Microeconomics Comprehensive Exam

June 2004

Instructions:

(1) The 4 problems on the exam are equally weighted. Use this information to help you maximize your score.

(2) Please answer each question on separate pieces of paper.

(3) When finished, please arrange you answers alphabetically.

A: A German producer has made an automated doughnut filler (ADF) worth $V > 0$ if sold at home and worth $V+s$ if sold abroad, $s > 0$. To ship the ADF abroad, the producer must use a shipper. If the shipper ships the ADF, they choose the probability, q, that the ADF arrives undamaged. This costs the shipper $c(q) = 100q^3$, $q \in [0,1]$. Any damage to the ADF is total (e.g. it rusts in the sea air and only produces horrible doughnuts) and reduces its value to 0. Both the producer and the shipper are risk neutral expected profit maximizers.

For parts 1-5, assume that the values of V and s are common knowledge. Further assume that the shipper makes a legally binding take-it-or-leave-it offer (p, r) to the producer. The producer accepts or rejects the offer, selling at home if the offer is rejected, paying p to the shipper if the offer is accepted, and being reimbursed r if the ADF is damaged in transit. If the offer is accepted, the shipper chooses q.

1. If an offer (p, r) is accepted, find the shipper's (subgame perfect) choice of q.
2. Find the efficient level of q.
3. Find the producers expected profits of accepting an offer (p, r).
4. If (p, r) is accepted by the producer, find the shipper's expected profits.
5. Find the subgame perfect equilibrium of this game.

For the rest of this problem, suppose that s is a random variable that takes on one of two possible values, 72 or 143, each with equal probability. Further, assume that the producer knows the realization of s, but that the shipper does not. The shipper now offers a take-it-or-leave-it menu of shipping options, (p_H, r_H) and (p_L, r_L), to the producer. The producer either rejects both menu choices and sells at home, or accepts one of the offers and ships. If an offer is accepted, the shipper chooses q.

6. Find, or explain how to find, a perfect Bayesian equilibrium of this modified game.

B: Consider a two-period ($t = 0, 1$) exchange economy with uncertainty in the second period represented by two states, $\omega = 1, 2$. As usual, we identify $\omega = 0$ with the initial date.There is a single good available for trade and consumption in each spot market (thus, the consumption space is identified with $\mathbb{R}^3_+$). There are I consumers characterized by their preferences

$$u_i : \mathbb{R}^3_{++} \to \mathbb{R} \quad (i = 1, \ldots, I),$$

and endowments $e_i \in \mathbb{R}^3_{++} \quad (i = 1, \ldots, I)$. Utility functions u_i are twice continuously differentiable, increasing, strictly quasi-concave, and the upper-contour sets $U_i(e_i) = \{x \in \mathbb{R}^3_{++} | \, u_i(x) \geq u_i(e_i)\}$ are closed in $\mathbb{R}^3_{++}$. There is a single firm that produces the consumption good in period 1. The production is described by exogenous output δ_ω in each future state $\omega = 1, 2$. The firm issues stock (or equity) and bond (or debt). Assume that the total amount of debt is d: $0 < \delta_1 < d < \delta_2$. Observe that the firm cannot return the debt if state 1 occurs, and can return the debt if state 2 occurs, hence the debt is defaultable. If there is no default, debt holders receive d, and stock holders receive the remaining part of the firm's output. In the case of default, debt holders grab the whole output, and the stock holders get nothing. All the payoffs are divided among the asset holders proportionally to their share holdings.

1. Is the asset market complete? Why or why not?

2. Define the financial equilibrium for this economy.

3. Suppose that there exists a financial equilibrium for this economy. Write the system of equations that characterize equilibria (you do not need to solve these equations!!)

4. Define the value of the firm (at date 0) as the value (price) of the stock plus the value (price) of the debt. Show that for this model, the value of the firm is independent of d, provided $d \in (\delta_1, \delta_2)$.

C: Consider a two-firm Cournot model with constant returns to scale. Suppose that the firms have different marginal costs of production: $c_1 > c_2 > 0$. Assume that the inverse demand curve is $p(q) = a - bq$, where $a > c_1$, and $b > 0$.

1. Derive the Nash equilibrium of this model. Will both firms always produce? Why or why not?

2. How do the equilibrium output, price and profits vary when c_1 changes?

3. Suppose now that there are $J \geq 2$ firms in the industry. Let $a > c_1 > \ldots > c_J$. Specify the conditions under which all the firms will produce. Find the Nash equilibrium in which all the firms produce. Let q^* be the equilibrium output of the industry, and q_j^* be the equilibrium output of firm j. Introduce $H = \sum_{j=1}^{J}(q_j^*/q^*)^2$, which is the *Herfindahl index of concentration.* Let ϵ be the elasticity of the market demand curve at the equilibrium price. Show that the ratio of the industry profit divided by the industry revenue in the Nash equilibrium is given by H/ϵ.

D: This question has three parts.

1. The production set of a price-taking firm exhibits additivity and non-increasing returns to scale. Show that the maximized value of the firms profits, if it exists, is weakly negative.

2. Nobel Savage's preferences over two goods can represented by the utility function $u(x, y) = xy + x$, where x denotes the amount of good 1 consumed and y denotes the amount of good 2. Nobel has wealth $w > 0$ and faces price $p_x > 0$ and $p_y > 0$. Being careful of boundaries, calculate Nobel's Slutsky matrix. Is it negative semidefinite? Explain.

3. State the Independence Axiom for preferences over lotteries. Explain why the Independence Axiom is a reasonable assumption for preferences over lotteries, but not for preferences (under certainty) over bundles of goods.