_i^1,\, s_i)$ in terms of an optimization problem.

(b) Argue that the consumer's optimization problem has a unique solution and characterize the solution.

(c) Find consumer i's demand functions. Will consumers always save at date 0?