Microeconomics Comprehensive Exam

August 2005

Instructions:

(1) Please answer each of the four questions on separate pieces of paper.

(2) When finished, please arrange you answers alphabetically.

A. The Fabulous Laszlo feeds his trained tigers on a mixture of fish and milk, depending on the prices of each. Assume that the technology for "producing" tigers is convex, and that it exhibits constant returns to scale. According to the data submitted by his assistant tamers, when the price of fish was \$10 per kilo of fish and the price of milk was \$10 per gallon, they fed 150 kilos of fish and 50 gallons of milk to each tiger. When the price of fish was \$20 per kilo of fish and the price of milk was \$10 per gallon, they fed each tiger no fish and 300 gallons of milk. When the price of fish was \$10 per kilo of fish and the price of milk was \$20 per gallon, they fed each tiger 250 kilos of fish and no milk.

1. Is there any evidence above that indicates that the Fabulous Laszlo's assistant tamers have not been minimizing costs? Explain.
2. If we assume that the Fabulous Laszlo's assistant tamers were always minimizing costs, briefly explain whether or not it is possible to produce a tiger by feeding it 50 kilos of fish and 150 gallons of milk.

B. Consider the following two-good, two-consumer exchange economy: Consumer 1 has endowment $(x, y) = (2, 0)$, and Consumer 2 has endowment $(x, y) = (0, 2)$. Their respective utility functions are

$$u_1(x, y) = \min\{x, y\}, \text{ and } u_2(x, y) = \min\{x, y^2\}.$$

1. Carefully define Pareto optimality. Describe the Pareto optimal allocations for this economy. Draw them in an Edgeworth box. Show that they meet your definition of Pareto optimality.
2. Carefully define a Walrasian equilibrium for an exchange economy. Find the Walrasian equilibrium (or equilibria) of this economy. Show that it (or they) meet your definition of a Walrasian equilibrium.

C. A shipping company can spend time, money, and other forms of effort making a ship seaworthy, making sure it follows safer routes, and training the crew in emergency procedures, and so forth. We, rather crudely, gather up all of these forms of efforts into a single scalar variable, e, that costs the company \$$c(e)$ where $c(\cdot)$ is a non-negative, increasing, convex function.

We set $S = 1$ if the ship sinks and $S = 0$ if it arrives in port with its cargo. As a function of e, the probability of safe arrival in port, $p(e) = P(S = 0|e)$ is a non-negative, increasing, concave function. If $S = 0$, the revenues that accrue to the company are \$$R$, if $S = 1$, the company suffers losses, \$$L$. Assume that $L < 0 < R$.

1. Assuming the owner of the shipping company is risk neutral, give a condition(s) under which the optimal effort under risk neutrality, e^*_{RN}, is strictly positive. Assuming this condition(s), explain how e^*_{RN} depends on R and L.
2. Assuming that the owner of the shipping company is an expected utility maximizer with a strictly increasing, concave utility function $u(\cdot)$, give a condition(s) under which the optimal effort under risk aversion, e^*_{RA}, is strictly positive. Assuming this condition(s), explain how e^*_{RA} depends on R and L.
3. Suppose that a risk neutral, monopoly insurance company can accurately monitor e and that the owner of the shipping company is a risk averse expected utility maximizer. Describe the optimal contract for the insurance company to offer to the owner of the shipping company.

D. After seeing a compensation package (s, ξ, η) offered by the principal (and described in more detail below), an agent takes an action $\mathbf{a} \in \mathbb{R}^n$ which is not observable by anyone else. The cost of the action is $C(\mathbf{a}) = \frac{1}{2}\mathbf{a}'\mathbf{a}$. The agent's utility outside of his/her relationship with the principal is u_0, the principal's outside profit is normalized to 0.

The agent's action gives rise to two Bernoulli (i.e. taking only the values 0 and 1) random variables, Y and X. $Y = 1$ corresponds to success for the principal, $Y = 0$ corresponds to failure. The value of Y is observed by the principal and the agent, but only by the principal and the agent. This means that no legally enforceable contract can be written on Y. If success occurs, the principal's (normalized) profit is 1.

$X = 1$ corresponds to a publicly verifiable performance measure being "success," $X = 0$ corresponds to it being "failure." Being publicly verifiable means that legally enforceable contracts can be written on it. X and Y are stochastically independent, and their distributions are given by

$$P(Y = 1|\mathbf{a}) = \mathbf{y}'\mathbf{a}, \quad \text{and} \quad P(X = 1|\mathbf{a}) = \mathbf{x}'\mathbf{a},$$

where $\mathbf{x}$ and $\mathbf{y}$ are normalized so that $\mathbf{x}'\mathbf{x} = \mathbf{y}'\mathbf{y} = 1$, and by assumption, the agent is restricted to a set $A \subset \mathbb{R}^n$ such that for all $\mathbf{a} \in A$, $\mathbf{y}'\mathbf{a}, \mathbf{x}'\mathbf{a} \in (0, 1)$.

The agent's compensation package (s, ξ, η) consists of: (1) a salary s which is not conditional on X, Y, nor anything else in the model, (2) an "objective" bonus, ξ, to be paid if $X = 1$, and (3) a "subjective" bonus, η, to be paid if $Y = 1$. The salary s and the objective bonus ξ are both legally enforceable, the subjective bonus is not. If the subjective bonus is to be paid, it must be sequentially rational for the principal to pay it.

To summarize, if the compensation package is (s, ξ, η) and action $\mathbf{a}$ is taken, the agent's expected utility is $u_A = s + \eta\mathbf{y}'\mathbf{a} + \xi\mathbf{x}'\mathbf{a} - \frac{1}{2}\mathbf{a}'\mathbf{a}$, and the principle's expected profits are $\mathbf{y}'\mathbf{a} - (s + \eta\mathbf{y}'\mathbf{a} + \xi\mathbf{x}'\mathbf{a})$.

Suppose throughout that the appropriate first order conditions determine the agent's actions.

1. As a function of u_0 and (s, ξ, η), give the agent's best responses and give the agent's indirect (or maximized) utility function. Verify that the second order conditions hold.
2. As a function of u_0, find the (perfect Bayesian) equilibrium contract. [Hint: Will a strictly positive η ever be paid in equilibrium?]
3. As a function of u_0, find the first best outcome, and show that it would be available if Y were publicly observable.
4. Show that the social surplus generated by the equilibrium contract is a function of $(\mathbf{x}'\mathbf{y})^2$, and that the equilibrium contract and the first best contract coincide when $(\mathbf{x}'\mathbf{y})^2 = 1$.