Microeconomics Comprehensive Exam

June 2006

Instructions:

(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1. N people may contribute to a public project in periods $t = 1, \ldots, T \le \infty$. In the first period in which the total of their present and past contributions is $K > 0$ or more dollars, the project is "complete" and each person receives a dollar benefit of V. They have the same discount factor $\delta \in (0, 1)$. If player i's sequence of contributions is $(z_i(1), z_i(2), \ldots, z_i(T))$, and the project is completed in period $T^* \le T$, his utility is

$$U_i = \delta^{T^*-1}V - \sum_{t=1}^{T} \delta^{t-1} z_i(t).$$

If the project is never completed,

$$U_i = -\sum_{t=1}^{T} \delta^{t-1} z_i(t).$$

Assume that contributions must be nonnegative, and that $K/N < V < K$. Past contributions are publicly observed, but in player i does not observe the period-t contributions of the other players until period $t + 1$.

a. Suppose that $N = 2$. Describe the pure-strategy Nash equilibria of the static, simultaneous-move ($T = 1$) game.

For the rest of the questions, suppose that $N = T$, and that player t can contribute only in period t. At the beginning of period t, let

$$X(t) = K - \sum_{s=1}^{t-1} z_s(s)$$

denote the amount that must be contributed to complete the project. (Let $X(1) = K$.)

b. There is an amount R for which it is a (conditionally) dominant strategy for player $t < T$ to complete the project in period t if $X(t) < R$. Find R. (Note that this question does not ask about equilibrium strategies.)

c. Suppose that $N = T = 2$, and that $\delta = 1$. Find the unique subgame perfect equilibrium.

d. Again supposing that $N = T = 2$ and that $\delta = 1$, find a Nash equilibrium in which the project is not completed.

2. Greta Gorilla, an expected-utility maximizer, lives next to a river. Every day during the banana season, which lasts for T days, a banana comes floating down the river. Banana t's quality q_t is drawn independently from the continuous distribution F, with support [0, 1].

Greta's utility is given by the sum of the quality levels of the bananas that she consumes. However, she is finicky. In particular, she cannot consume a banana unless its quality level is higher than the quality level of every banana that she has consumed previously. That is, she can consume banana t only if $q_t > x_t$, where

$$x_t = \max\{q_s : s = 1,\dots,t-1\}.$$

a. Suppose that $T = 2$, and that the realized quality of the banana on the first day is q_1. What is Greta's expected (total) utility if she eats banana 1? What is her expected utility if she does not eat it?

b. Again suppose that $T = 2$. Show that in order to maximize her expected utility, Greta should eat banana 1 regardless of its quality.

c. Now suppose that $T = 3$, and that the realized quality of the banana on the first day is q_1. What is Greta's expected utility if she eats banana 1? What is her expected utility if she does not eat it? (You may assume that she will eat banana 2, if possible, regardless of its quality.)

3. Consider a simple duopoly problem, with two firms that are identical except in one important sense: one is run by the owner and the other – by a manager working under a contract specified below. The two firms produce an undifferentiated good, the market demand for which is given by the inverse demand curve $p = \max\{A - q, 0\}$, where p is the price, q is the total quantity produced, and $A > 0$ is a constant. The firms compete as Cournot competitors; each names a quantity supplied simultaneously and independently of the other, and the market price is set so that that the demand equals the sum of the two supplies. Both firms have zero marginal costs of production.

One firm is owner managed. The quantity decision for this firm is made by the owner, who retains any profits. This owner is risk neutral.

The second firm is not managed by its owner. The owner of the second firm has hired a manager and has given this manager a contract wherein the compensation paid to the manager is a linear function of the profits the firm shows, π_2, and the quantity the firm sells, q_2. Specifically, this contract calls for the manager to be paid $\alpha\pi_2 + \beta q_2$, where α and β are parameters chosen by the owner of the firm. The owner retains any profits made by the firm net of pay to the manager. Note well that the manager makes the quantity decision to maximize her compensation and not the net profits to the owner. The owner-manager of the first firm knows the contract terms under which the manager of the second firm operates.

a. What, in this case (for given α and β), is the Cournot equilibrium? What (as a function of α and β) are the profits of the first firm, the manager's compensation, and the net profits to the owner of the second firm?

b. The owner of the second firm is interested in designing the "optimal" contract for her manager. The contract must, in equilibrium, provide the manager with a reservation level of income, w. Other than that, the owner can choose any *linear* contract she wishes; α and β are chosen to maximize her net profits. The owner-manager of the first firm and the manager of the second firm will behave in Cournot fashion after this contract is chosen. What is the optimal contract? [*Hint: It is convenient to re-write all equilibrium variables you found in part a. as functions of* α *and* $\gamma := \beta/\alpha$, *and to make the owner choose* α *and* γ *instead of* α *and* β.]

4. In a certain economy, there are two commodities, education, E, and food, F, produced by using labor, L, and land, T, according to the production functions

$$E = (\min\{L, T\})^2 \quad \text{and} \quad F = (LT)^{1/2}.$$

There is a single consumer with the utility function

$$u(E, F) = (EF)^{1/2},$$

and the endowment (e_L, e_T). To ease the calculations, take $e_L = e_T = 1$.

a. Find the optimal allocation of the endowments to their productive uses.

b. Suppose that the consumer can choose any amount of education and food desired at the going prices (which she takes as given). Both the food and education industries take output and input prices as given. The food industry maximizes its profit. The education industry minimizes its cost and produces the level of education so as to meet the consumer's demand. Let food be the numeraire good. Find the equilibrium prices and allocations in this economy. Compare the equilibrium allocation and the optimal allocation in part a.

c. Recognizing that the production of education entails increasing returns to scale, the government of this economy decides to control the education industry by setting the price for education and finance its operation with a lump-sum tax on the consumer. Suppose that the government sets the price of education to equal the marginal cost of education at the optimal level (derived in part a). What is the lump-sum tax necessary to decentralize the optimal allocation in part a?